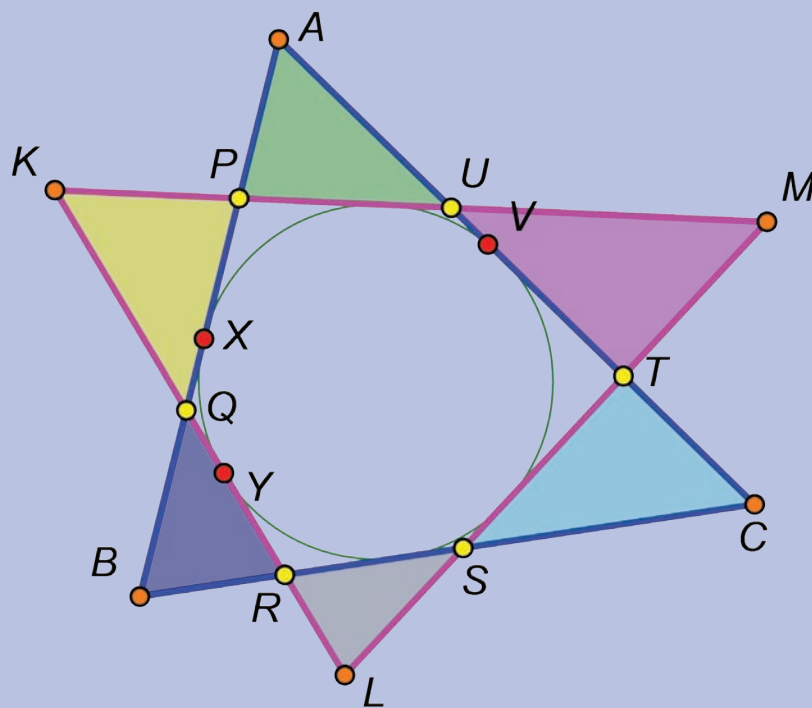


# Learning & Teaching Mathematics

A Journal of



# A Quick Tool for Tracking Procedural Fluency Progress in Grade 2, 3 and 4 Learners

**Debbie Stott**

**South African Numeracy Chair, Rhodes University, Grahamstown**

**d.stott@ru.ac.za**

## INTRODUCTION AND CONTEXT

The South African Numeracy Chair (SANC) project works with fifteen schools in the broader Grahamstown area in the Eastern Cape. Among other things, the SANC project works toward improving numeracy proficiency among learners, basing its notion of numeracy proficiency on Kilpatrick, Swafford & Findell's (2001) definition of mathematical proficiency. This definition comprises five intertwined and interrelated strands: *Conceptual Understanding*, *Procedural Fluency*, *Strategic Competence*, *Adaptive Reasoning* and *Productive Disposition*. As part of the SANC project we run a number of regular after-school maths clubs for learners<sup>9</sup>, and in the club activities we strive to develop numeracy proficiency in the learner participants in each of these five strands.

## PROCEDURAL FLUENCY

This article specifically focuses on developing and tracking learner progress in one of the five strands, namely *Procedural Fluency*. Kilpatrick et al. (2001) describe procedural fluency as “skill in carrying out procedures flexibly, accurately, efficiently and appropriately” (p. 116). Although this article focuses only on the procedural fluency strand, this strand should not be seen in isolation. Rather, the five strands should be seen to complement each other, providing an interwoven conceptualization of numeracy proficiency. This is particularly so in the case of procedural fluency and conceptual understanding. Russell (2000) explains that there is a need to balance both skills and understanding and to make sure the learners develop both procedural competence and understanding where one should strive for a “connection between conceptual understanding and computational proficiency” (NCTM, as cited in Russell, 2000, p. 156). Baroody, Feil & Johnson (2007) point out that one of the reasons that Kilpatrick et al. (2001) recommend that the strands of mathematical proficiency be taught in an interwoven manner is because “linking procedural to conceptual knowledge can make learning facts and procedures easier, provide computational shortcuts, ensure fewer errors, and reduce forgetting (i.e., promote efficiency)” (p. 127).

## TRACKING PROGRESS IN PROCEDURAL FLUENCY

There are many ways of developing procedural fluency in young learners, and in the after-school maths clubs we focus on developing procedural fluency both explicitly and implicitly through carefully crafted learner activities. Askew (2010) believes that developing procedural fluency is “best done little and often rather than in less frequent, longer blocks of time” (p. 27). He argues that practice in procedural fluency needs to (a) be simple to set up and carry out, (b) be done little and often, (c) keep learners focused on the mathematics, and (d) help each learner see their own progress (p. 28).

This article presents a series of activities that I developed in collaboration with Mellony Graven. These activities are used in the after-school maths clubs to monitor learners' procedural fluency progress, specifically with respect to speed and accuracy. The activities are quick and easy to administer and mark, and they allow one to see how quickly learners are answering within the allocated time for each activity as

---

<sup>9</sup> For additional information about these clubs see Graven and Stott (2012) as well as Graven (2011).

well as how accurately they are answering each activity within that time. The changes in these scores over time provide a picture of each learner's progress.

### THE ACTIVITIES

There are seven different activities to choose from, and learners are given a specified amount of time to attempt each activity. Table 1 describes and provides a sample of each activity along with details of the time and mark allocation for each activity. The activities are all freely available from the South African Numeracy Chair Project website<sup>10</sup>.

**TABLE 1:** Description, details and sample of each of the seven activities.

| Activity type          | Description & sample  | Time allocation | Total marks <sup>11</sup> |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
|------------------------|---|-----------------|---------------------------|-----|---|----|---|-----------|-----|----------|----|-----|--|-------|-----|----------|----|---|--|---|--|--|---|--|--|--|----|
| Add and subtract to 10 | <p>Numbers range up to 10.</p> <p>Use the numbers in the shaded header rows and shaded columns to add/subtract e.g. <math>2 + 3 = 5</math> and <math>10 - 2 = 8</math></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>Add</td><td>2</td><td>4</td></tr> <tr><td>+</td><td>5</td><td></td></tr> <tr><td>3</td><td></td><td></td></tr> <tr><td>5</td><td></td><td></td></tr> </table> <table border="1" style="display: inline-table;"> <tr><td>Minus</td><td>10</td><td>8</td></tr> <tr><td>-</td><td>8</td><td></td></tr> <tr><td>2</td><td></td><td></td></tr> <tr><td>4</td><td></td><td></td></tr> </table> | Add             | 2                         | 4   | + | 5  |   | 3         |     |          | 5  |     |  | Minus | 10  | 8        | -  | 8 |  | 2 |  |  | 4 |  |  | <p>1 minute for add</p> <p>1 minute for subtract</p> | 48 |
| Add                    | 2   | 4               |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| +                      | 5   |                 |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| 3                      |   |                 |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| 5                      |   |                 |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| Minus                  | 10  | 8               |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| -                      | 8   |                 |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| 2                      |   |                 |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| 4                      |   |                 |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| Doubling               | <p>Double the shaded number, e.g. double 4 is 8, double 2 is 4</p> <table border="1" style="display: inline-table;"> <tr><td>4</td><td></td><td>2</td><td>4</td></tr> <tr><td>11</td><td></td><td>12</td><td></td></tr> </table>  | 4               |                           | 2   | 4 | 11 |   | 12        |     | 1 minute | 17 |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| 4                      |   | 2               | 4                         |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| 11                     |   | 12              |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| Halving                | <p>Halve the shaded number, e.g. half 4 is 2, half 2 is 1</p> <table border="1" style="display: inline-table;"> <tr><td>4</td><td></td><td>2</td><td>1</td></tr> <tr><td>14</td><td></td><td>12</td><td></td></tr> </table>   | 4               |                           | 2   | 1 | 14 |   | 12        |     | 1 minute | 17 |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| 4                      |   | 2               | 1                         |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| 14                     |   | 12              |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| Add/subtract 10        | <p>Add 10 to or subtract 10 from the shaded number, e.g. <math>5 + 10 = 15</math>, <math>12 - 10 = 2</math></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>Add 10</td><td>5</td><td>15</td></tr> <tr><td>2</td><td></td><td>9</td></tr> </table> <table border="1" style="display: inline-table;"> <tr><td>Minus 10</td><td>10</td><td>12</td><td>2</td></tr> <tr><td>19</td><td></td><td>25</td><td>65</td></tr> </table>   | Add 10          | 5                         | 15  | 2 |    | 9 | Minus 10  | 10  | 12       | 2  | 19  |  | 25    | 65  | 1 minute | 20 |   |  |   |  |  |   |  |  |  |    |
| Add 10                 | 5   | 15              |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| 2                      |   | 9               |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| Minus 10               | 10  | 12              | 2                         |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| 19                     |   | 25              | 65                        |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| Add/subtract 100       | <p>Add 100 to or subtract 100 from the shaded number, e.g. <math>5 + 100 = 105</math>, <math>102 - 100 = 2</math></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>Add 100</td><td>5</td><td>105</td></tr> <tr><td>3</td><td></td><td>9</td></tr> </table> <table border="1" style="display: inline-table;"> <tr><td>Minus 100</td><td>100</td><td>102</td><td>2</td></tr> <tr><td>109</td><td></td><td>125</td><td>165</td></tr> </table>   | Add 100         | 5                         | 105 | 3 |    | 9 | Minus 100 | 100 | 102      | 2  | 109 |  | 125   | 165 | 1 minute | 20 |   |  |   |  |  |   |  |  |  |    |
| Add 100                | 5   | 105             |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| 3                      |   | 9               |                           |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| Minus 100              | 100   | 102             | 2                         |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |
| 109                    |   | 125             | 165                       |     |   |    |   |           |     |          |    |     |  |       |     |          |    |   |  |   |  |  |   |  |  |  |    |

<sup>10</sup> <http://www.ru.ac.za/sanc/numeracyresources/miscresources>

<sup>11</sup> Note: Each answer is worth 1 mark. This column therefore also represents the number of required responses.

|                        |   |            |    |   |   |   |   |  |  |   |  |  |  |            |    |    |    |   |   |  |  |    |  |  |  |   |    |
|------------------------|---|------------|----|---|---|---|---|--|--|---|--|--|--|------------|----|----|----|---|---|--|--|----|--|--|--|---|----|
| Add and subtract to 20 | <p>Numbers range up to 20</p> <p>Use the numbers in the shaded header rows and columns to add or subtract e.g. <math>3 + 3 = 6</math> and <math>10 - 3 = 7</math></p> <div style="display: flex; justify-content: space-around; align-items: center;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td style="background-color: #d3d3d3;">Add<br/>+</td><td style="background-color: #d3d3d3;">3</td><td style="background-color: #d3d3d3;">2</td><td style="background-color: #d3d3d3;">7</td></tr> <tr><td style="background-color: #d3d3d3;">3</td><td>6</td><td></td><td></td></tr> <tr><td style="background-color: #d3d3d3;">0</td><td></td><td></td><td></td></tr> </table> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td style="background-color: #d3d3d3;">Minus<br/>-</td><td style="background-color: #d3d3d3;">10</td><td style="background-color: #d3d3d3;">20</td><td style="background-color: #d3d3d3;">11</td></tr> <tr><td style="background-color: #d3d3d3;">3</td><td>7</td><td></td><td></td></tr> <tr><td style="background-color: #d3d3d3;">10</td><td></td><td></td><td></td></tr> </table> </div> | Add<br>+   | 3  | 2 | 7 | 3 | 6 |  |  | 0 |  |  |  | Minus<br>- | 10 | 20 | 11 | 3 | 7 |  |  | 10 |  |  |  | 2 minutes for add<br><br>2 minutes for subtract | 48 |
| Add<br>+               | 3   | 2          | 7  |   |   |   |   |  |  |   |  |  |  |            |    |    |    |   |   |  |  |    |  |  |  |   |    |
| 3                      | 6   |            |    |   |   |   |   |  |  |   |  |  |  |            |    |    |    |   |   |  |  |    |  |  |  |   |    |
| 0                      |   |            |    |   |   |   |   |  |  |   |  |  |  |            |    |    |    |   |   |  |  |    |  |  |  |   |    |
| Minus<br>-             | 10  | 20         | 11 |   |   |   |   |  |  |   |  |  |  |            |    |    |    |   |   |  |  |    |  |  |  |   |    |
| 3                      | 7   |            |    |   |   |   |   |  |  |   |  |  |  |            |    |    |    |   |   |  |  |    |  |  |  |   |    |
| 10                     |   |            |    |   |   |   |   |  |  |   |  |  |  |            |    |    |    |   |   |  |  |    |  |  |  |   |    |
| Add/subtract tasks     | <p>Mixture of horizontal addition and subtraction sums from 1 digit up to 3 digits</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>1.1 <math>3 + 4 =</math> <input type="text"/></p> <hr style="width: 100%;"/> <p>1.3 <math>23 + 18 =</math> <input type="text"/></p> </div> <div style="text-align: center;"> <p>2.1 <math>8 - 2 =</math> <input type="text"/></p> <hr style="width: 100%;"/> <p>2.3 <math>23 - 18 =</math> <input type="text"/></p> </div> </div>  | 15 minutes | 10 |   |   |   |   |  |  |   |  |  |  |            |    |    |    |   |   |  |  |    |  |  |  |   |    |

These activities are administered at least once a term in each of the clubs that we work with, and the results are captured in a spreadsheet. With repeated administration of the activities over time, this allows one to track each learner's progress.

### SCORING THE ACTIVITIES AND EVALUATING PROGRESS

The results from the seven activities allow one to see how quickly learners are working as well as how accurately they are working within the time limit for each activity. For each activity three different scores are calculated:

- **Actual mark:** The number of items a learner correctly answered.
- **Completion %:** The number of items answered by a learner (either correctly or incorrectly) as a percentage of the total number of items in the activity.
- **Accuracy %:** The number of items a learner correctly answered as a percentage of the number of items answered.

The *completion* and *accuracy* percentages allow one to track two things: (a) the *speed* at which learners are answering in the given time period (a *completion rate*), and (b) the *accuracy* of their work in that time (an *accuracy rate*). While these two rates provide useful information about each learner, they should of course not be interpreted in isolation. A learner who provides answers for every item in a particular activity, but who guesses these answers, would score 100% for completion but very low for accuracy. Similarly, a learner who only answers one item in a particular activity, but who answers that item correctly, would score very low on the completion rating while scoring 100% for accuracy. It is important then to review both the completion and the accuracy scores together in order to get a meaningful picture of a learner's progress over time.

By way of example let us consider the doubling activity administered to a hypothetical learner in two different terms. The scenario is shown in Table 2.

**TABLE 2:** Doubling activity administered to a hypothetical learner in two different terms.

|               | <b>Total marks for activity</b> | <b>Actual mark</b>                                | <b>Completion %</b>                              | <b>Accuracy %</b>   |
|---------------|---------------------------------|---|--|---------------------|
| <b>Term 1</b> | 17                              | 7 items correctly answered<br>7 out of 17 = 41%   | 10 items answered in total<br>10 out of 17 = 59% | 7 out of 10<br>70%  |
| <b>Term 2</b> | 17                              | 12 items correctly answered<br>12 out of 17 = 71% | 15 items answered in total<br>15 out of 17 = 88% | 12 out of 15<br>80% |

The doubling activity comprises 17 marks, one mark per item/response. In the first term the learner answered 7 of the 17 items correctly, giving an actual mark of 41%. However, on closer inspection we see that the learner only answered 10 of the 17 items (59% completion) and that of these 10 answers, 7 were correct (70% accuracy). This more nuanced analysis, i.e. the completion and accuracy scores, gives a significantly different perspective to the 41% overall score.

In the second term, when the doubling activity was administered again, the learner answered 12 of the 17 items correctly, giving an actual mark of 71%. With respect to completion and accuracy we see that the learner answered 15 of the 17 items (88% completion) and that of these 15 answers, 12 were correct (80% accuracy). Comparing the second term to the first we can now see that the actual score for the doubling activity increased from 41% to 71%, the completion score increased from 59% to 88%, and the accuracy score increased from 70% to 80%. This provides a rich picture of the learner's progress.

#### **SUGGESTIONS FOR USING THE ACTIVITIES**

The seven different activities described in Table 1 are available from the SANC website and can be downloaded from <http://www.ru.ac.za/sanc/numeracyresources/misresources>. The activities could perhaps effectively be used in Grade 2, 3 and 4 classrooms and could be administered at the beginning of the 2<sup>nd</sup> and 4<sup>th</sup> terms. Learners could be involved in marking the activities if required, as well as logging their own scores on a special chart or sheet in order to see their own progress.

#### **ACKNOWLEDGEMENT**

The work of the South African Numeracy Chair Project, Rhodes University is supported by the FirstRand Foundation (with the RMB), Anglo American Chairman's fund, the Department of Science and Technology and the National Research Foundation.

#### **REFERENCES**

- Askew, M. (2010). Do it again. *Teach Primary*, 27–28. Retrieved February 11, 2013, from [http://www.teachprimary.com/resource\\_uploads/do-it-again.pdf](http://www.teachprimary.com/resource_uploads/do-it-again.pdf)
- Baroody, A. J., Feil, Y., & Johnson, A. R. (2007). An alternative reconceptualization of procedural and conceptual knowledge. *Journal for Research in Mathematics Education*, 38(2), 115–131.
- Graven, M. (2011). Creating new mathematical stories: Exploring opportunities within maths clubs. In H. Venkat & A. A. Essien (Eds.), *Proceedings of 17th National Congress of the Association for Mathematical Education of South Africa (AMESA)* (pp. 161–170). Johannesburg: University of the Witwatersrand.
- Graven, M., & Stott, D. (2012). Design issues for mathematics clubs for early grade learners. In D. Nampota & M. Kazima (Eds.), *Proceedings of the 20th Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education* (pp. 94–105). Lilongwe: University of Malawi.
- Kilpatrick, J., Swafford, J., & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics*. Mathematics Learning Study Committee. Washington, DC: National Academy Press.
- Russell, S. J. (2000). Developing computational fluency with whole numbers. *Teaching Children Mathematics*, 7(3), 154–158.