"Pushing for Progression" in number sense and fluency
Maths Club Development Programme

Multiplication and Division

Session Three
Teacher Handbook

Name

School

District

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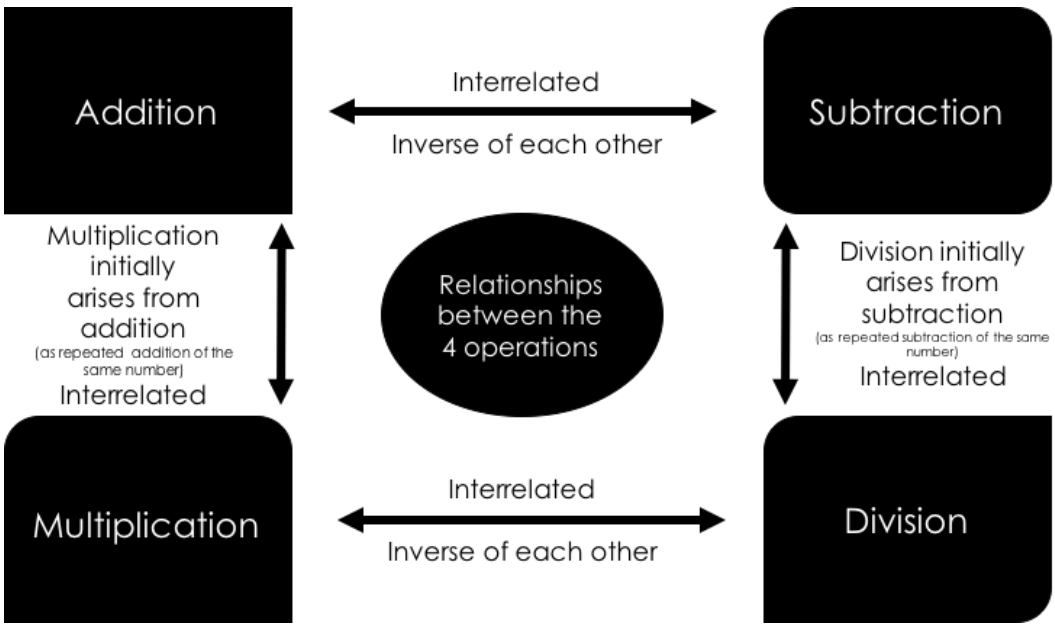
Big ideas in multiplication and division

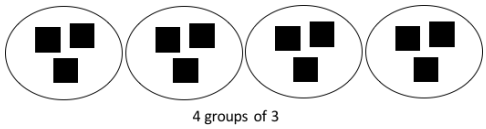
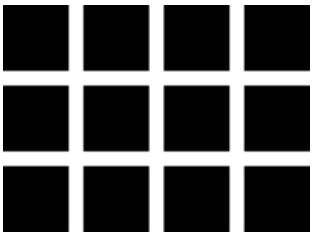
"Multiplication and division are critical foundations for more difficult concepts in number, algebra, measurement, and statistics. Thinking multiplicatively involves many different mathematical ideas as well as constructing and manipulating factors (the numbers that are multiplied) in response to a variety of contexts" (Numeracy Professional Development Projects, 2007, p. 3)

In this series of club sessions, we focus on some of the big ideas in multiplication and division.

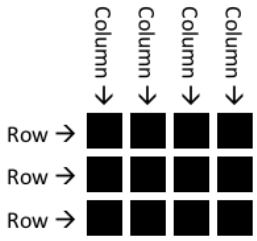
- Developing the ability to "unitise".
  "coming to regard numbers as units, that is, as single whole objects that can be counted. For example, when a student counts how many 3s in 12 as one 3, two 3s, three 3s, four 3s; the 3s are regarded as units. Unitizing can involve, for example, reasoning that if there are four 3s in 12, then there are eight 3s in 24, which involves counting units of units. Tasks to elicit unitizing include counting rows in arrays, and drawing attention to the unitary aspect alongside the composite aspect of numbers" (Ellemor-Collins & Wright, 2011, p. 5).
- Understanding the commutative, associative and distributive properties in multiplication
- Recognising multiplication and division as inverse operations and using this to solve problems.
- Understanding the place value patterns that occur when multiplying and dividing by ten or 100
- Looking for and recognising patterns and connections within and between tables. The recognition of pattern in multiplication helps learners remember facts.
  For example 5x is half of 10x OR doubling twice is the same as multiplying by four
- Making connections and understanding relationships between the other operations



Using arrays to teach multiplication and division

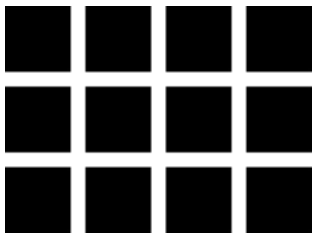
 4 groups of 3	"Groups of" or "count in…" models are additive i.e. we add another three each time. Skip counting and the "Groups of" model is often used to introduce multiplication, yet these are both additive or linear models. Issues arise when numbers begin to get bigger and they cannot assist in visualising the commutative properties of multiplication.
	Arrays are a powerful model. By arranging the groups into a grid we can simultaneously see the individual items and the total. We can still skip count in either 3's or 4's but we can also see the link between the two.

Arrays are defined as a set of numbers or shapes laid out in a rectangle. For early multiplication purposes, an array will consist of shapes or dots. Arrays are a simple yet powerful visual aid for helping children to understand how multiplication (and fractions) work by encouraging multiplicative thinking.



It is convention to read the array with **rows first** and then **columns**.

The array to the right represents 3 x 4 (3 rows of four), with 12 squares altogether.



Key ideas about arrays

Additive or linear models of multiplication (such as the "groups of" model) do not help develop further understandings of the underlying patterns of multiplication, which is key to future work in mathematics. Arrays lend themselves to multiplicative understandings rather than additive understanding.

- Arrays are an important conceptual, two-dimensional step between modelling multiplication with concrete objects, to using an area model for multiplication and finally onto algorithms. (More on the area / grid model below.)
- They are important for understanding:
  - Factors and products
  - Both the *communicative* and *distributive* properties
  - conservation and visualisation of the area of any 2D shape
  - conservation and visualisation of fractions (e.g. understand that one half cut vertically and one half cut horizontally really are the same size)
  - algebraic concepts such as x squared. This means a square with a side length of x. This also means that 3x is a rectangle with one side of 3 and the other of x.

FP - Introducing multiplication with a context that lends itself to arrays

Mike Askew indicates that research suggests this process for introducing multiplication.

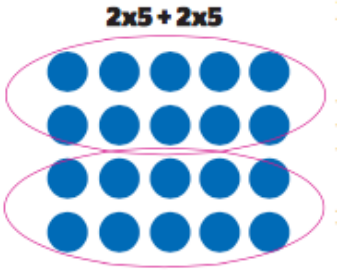
Rather than starting off with the abstract calculation and then setting up a physical model or diagram to represent it, we are better off starting with simple contexts that can be described and talked about and then represented by a multiplication sentence. In other words, rather than starting from what children do not know (what '4 x 5' means) we begin with something they are familiar with and help them move to the symbolic.

Take, for example, this context problem:

> *A baker is putting muffins on a tray to put in the oven. A tray holds four rows of five muffins. How many muffins can the baker put onto a full tray?*

This context lends itself nicely to children modelling it as a four by five array, either with physical objects or drawings. The discussion would then be around how many muffins there are altogether and whether or not anyone had a quick way to find the total that did not involve counting each muffin singly. Taking the children's explanations can lead to introducing the notation of multiplication out of what they saw and did. For example, a child might say that they added five and five and five and five, recorded as 5+5+5+5.

Another may say that they saw two groups of 10, each arising from pairs of fives. Marking this up on an image, children can see this recorded as:

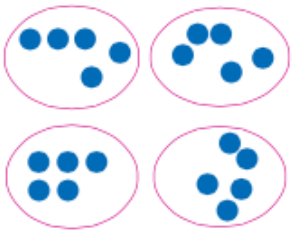


Similarly, other 'seeings' might include 4 + 4 + 4 + 4 + 4. From here it is a short step to introducing 4 x 5 or 5 x 4 as a quick way of recording the repeated additions.

The context of muffins on a tray is not arbitrary. Compare this with, say:

> *The baker is putting muffins into bags. She puts five muffins into each bag and fills four bags. How many muffins is that?*

The model or image that this lends itself to is something like:



While this is fine, because it is less structured it does not lend itself to the variety of ways of seeing, describing and finding the total as the array does. It is also unlikely to lead to a conversation about 4 x 5 being the same as 5 x 4. The equivalence of these two statements is self-evident in the tray of muffins, but far from obvious in the context of bags of muffins. Rotating the array through a quarter turn clearly leaves the number of muffins unchanged, but it is not so immediately obvious that four bags of five must contain the same number of muffins as five bags of four.

Other contexts that lend themselves to being modelled as arrays include rows of chairs, square tiles on a floor, and windows made up of small panes. A search on the internet can produce many suitable images that, introduced one at a time over a series of days, will provoke rich conversations about 'shortcuts' to counting the total number of items and allow you to drip-feed the notation of multiplication to represent the array.

Source: Signs of the times article (by Mike Askew)
http://www.teachprimary.com/resource_uploads/signs-of-the-times.pdf

By introducing multiplication through simple contexts, we can help children to understand the **close connection between multiplication and division**.

Problems such as these help develop this understanding:

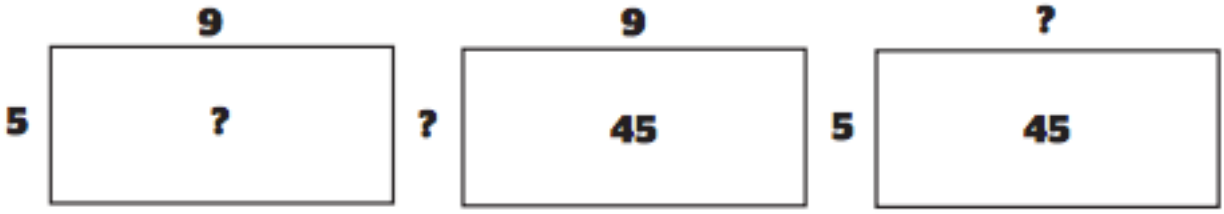
> *Bulewa the grocer is putting apples into bags. Bulewa puts nine apples in each bag and fills five bags altogether. How many apples does Bulewa put into bags?*
>
> *Bulewa also bags up some pears. Bulewa puts nine pears in each bag and has 45 pears to put into bags. How many bags of pears can Bulewa fill?*
>
> *Bulewa is putting oranges into bags. Bulewa has 45 oranges and five bags. If Bulewa puts the same number of oranges into each bag, how many oranges are in one bag?*

The same context - putting fruit into bags - described in these different ways provides a **context to talk about the relationship between multiplication and division**.

Any multiplication calculation can give rise to essentially two different types of division problems. When Bulewa is putting pears into bags, the number of pears in each bag is already known - this is a division as repeated subtraction (or quotitioning) problem. The size of the 'share' (or quota) is known - nine pears. What we do not know is how many of these 'shares' can be made from 45 pears.

In the case of the oranges, the number of shares (bags) is known - five - but we don't know the number to put into each bag. This is an example of division as sharing or partitioning (the 45 oranges need to be partitioned into five groups, each containing an equal number).

The array can help make clear the connections here.



Source: (Askew, 2011)
http://www.teachprimary.com/resource_uploads/signs-of-the-times.pdf

Arrays in the intermediate phase

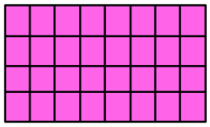
If arrays are introduced in the Foundation Phase as contexts for doing multiplication and division, they can assist in deepening understanding in the intermediate grades with a specific focus on the distributive and commutative properties. Additionally, when used in a more abstract way, they can be used to model and practice multi-digit multiplication before the introduction of the long multiplication algorithm.

The distributive and commutative properties with arrays

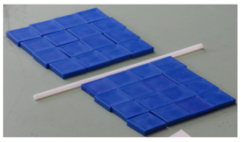
The Grade 4 to 6 Mathematics CAPS document indicates that learners need to understand these properties but it is not necessary to know the terms.

The distributive property

An array can be split into smaller parts based on place value or other facts to represent the distributive property.



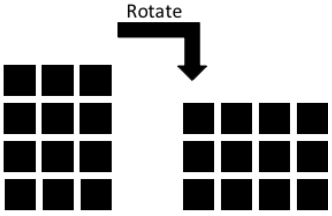
How might you split this array to work out 4 × 8?



8 × 4 = (4 × 4) + (4 × 4)

The commutative property

Any array can be turned by 90° to illustrate the commutative property.



4 rows of 3
4 x 3
4 threes

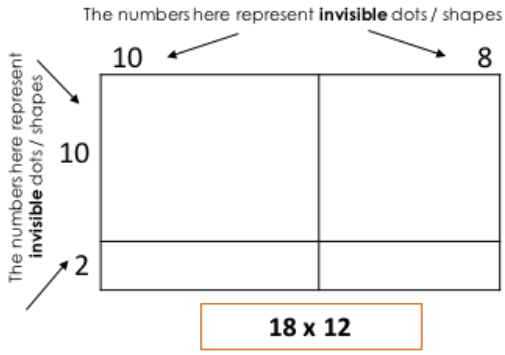
3 rows of 4
3 x 4
3 fours

Using arrays to do multi-digit multiplication

The area model provides a transition from concrete representations of arrays to a more abstract representation, which discourages learners from counting the dots/ shapes in the array. It is a useful step between arrays and more formal long multiplication algorithms.

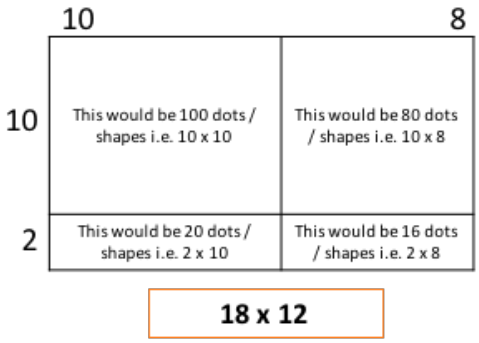
Instead of using (and drawing) all the dots / shapes in an array, it is easier to represent it conceptually using an *area model* by breaking each number into its *place value components* and seeing invisible dots / shapes, as shown with this example for 18 x 12:

1 – Break the numbers into sectors by place value

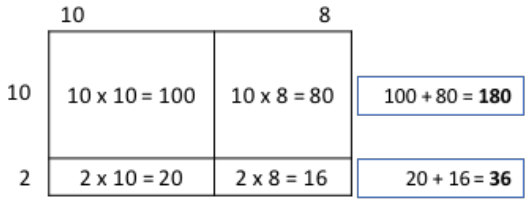


18 is broken into place value components of 10 and 8 across the top (representing columns)
12 is broken into place value components of 10 and 2 down the side (representing rows)

2 – Multiply the rows and columns for each sector of the grid

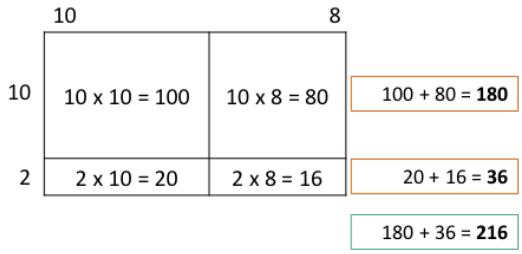


3 – Add the values from the individual sectors in rows



The invisible dots / shapes in each separate array are multiplied and added across the rows to arrive at 180 and 36 respectively.

4 – Add the totals from the two rows to arrive at the answer



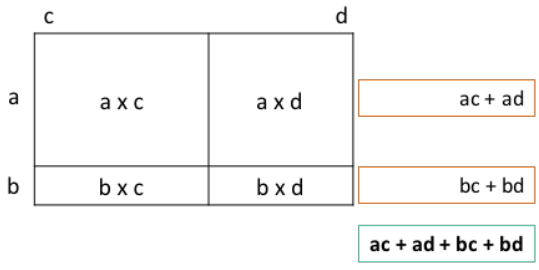
The row values are added together to give a final answer of **216**.

This method of multiplying has direct links to the long multiplication algorithm and to algebraic multiplication as shown below.

Linking to long multiplication

18	
X 12	
100	(10x10)
80	(10x8)
20	(2x10)
16	(2x8)
216	

Linking to algebraic multiplication



Progression in multiplication and division

As with addition and subtraction, the focus here is to progress learners from using tallies and drawings (and seeing items as individual pieces) to unitising, using repeated addition and skip counting, through to arrays, flexible use of multiplication facts and appropriate use of algorithms.

Constrained methods	Less constrained	Semi fluent methods	Flexible fluency
Inefficient (I)	*Somewhere in between (IE)*		*Efficient (E)*
Use of fingers, tally marks, circles, drawings of any kind	Skip counting, repeated addition	Arrays, breaking down into expanded notation	Use of known multiplication and division facts, appropriate use of algorithms for 2 and 3 digit problems
Counting in 1s No sense of groups	Recognising groups / counting in groups	Recognising arrays of rows and columns Recognising patterns of multiplication and knowing key tables (x5, x10, x100 doubling etc.)	Solving multiplicative problems using efficient strategies including for example: * expansion (or grid method) * combinations of known facts (e.g. times 12 is x 10 and add double) * standard algorithm (short & long forms)

Strategies for learning tables

It is possible for learners to learn other tables by working from ones that are easier to learn (such as 2, 3 and 10) by using doubling strategies and seeing patterns.

10 x table

Draw on place value to help learners understand what happens when multiplying by 10.

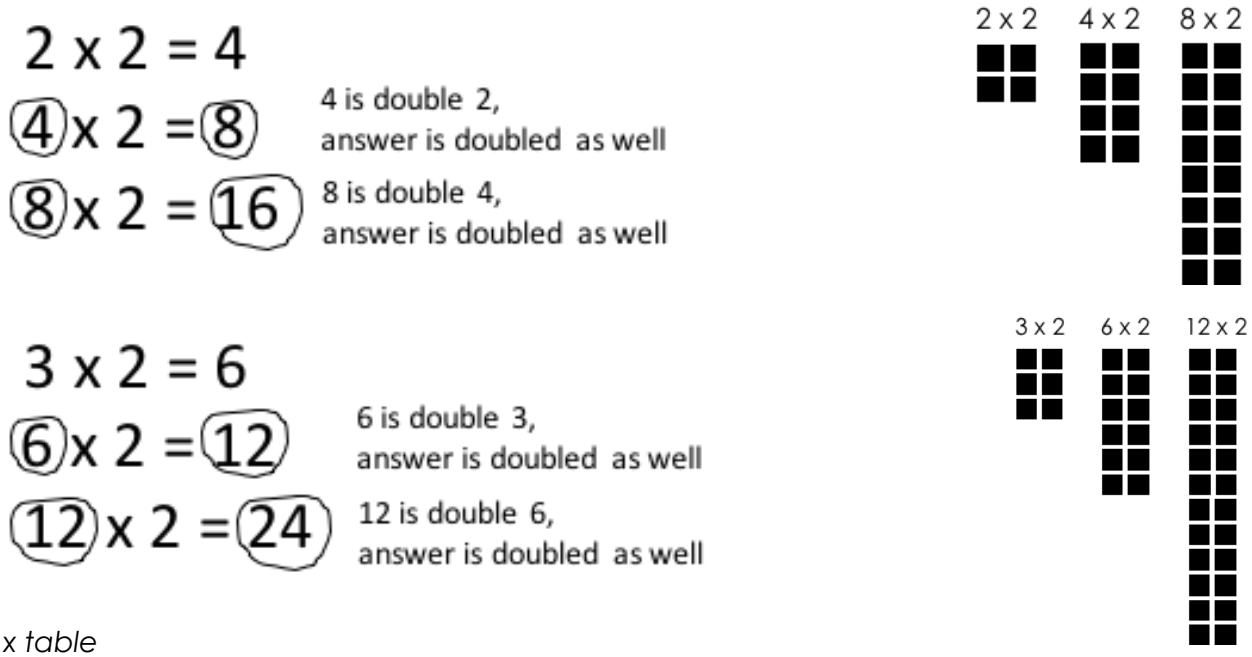
5 and 10 x tables

It is possible to work out the 5 times table by multiplying by 10 first and then having the answer. This works because 5 is half of 10.

e.g. 14 x 5. 14 x 10 = 140, half of 140 is 70, so 14 x 5 is 70.

2, 4 and 8 x tables and 3, 6 and 12 x tables

Use a double and double again strategy. Use arrays to model how this works if necessary



7 x table

Encourage learners to use commutativity to learn this one. Make the connections between the 7 and other multiplication tables.

References

Askew, M. (2011, September). Signs of the times. *Teach Primary*, 97(26), 34; 35; 37. Retrieved from http://www.teachprimary.com/resource_uploads/signs-of-the-times.pdf

Ellemor-Collins, D., & Wright, R. J. (2011). Unpacking mathematisation: An experimental framework for Arithmetic Instruction. In *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 1–8). Ankara, Turkey.

Numeracy Professional Development Projects. (2007). *Book 6: Teaching multiplication and division. Revised Edition 2007*. Wellington, New Zealand: Ministry of Education, New Zealand.

Club sessions 10 to 15: mathematical focus

The overall object of learning for this series of clubs is detailed on this page. The activities detailed in this booklet help to focus on these big ideas and are intended to help you as the club leader to encourage learners to progress from using tallies to more efficient strategies for multiplication and division.

At the start of each session, check the **PURPOSE OF THE SESSION / OBJECT OF LEARNING** and **APPROACH TO RUNNING THE SESSION** boxes at the top of each planning sheet to set your focus for each session.

Club Session Planning

Club Overviews: Session 10 to 15 *Page: 12*	
Foundation Phase	**Intermediate Phase**
Session Ten *Page: 13*	**Session Ten** *Page: 20*
Session Eleven *Page: 14*	**Session Eleven** *Page: 22*
Session Twelve *Page: 16*	**Session Twelve** *Page: 23*
Session Thirteen *Page: 17*	**Session Thirteen** *Page: 24*
Session Fourteen *Page: 18*	**Session Fourteen** *Page: 25*
Session Fifteen *Page: 19*	**Session Fifteen** *Page: 26*

Object of learning for all these sessions:

- Working with the array model to develop multiplicative thinking in learners
- Emphasis on using doubling strategies for learning multiplication facts in 4,and 8,6 and 12 times tables.
- Developing number sense across a range of numbers using different operations
- All card and dice games are intended to promote learners' fluency in using number facts, especially multiplication tables

Overviews

The session overviews are shown here for Grade 1 through to the IP grades. This means that if you encounter a learner who needs to be extended or remediated in your clubs, you have access to other activities that can be useful.

Grade 1

	Session 10	Session 11	Session 12	Session 13	Session 14	Session 15
	Timings based on a 60 minute club					
Mental warmup	"TEN"	SKIP COUNTING IN 2s, 5s and 10s	DOUBLES/HALVES NUMBER SENSE GRID	FIZZ POP DOUBLING and x2		"TWELVE"
Time	*5 mins*	*5 mins*	*5 mins*	*10 mins*		*15 mins*
Games	NAUGHTY THREES	TOTAL THREE	BUILD ARRAYS GAME	BEE ARRAYS	HOW CLOSE TO 100	ASSESSMENT
Time	*15 mins*	*20 mins*	*20 mins*	*25 mins*	*30 mins*	*35 mins*
Activities	PIES AND MUFFINS	MORE ARRAYS	ARRAY CARD LAYOUTS 12 and 18	FIND ARRAYS	ARRAYS HOORAY	SKIP COUNTING MAZES
Time	*30-40 mins*	*20 mins*	*25 mins*	*20 mins*	*30 mins*	*10 mins*
Pay it Forward	NAUGHTY THREES	TOTAL THREE	BUILD ARRAYS GAME			
Take home work	Homework book(s)					

Grades 2 and 3

	Session 10	Session 11	Session 12	Session 13	Session 14	Session 15
	Timings based on a 60 minute club					
Mental warmup	"EIGHTEEN"	I HAVE, WHO HAS: DOUBLES AND HALVES	DOUBLES/HALVES NUMBER SENSE GRID	FIZZ POP DOUBLING and x2		"TWENTY FOUR"
Time	*15 mins*	*15 mins*	*10 - 15 mins*	*10 mins*		*15 mins*
Games	NAUGHTY THREES	TOTAL THREE	BUILD ARRAYS GAME	BEE ARRAYS	HOW CLOSE TO 100	ASSESSMENT
Time	*15 mins*	*20 mins*	*20 mins*	*25 mins*	*30 mins*	*35 mins*
Activities	PIES AND MUFFINS	ARRAY SCAVENGER HUNT	ARRAY CARD LAYOUTS 12 and 18	FIND ARRAYS	ARRAYS HOORAY	SKIP COUNTING MAZES
Time	*30 mins*	*20 mins*	*25 mins*	*20 mins*	*30 mins*	*10 mins*
Pay it Forward	NAUGHTY THREES	TOTAL THREE	BUILD ARRAYS GAME			
Take home work	Homework book(s)					

Intermediate Phase

	Session 10	Session 11	Session 12	Session 13	Session 14	Session 15
	Timings based on a 60 minute club					
Mental warmup	"TWENTY FOUR"	I HAVE WHO HAS 2, 4, 8 TABLES	NUMBER SENSE MULTIPLICATION GRID	FIND 90 + AND x	I HAVE WHO HAS 3, 6, 12 TABLES	FIZZ POP X 10, X 100
Time	*10 mins*	*10 mins*	*10 mins*	*20 mins*	*15 mins*	*5 mins*
Games	NUMBER PATTERN INVESTIGATIONS	2, 4, 8 CARD GAME	HOW CLOSE TO 100		MULTIPLCATION DICE GAME	ASSESSMENT
Time	*20 mins*	*20 mins*	*25 mins*		*20 mins*	*35 mins*
Activities	ARRAY SCAVENGER HUNT	ARRAY CARD LAYOUTS 18, 24 and 36	ARRAYS HOORAY	GRID METHOD (1)	GRID METHOD (2)	ADDITION AND MULTIPLICATION PUZZLES
Time	*30 mins*	*30 mins*	*25 mins*	*40 mins*	*25 mins*	*20 mins*
Pay it Forward		2, 4, 8 CARD GAME				
Take home work	Homework book(s)					

FP	Maths Club Whole Session Planning Sheet	Session Ten
Purpose of the session / object of learning	Key focus is the introduction of arrays to the FP learners using familiar contexts	

What resources / manipulatives will you need?	Home sharing/ Pay It Forward task
• **NUMBER SENSE**: white/blackboard or flipchart • **NAUGHTY THREES**: 2 dice per pair • **NUMBER SENSE**: pencils, scrap paper • **MUFFINS & PIES**: activity sheets in plastic sleeves, egg boxes, ice trays etc	Learners can play NAUGHTY THREES at home. Learners can look for and work out arrays at home.
Organisational requirements	**Your approach to running the session**
• **NUMBER SENSE**: individual then whole group • **NAUGHTY THREES**: play in pairs / groups of 3 • **NUMBER SENSE**: individual work • **MUFFINS & PIES**: Whole group	The most important aspect of this session is introducing the arrays through the Muffins & Pies activity. If time is short, leave out the dice game.

Number sense: "TEN" or "EIGHTEEN" – 15 minutes

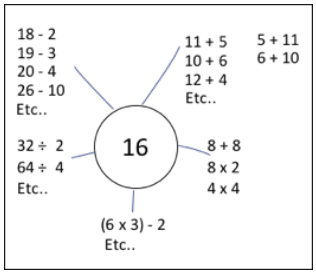
GRADE 1

- Write the number **10** on the board or flipchart
- Give learners 5 minutes to come up with at least two ways to make 10 using addition and subtraction. They can come up with more if they wish.

GRADES 2 & 3

- Write the number **18** on the board or flipchart
- Give learners 5 minutes to come up with at least one way to make 18 using addition, subtraction and if possible multiplication. They can come up with more if they wish.

- Ask for contributions and write them on the board in a way that emphasises patterns (see example for 16 to the right), asking learners if they agree with the contribution.
- When you write a contribution on the board, always get them to check their list and tick off if they have the same idea.



Game: NAUGHTY THREES – 15 minutes (GRADES 1, 2 AND 3)

Practice addition / skip counting in 5s

- Take turns to throw both dice
- Players only score when two identical numbers are thrown e.g. two 1's, two 2's and so on
- First player to a score of 30 wins

VARIATIONS:

- Change values of the dice to practice other skip counting sequences e.g. 2s, 3s and 4s
- Change the target score to a bigger or smaller number

Throw…

Two 1s	5 points
Two 2s	5 points
Two 3s	wipe out score and start again
Two 4s	5 points
Two 5s	5 points
Two 6S	25 points

Activity: MUFFINS & PIES – 30 minutes (GRADES 1,2 and 3)

Use this time to introduce learners to the ideas of arrays using familiar contexts.
Work through the example on page 5.

FP	Maths Club Whole Session Planning Sheet	Session Eleven
Purpose of the session / object of learning	Once again the focus is on working with arrays as a means of understanding multiplication	

What resources / manipulatives will you need?	Home sharing/ Pay It Forward task
• **GR2,3 MENTAL:** Double and halving I have who has loop cards • **TOTAL THREE**: Scrap paper, pencils, 2 dice per pair • **GR1 MORE ARRAYS:** Activity in plastic sleeve, kokis, cloth • **GR2,3 ARRAY SCAVENGER HUNT**: black/whiteboard or flipchart	Learners play TOTAL THREE at home
Organisational requirements	**Your approach to running the session**
• **TOTAL THREE**: Pair work • **MORE ARRAYS**: Individual • **ARRAY SCAVENGER HUNT**: whole group	The most important aspect of the session is the work with arrays. If time is short, leave out the dice game.

Mental: SKIP COUNTING – 5 minutes (GRADE 1)

Practice counting in 2s, 5s and 10s

- Organise the learners into a circle

Skip counting in 2s:	Skip counting in 5s:	Skip counting in 10s:
each learner puts up their hands in a fist, count around the circle in 2s (i.e. counting 2 hands at a time)	each learner shows all fingers on both hands, count around the circle in 5s (i.e. counting 5 fingers at a time)	each learner shows all fingers on both hands, count around the circle in 10s (i.e. counting 10 fingers on both hands at a time)

Mental: I HAVE, WHO HAS CARDS FOR DOUBLES AND HALVES – 10 minutes (GRADE 2 and 3)

Use the Double and Halves I have, who has loop cards for this session.

- Hand out one card to each learner, including yourself. If you have extras, give a second to a number of learners.
- The person who has the card labelled "START" begins by reading what is on their card. E.g. "who has Double 6?"
- Learners must check their cards for the answer to Double 6 and read what is on their card e.g. "I have 12. Who has Double 3?"
- The game continues until play returns to the person who started.

Game: TOTAL THREE –20 minutes (Grades 1, 2 and 3)

AIM: Practice addition, subtraction, multiplication

Take turns to roll the two dice. Do the following calculations each time you roll:

- Add the two numbers shown on the dice
- Find the difference between the two numbers
- Multiply the two numbers OR skip count (Grade 1)
- Add the three numbers to get a score for that round
- After 10 rounds the player with the highest total is the winner

VARIATION

To make the activity more challenging use 8, 10, 12 or 20-sided dice

For example
Roll ⚃ and ⚄
6 + 3 = 9
6 – 3 = 3
6 x 3 = 18 OR
6, 12, 18 (3, 6, 9, 12, 15, 18)
Score is:
9 + 3 + 18 = 30

Activity: MORE ARRAYS – 30 minutes (GRADE 1)

Work through the AN APPLE A DAY activity

Activity: ARRAY SCAVENGER HUNT – (GRADE 2 and 3)

On the next page → or AN APPLE A DAY

Activity: ARRAY SCAVENGER HUNT – 30 minutes (GRADE 2 and 3)

- Draw a rectangle on the board. Ask the learners what shape it is, and discuss the differences between squares and rectangles. If the learners are unsure, ask them what they can see that is different and the same.

- Draw a gird of rows and columns and ask them if this is still a rectangle. Then look at how many rows and columns the rectangle has, introducing the vocabulary or rows, columns and array as you go along.

- Ask the learners how many squares in the grid and ask how they worked it out. Also ask them if they could share a sum to show how they worked it out. At this point I often get the typical mix of responses as shown to the right. We talk about the multiplication sums as being a quicker way to represent the repeated addition.

3 + 3 + 3 + 3
4 + 4 + 4
3 x 4
4 x 3

- Then go outside the classroom and I look for an array in the vicinity, normally a window or gate. I point out to the learners that this is an array and physically point out the rows and columns. Ask them how many shapes in the array.

- In a window such as the example shown, we discuss why the two panes at the top are not considered to be an array as they do not make a rectangle or square and they have curved sides.

- Go on a scavenger hunt around the school or the local area to find as many arrays as possible, each time working out the rows and columns and the total.

FP	Maths Club Whole Session Planning Sheet	Session Twelve
Purpose of the session / object of learning	More exposure to arrays.	

What resources / manipulatives will you need?	**Home sharing/ Pay It Forward task**
• **BUILD ARRAYS**: 1 dice per pair, counters (bottle tops), scrap paper and pencils • **NUMBER SENSE**: Whiteboard, blackboard or flipchart • **ARRAY HOORAY:** Playing cards (or counters), scrap paper and pencils	Play BUILD ARRAYS at home
Organisational requirements	**Your approach to running the session**
• **NUMBER SENSE**: individual then whole group • **BUILD ARRAYS**: Learners work in pairs • **ARRAY HOORAY**: Learners work in pairs	

Number sense: DOUBLES AND HALVES (x2 and ÷2) – 10 to 15 minutes (GRADES 1, 2 and 3)

Draw this grid on the board or flipchart. Ask these questions.

12	4	8	2
7	9	48	24
11	6	5	3

- Find numbers which are double another number on the grid
- Find numbers which are half of another number on the grid
- Which numbers do not have a double?
- Which numbers do not have a half in the grid?

Game: BUILD ARRAYS – 20 minutes (GRADE 1,2 and 3)

- One learner rolls the dice twice
- The first number rolled tells how many rows to make in an array.
- The second number rolled tells how many counters to put in each row of your array (to make up the columns)
  FOR EXAMPLE: If a learner rolls a 2 and then a 5, they might make an array like this →

- Learners must draw each array made, recording how many rows, how many columns, and how many counters altogether. Encourage them to write either a repeated addition sum or a multiplication sum for each array
  FOR EXAMPLE
  2 x 5 = 10; 5 x 2 = 10; 2 + 2 + 2 + 2 + 2 = 10; 5 + 5 = 10

Activity: ARRAY CARD LAYOUTS – 20 minutes (GRADES 1, 2 and 3)

- Divide a pack of cards into piles of 12.
  If you have sufficient counters or bottle tops, use those instead.
- Learners works in pairs
- Remind learners about the structure of an array: always a rectangle or square, equal items in each row and column
- Learners must build as many different arrays as possible with ALL of their 12 cards
- After building each array, they must write down the number of rows and columns the array has, and the total number of cards, which should always be 12.
- After all the arrays of 12 have been explored, and if there is time, split the cards into piles of 18 or 24 and repeat.

Arrays with 12 cards	**Arrays with 18 cards**	**Arrays with 24 cards**
12 x 1; 1 x 12 6 x 2; 2 x 6 4 x 3; 3 x 4	18 x 1; 1 x 18 9 x 2; 2 x 9 6 x 3; 3 x 6	24 x 1; 1 x 24 12 x 2; 2 x 12 8 x 3; 3 x 8 6 x 4; 4 x 6

FP	Maths Club Whole Session Planning Sheet	Session Thirteen
Purpose of the session / object of learning	Further work with arrays in different formats. Grade 2 and 3 work with drawn representations on the ARRAY HOORAY activity which introduces some early division using arrays.	

What resources / manipulatives will you need?	Home sharing/ Pay It Forward task
• **BEE ARRAYS**: 2 dice, game board in plastic sleeves, kokis, cloths • **ARRAYS HOORAY**: Copies of the activities in plastic sleeves, kokis and cloth	Homework books
Organisational requirements	**Your approach to running the session**
• **BEE ARRAYS**: Play in pairs • **ARRAYS HOORAY**: Individual	

Mental: FIZZ POP WITH DOUBLING and 2 x TABLE – 10 minutes (GRADES 1, 2 and 3)

- Start with ***DOUBLING***. Say "I will say a number and you must double it".
  - The game starts with leader saying "FIZZ", club responds with "POP"
  - Say the number and club responds. E.g. "5" and club responds with "10"
  - These are good sequences to use: 2, 4, 8, 16, 32 … or 3, 6, 12, 24, 48 … keep going until the learners cannot go any further.
  - If you get an answer with the harder numbers, ask the learner to share their method, then ask group to try the method for the next number.

- Repeat with ***2x table*** (Optional for Grade 1) Say "I will say a number and you must times it by two".
  - Use the above sequence
  - Can learners see the connection between doubling and multiplying by two?

Game: BEE ARRAYS – 25 minutes (GRADES 1,2 and 3)

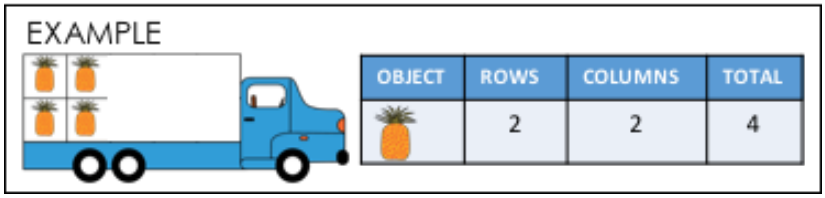
AIM: to get 3 or 4 hexagons touching to each other in a line or other configuration as shown on the game board (3 will take less time and may be more accessible for Grade 1 learners)

- 1st learner throws both dice e.g. 2 and 3
- Learner looks on the game board for a 2 x 3 or 3 x 2 array. If there is one available, mark the hexagon with initials or name
- Next learner takes a turn
- 1st learner to mark 3 or 4 hexagons in a line or other configuration, wins

Activity: FIND ARRAYS– 25 minutes (GRADES 1,2 and 3)

AIM: learners identify the arrays in the pictures and complete the tables on each activity sheet

EXTENSION
If there is time, you could follow up with some questions to extend the activity:



SHEET 1
- What is the same about each array on this page? (Looking for a connection that each one has two rows)
- How much fruit altogether on the page? (Encourage the learners to add up the totals for all the fruit items)
- Can you find a quick way to work out how many wheels there are on the page?

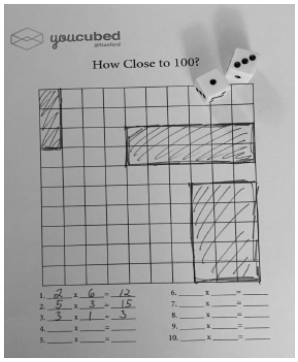
SHEET 2
- How many animals altogether on the page? (Encourage the learners to add up the totals for all the animals, including the frog)
- How many insects altogether on the page? (Encourage the learners to add up the totals for all the insects)
- Can you find a quick way to work out how many human legs there are on the page? How many monkey / frogs / chicken / fly legs?
- Can you find a quick way to work out how many eyes (human, animal and insect) there are on the page?

FP	Maths Club Whole Session Planning Sheet	Session Fourteen
Purpose of the session / object of learning	The focus here is fluency with basic multiplication facts and working with patterns in the 100 grid	

What resources / manipulatives will you need?	**Home sharing/ Pay It Forward task**
• **NUMBER PATTERN INVESTIGATIONS**: Copies of the grids in plastic sleeves, kokis and cleaning cloth • **HOW CLOSE TO 100**: Copies of the grids in plastic sleeves, kokis and cleaning cloth	Homework books
Organisational requirements	**Your approach to running the session**
• **HOW CLOSE TO 100**: Pair work • **NUMBER PATTERN INVESTIGATIONS**: Individual	Grade 1 learners can either play the How Close to 100 game for the whole session, or try the Number Pattern Investigations (or at least the 5 and 10 pattern)

Game: HOW CLOSE TO 100 – 30 minutes (GRADES 1, 2 and 3)

By drawing arrays, the goal is to fill up the grid to get it as full as possible, using the space as efficiently as possible. How close to 100 can you get? How many squares do you have empty?



- The first player rolls the dice and uses those two numbers to make an array on the 100 grid.
  FOR EXAMPLE
  Roll a  and  so draw a 2 x 6 or 6 x 2 array on the grid
- After the player draws the array on the grid, write in the multiplication sum that describes the array at the bottom of the page.
  FOR EXAMPLE
  2 x 6 = 12 or 6 x 2 = 12
- The second player rolls the dice, draws the array and writes their multiplication sum
- The game ends when both players have rolled the dice and cannot put any more arrays on the grid.

VARIATIONS
- Each child has a grid of its own. Who can get closest to 100?
- Make grids of 400 and add more dice, or use dice with more sides
- Adapt the game to let the grid represent 100% for older learners

Source: Youcubed (https://www.youcubed.org/task/how-to-close-100/).

Activity: ARRAY HOORAY – 30 minutes (GRADES 1,2 and 3)

Sheet One: Match the multiplication fact to the array

Choose a multiplication fact to match the arrays. Then say how many altogether in each array.

Fact	How many?	Fact	How many?
2 x 4	8	4 x 2	8

Sheet Two: Use the arrays to write different types of facts

This array shows...	Write an addition fact for this array	Write a multiplication fact for this array	Write another multiplication fact for this array	Show how to split the array into equal parts
EXAMPLE 3 rows x 5 columns	3 + 3 + 3 + 3 + 3 = 15	3 x 5 = 15	5 x 3 = 15	Split into 3 equal parts

FP	Maths Club Whole Session Planning Sheet	Session Fifteen
Purpose of the session / object of learning	In this final session, the 4 operations assessment is re-administered and the session finishes with something fun.	

What resources / manipulatives will you need?

- **NUMBER SENSE**: Black/whiteboard or flipchart, scrap paper and pencils
- **ASSESSMENT**: 1 copy of assessment scripts per learner, plus 1 for yourself
- **MAZES**: Activity in plastic sleeves, kokis, cleaning cloths

Organisational requirements	Your approach to running the session
• **NUMBER SENSE**: individual then whole group • **ASSESSMENT**: individual work • **MAZES**: individual work	This is the final club session, so do the assessment and then finish off with something fun.

Number sense: "TWELVE" or "TWENTY FOUR" – 15 minutes

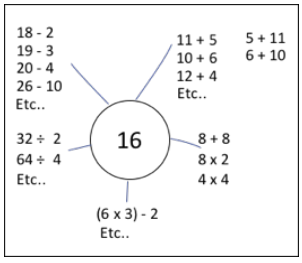
GRADE 1

- Write the number **12** on the board or flipchart
- Give learners 5 minutes to come up with at least two ways to make 12 using addition, subtraction and if possible multiplication or an array. They can come up with more if they wish.

GRADES 2 & 3

- Write the number **24** on the board or flipchart
- Give learners 5 minutes to come up with at least one way to make 24 using addition, subtraction and if possible multiplication and division. They can come up with more if they wish.

- Ask for contributions and write them on the board in a way that emphasises patterns (see example for 16 to the right), asking learners if they agree with the contribution.
- When you write a contribution on the board, always get them to check their list and tick off if they have the same idea.



Assessment: 35 minutes

Re-administer the 4 operations assessment under the same conditions used in the first club.

Activity: SKIP COUNTING MAZES – 10 minutes

AIM: Learners must find a way through the mazes by counting in 2s, 5s, 10s and 3s. NOTE: the numbers may be next to, above / below or diagonal.

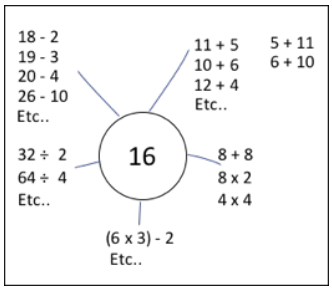
0 4 6 8 9 10 3 8 6  13 18 2 20 2 10 8 6 4 START 20 18 16 14 15 7 10 2  **2** 18 17 14 1 **14** 7 0 END **4** 5 1 13 **12** 9 **16** **18** **20** 1 12 **6** 0 **10** 11 10 21 4 3 0 15 16 **8** 1 3 19 11 13 12 2 4 5 7 20 9 1 2 8 19	8 4 6 8 9 10 3 8 6  6 18 2 20 2 11 6 5 20 **5** **10** 6 18 16 14 **40** 10 8 END START 24 **15** 17 14 **35** 10 **45** **50** 0  4 **20** 1 **30** 5 4 6 4 2 12 6 **25** 12 11 10 17 18 9 0 3 16 3 1 3 19 18 13 12 2 4 5 7 8 9 1 2 8 19
29 4 6 8 18 17 16 8 60  25 23 10 9 19 11 15 5 10 24 **20** **30** 21 20 10 14 **90** **100** END **10** 73 15 **40** **50** 100 19 **80** 21 10 START 140 10 100 51 **60** **70** 40 30 20  130 120 110 85 32 12 120 130 10 80 90 60 50 40 14 100 40 21 90 40 22 20 90 60 50 40 90 100	 4 6 8 9 16 14 12 10 0 6 10 9 8 **18** 6 5 8 5 14 13 11 **15** 20 **21** 10 6 4 38 36 **12** 6 22 8 **24** 2 2 START 4 **9** 1 26 24 3 6 **27** **30** **3** **6** 32 30 28 11 2 1 0 END 0 4 16 3 1 3 19 18 10  2 4 5 7 8 9 1 2 8

IP	Maths Club Whole Session Planning Sheet	Session Ten
Purpose of the session / object of learning	The object of learning in this session is to help learners to explore what arrays are and how they can be useful for multiplication	

What resources / manipulatives will you need?	Home sharing/ Pay It Forward task
• **NUMBER SENSE**: White/ blackboard or flipchart for showing learner contributions, scrap paper and pencils • **NUMBER PATTERN INVESTIGATIONS**: Copies of activity in A4 plastic sleeves, kokis, cleaning cloths	Learners continue the Scavenger Hunt at home looking for and working out arrays
Organisational requirements	**Your approach to running the session**
• **NUMBER SENSE:** individual, then whole group • **NUMBER PATTERN INVESTIGATIONS**: Individual work • **SCAVENGER HUNT**: whole club	The most important aspect of this session is introducing the arrays through the Scavenger Hunt. If time is short, leave out the pattern investigation.

Number sense: "TWENTY FOUR" – 10 minutes

- Write the number **24** on the board or flipchart
- Give learners 5 minutes to come up with at least one way to make 24 for each operation. They can come up with more if they wish.
- Ask for contributions and write them on the board in a way that emphasises patterns (see example for 16 to the right), asking learners if they agree with the contribution.
- When you write a contribution on the board, always get them to check their list and tick off if they have the same idea.



Activity: NUMBER PATTERN INVESTIGATIONS – 25 minutes

AIM: the activity sheet shows 3 100 grids. The aim is for learners to skip count and complete the grids in such a way as to see the relationships between different multiplcation tables.

- Follow the instructions on the activity sheet
- Ask questions when the learners are done:
  - What do you notice about the bocks that you have coloured in? Do they make a patterns on the grid?
  - What about the blocks with a X? Do they make a pattern on the gird?
  - Is there a connection between the two patterns?
  - Can you think of a reason this might be?

If there is time, learners can do this puzzle. **Note that this is not a 100 grid.** Learners must find the patterns and fill in the white blocks. These are the answers.

IP answers

3	6	9	12	15	18	21	24
	8			20		26	
5	10	15	20	25		31	
	12			30	33	36	39
	14			35			49
8	16	24	32		55	57	59
4		21					69
0	8	18	28	38	48	58	

Activity: ARRAY SCAVENGER HUNT – 25 MINUTES

On the next page →

IP	Maths Club Whole Session Planning Sheet	Session Ten cont.

Activity: ARRAY SCAVENGER HUNT – 25 minutes

- Draw a rectangle on the board. Ask the learners what shape it is, and discuss the differences between squares and rectangles.

- Draw a gird of rows and columns and ask them if this is still a rectangle. Then look at how many rows and columns the rectangle has, introducing the vocabulary or rows, columns and array as you go along.

- Ask the learners how many squares in the grid and ask how they worked it out. Also ask them if they could share a sum to show how they worked it out. At this point I often get the typical mix of responses as shown to the right. We talk about the multiplication sums as being a quicker way to represent the repeated addition.

3 + 3 + 3 + 3
4 + 4 + 4
3 x 4
4 x 3

- Then go outside the classroom and I look for an array in the vicinity, normally a window or gate. I point out to the learners that this is an array and physically point out the rows and columns. Ask them how many shapes in the array.

- In a window such as the example shown, we discuss why the two panes at the top are not considered to be an array as they do not make a rectangle or square and they have curved sides.

- Go on a scavenger hunt around the school or the local area to find as many arrays as possible, each time working out the rows and columns and the total.

IP	Maths Club Whole Session Planning Sheet	Session Eleven

Purpose of the session / object of learning	The focus here is on practising fluency with 2, 4 and 8 x tables and using array layouts to explore factors.

What resources / manipulatives will you need?	**Home sharing/ Pay It Forward task**
• **MENTAL GAME**: set of 2,4 and 8 I have, who has loop cards • **2, 4, 8 GAME**: One deck of playing cards per pair, using just 2s, 4s and 8s, pencils, scorecards • **ARRAY CARD LAYOUTS**: Packs of cards (or counter, bottle tops), scrap paper, pencils	Play 2, 4, 8 game with people at home and practice the doubling strategy.
Organisational requirements	**Your approach to running the session**
• **MENTAL GAME**: Whole group • **ARRAY CARD LAYOUTS**: work in pairs	**Discuss the doubling strategy for the 2, 4 and 8 x tables** • After the mental game, ask the learners if they noticed anything about the way the questions on the cards related to each other. • If not, continue to explain how it is possible to work out the 4 and 8 times tables by knowing the 2 x table. (See discussion of strategies on page 9 above to guide you)

Mental: I HAVE, WHO HAS CARDS FOR 2,4,8 x TABLES – 10 minutes

- Use the *2,4, 8 I have, who has loop cards* for this session.
- Hand out one card to each learner, including yourself. If you have extras, give a second to a number of learners.
- The person who has the card labelled "START" begins by reading what is on their card. E.g. "who has 6 x 4?"
- Learners must check their cards for the answer to 6 x 4 and read what is on their card e.g. "I have 24. Who has 6 x 8?"
- The game continues until play returns to the person who started.

Game: 2, 4, 8 – 20 minutes

- Place the cards face down on table between players
- Throw the dice e.g. 
- Pick up top card 

Scores If the answer is…			e.g.
LESS THAN < 30	BETWEEN 31 and 79	MORE THAN > 80	
2 points	1 point	3 points	

- Work out the multiplication sum and write it on score card e.g. e.g. 4 x 4 = 16
- Place card to one side. Next player takes a turn. Continue until all cards have been used.
- After all the cards are used (6 turns each)
  - Look at each sum's answer and work out the score using the table above.
  - FOR EXAMPLE: 6 x 8 = 48 will score 1 point as it falls between 31 and 79
  - Write the score next to each sum on the score card

Activity: ARRAY CARD LAYOUTS 18, 24 & 36 – 30 minutes

AIM: to use card array layouts to explore factors. If you wish, you can bring in the correct mathematical terms

- Divide a pack of cards into piles of 18. If you have sufficient counters or bottle tops, you can use those instead.
- Remind learners about the structure of an array: always a rectangle or square, equal items in each row and column
- Learners must build as many different arrays as possible with ALL of their 18 cards
- After building each array, learners must write down the number of rows and columns the array has, and the total number of cards, which should always be 18.
- After all the arrays of 18 have been explored, split the cards into piles of 24 or 36 and repeat.

Arrays with 18 cards	**Arrays with 24 cards**	**Arrays with 36 cards**
18 x 1; 1 x 18 9 x 2; 2 x 9 6 x 3; 3 x 6	24 x 1; 1 x 24 12 x 2; 2 x 12 8 x 3; 3 x 8 6 x 4; 4 x 6	36 x 1; 1 x 36 18 x 2; 2 x 18 9 x 3; 3 x 9 6 x 6

IP	Maths Club Whole Session Planning Sheet	Session Twelve

Purpose of the session / object of learning	The focus here is fluency with basic multiplication facts and working with arrays and the distributive property

What resources / manipulatives will you need?	Home sharing/ Pay It Forward task
• **NUMBER SENSE**: white/blackboard or flipchart • **HOW CLOSE TO 100**: two six-sided dice, the recording sheet over the page and two different colour markers for each pair of learners, grids in plastic sleeves • **ARRAY HOORAY**: activity sheets inside plastic sleeves, kokis, cloth	Homework books
Organisational requirements	**Your approach to running the session**
• **NUMBER SENSE:** individual, then whole group • **HOW CLOSE TO 100**: pair work • **ARRAY HOORAY**: individual work	ARRAY HOORAY activity. Learners may need support with: • Sheet 1 to see the connection to doubling and the doubling strategy • Sheet 2 to be encouraged to work with the distributive property.

Number sense: MULTIPLICATION GRID – 10 minutes

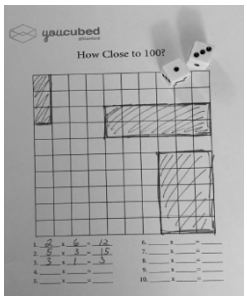
Draw this grid on the board or flipchart. Ask these questions. Answers in []

- Find 3 ways to multiply 2 numbers to get 24. *[3 and 8; 2 and 12; 6 and 4]*
- Find 2 ways to multiply 3 numbers to get 48. *[6, 4 and 2; 3, 8 and 2]*
- Which numbers are 4 times bigger than another? *[2/8; 6/24; 12/48]*
- Find 2 ways to divide 2 numbers to get 4? *[48÷12; 24÷6;12÷3;8÷2]*
- Which numbers are 3 times smaller than another? *[8/24; 12/4; 36/12]*

4	8	2
12	48	24
6	36	3

Game: HOW CLOSE TO 100 – 25 minutes

By drawing arrays, the goal is to fill up the grid to get it as full as possible, using the space as efficiently as possible. How close to 100 can you get? How many squares do you have empty?



- The first player rolls the dice and uses those two numbers to make an array on the 100 grid. FOR EXAMPLE

  Roll a  and  so draw a 2 x 6 or 6 x 2 array on the grid

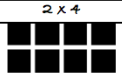
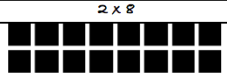
- After the player draws the array on the grid, write in the multiplication sum that describes the array at the bottom of the page. FOR EXAMPLE
  2 x 6 = 12 or 6 x 2 = 12
- The second player rolls the dice, draws the array and writes their multiplication sum
- The game ends when both players have rolled the dice and cannot put any more arrays on the grid.

VARIATIONS
Each child has a grid of its own. Who can get closest to 100?
Make grids of 400 and add more dice, or use dice with more sides
Adapt the game to let the grid represent 100% for older learners

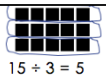
Source: Youcubed (https://www.youcubed.org/task/how-to-close-100/).

Activity: ARRAY HOORAY - 25 minutes

Sheet One: Match the multiplication fact to the array
Choose a multiplication fact to match the arrays. Then say how many altogether in each array.

Fact	How many?	Fact	How many?	Fact	How many?	What do you notice?
2 x 2 	4	2 x 4 	8	2 x 8 	16	That the array gets bigger each time Each array is double the one before

Sheet Two: Use the arrays to write different types of facts (multiplication, division and distributive)

This array shows...	Write a multiplication fact for this array	Write another multiplication fact for this array	Split the array into 2 smaller arrays and add them together. Use brackets to split up your sums.		Use the array to draw and write a division fact for this array
EXAMPLE  3 rows x 5 columns	3 x 5 = 15	5 x 3 = 15	 (3 x 2) + (3 x 3) 6 + 9 = 15	 (2 x 5) + (1 x 5) 10 + 5 = 15	 15 ÷ 3 = 5

IP	Maths Club Whole Session Planning Sheet	Session Thirteen
Purpose of the session / object of learning	The focus here is on introducing learners to the grid method of multiplication	

What resources / manipulatives will you need?	Home sharing/ Pay It Forward task
• **NUMBER SENSE**: white/blackboard or flipchart • **GRID METHOD**: Copies of the grids in plastic sleeves, kokis and cleaning cloth	Homework books, particularly Multiplication and Division book to practise the grid method.
Organisational requirements	**Your approach to running the session**
• **NUMBER SENSE:** individual, then whole group • **GRID METHOD**: Learners work alone	Learners will need a great deal of practice with using the grid method to multiply, starting with 2-digit by 1-digit problems, working up to 3-digit by 3-digit problems. Master copies of worksheets for practicing this method can be found on the SANC Project website[1].

Number sense: COMBINATIONS THAT MAKE 90 USING + and x – 20 minutes

Put these numbers up on the board / flipchart for the learners

Learners must look for combinations of numbers that **make 90 using + and x**
Example: 15 + 45 + 20 + 10

16	20	18	2
43	10	45	4
24	15	27	5

After 10 minutes, gather contributions from learners and write them on the board. Try not to judge if right or wrong – let the club do that.

Activity: GRID METHOD – 40 minutes

Introduce the GRID METHOD of multiplication

Using the information on page 8, introduce the learners to the grid method. If your learners already know how to use the long multiplication algorithm, make the connections between the grid method and the algorithm by noticing the numbers and using the array (grid) to see where these numbers arise from.

We suggest starting with 2-digit by 1-digit problems such as the one shown in the picture e.g. 16 x 6

Example
16 x 6 = 96

X	6
10	60
6	36
Answer →	96

Example
16 x 26 = 416

X	10	6	Add up ...
20	200	120	320
6	60	36	96
Answer →			416

Then allow the learners to practice on the activity sheet with these examples, or make up your own:
26 x 7
34 x 4
46 x 3
76 x 6

If learners are confident with these, you could introduce 2-digit by 2-digit examples, starting with the one shown e.g. 16 x 26. Allow them to practice with some of your examples on the activity sheet.

[1] http://www.ru.ac.za/sanc/teacherdevelopment/niclegr3-42011-2015/nicle2014/nicle2-14

IP	Maths Club Whole Session Planning Sheet	Session Fourteen
Purpose of the session / object of learning	Further practise with the grid method and practising fluency with the 3, 6 and 12 x tables using the doubling strategy.	

What resources / manipulatives will you need?	**Home sharing/ Pay It Forward task**
• **MENTAL**: set of 3, 6 and 12 I have, who has loop cards • **GRID METHOD**: Copies of the grids in plastic sleeves, kokis and cleaning cloth • **MULTIPLICATION GAME**: copies of the laminated board, 2 6-sided dice, 1 wooden dice with numbers 7 to 12	Homework books, particularly Multiplication and Division book to practise the grid method.
Organisational requirements	**Your approach to running the session**
• **MENTAL**: Whole group • **MULTIPLICATION GAME**: work in pairs or threes • **GRID METHOD**: Individual work	Learners will need a great deal of practice with using the grid method to multiply, starting with 2-digit by 1-digit problems, working up to 3-digit by 3-digit problems. Master copies of worksheets for practicing this method can be found on the SANC Project website.

Mental: I HAVE, WHO HAS CARDS FOR 3,6,12 x TABLES – 15 minutes

- Use the 3, 6, 12 I have, who has loop cards for this session.
- Hand out one card to each learner, including yourself. If you have extras, give a second to a number of learners.
- The person who has the card labelled "START" begins by reading what is on their card. E.g. "who has 3 x 3?"
- Learners must check their cards for the answer to 3 x 3 and read what is on their card e.g. "I have 9. Who has 3 x 6?"
- The game continues until play returns to the person who started.

- **NB**: After the game, remember to make the connections between these tables using the doubling strategy discussed on page 9 and in session 11.

Game: MULTIPLICATION GAME – 20 minutes

AIM: to get either an entire row/column or diagonal on the board OR 6 answers

- There are 2 different boards
- Learners share a board and 3 dice.
- 1st learner throws the dice and uses addition and multiplication only to find an answer that is on the board
- if they find an answer, they mark that answer for themselves. If they cannot find an answer pass dice to the other player
- The winning player is first to get either an entire row/column or diagonal on the board OR 6 answers

Example: throw 8, 5 and 4 with the dice
Possible answers could be:
Add 5 & 4 = 9, multiply by 8 to get **72**
Add 8 & 4 = 12, multiply by 5 to get **60**
Add 8 & 5 = 13, multiply by 4 to get **52**

Activity: GRID METHOD – 25 minutes

Work further with the GRID METHOD of multiplication

Depending on how the learners grasped the method last week, continue where you left off, or move onto 3-digit by 1-digit or 3-digit by 3-digit.

Continue to make the connection between this method and the long multiplication algorithm.

Example

163 x 6 = 978

X	100	60	3
6	600	360	18
Answer →			978

Example

163 x 16 = 2608

X	100	60	3	Add up ...
10	1000	600	30	1630
6	600	360	18	978
Answer →				2608

IP	Maths Club Whole Session Planning Sheet	Session Fifteen
Purpose of the session / object of learning	In this final session, learners will re-do the 4 operations assessment and engage with some multiplication puzzles	

What resources / manipulatives will you need?	**Home sharing/ Pay It Forward task**
• **ASSESSMENT**: 1 copy of the 4 operations script for each learner, plus 1 for yourself • **ACTIVITY**: Activity sheets in plastic sleeves, kokis, cloths, scrap paper and pencils	Homework books and puzzles
Organisational requirements	**Your approach to running the session**
• **ASSESSMENT**: Individual • **ACTIVITY**: Individual	This is the final club session, so do the assessment and then finish off with something fun.

Mental warmup: FIZZ POP WITH x10 and x100 – 5 minutes

- Start with ***x10***. Say "I will say a number and you must multiply it by 10".
  - The game starts with leader saying "FIZZ", club responds with "POP"
  - Say the number and club responds. E.g. "5" and club responds with "50"
  - Use any sequence including numbers in the hundreds.

- Repeat with ***x100 table***. Say "I will say a number and you must multiply it by 100".
  - Use any sequence
  - **NB**: Can learners see the connection between x10 and x100? What happens to the numbers? How many zeros?

Assessment – 35 minutes

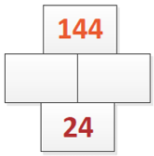
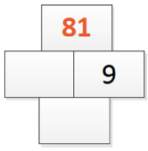
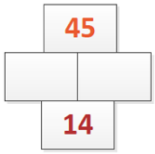
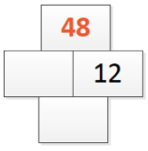
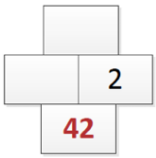
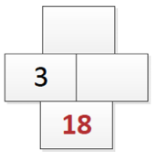
Re-administer the 4 operations assessment under the same conditions used in the first club.

Activity: ADDITION AND MULTIPLICATION PUZZLES – 20 minutes

AIM: to work out two numbers that add to bottom number and are factors of the top number.

- Learners follow instruction on the activity sheet
- Learners should be encouraged to write both the bottom and top sums on the sheet
- Once they have finished the first set, encourage the learners to try and make up some of their own.
- They can then swap these with a friend

ANSWERS

 3 x 8 = 34 3 + 8 = 11	 12 x 12 = 144 12 + 12 = 24	 9 x9 = 81 9 + 9 = 18	 9 x 5 = 45 9 + 5 = 14	 4 x 12 = 48 4 + 12 = 16	 40 x 2 = 80 40 + 2 = 42	 3 x 15 = 45 3 + 15 = 18

ACTIVITY MASTER COPIES

In this section you will find the master copies for the activities used in the planning sheets above for both the Foundation and Intermediate Phases. You may photocopy these.

To save paper, it is suggested that you copy a set for the club:

- 12 if the activity is for individual work
- 6 if the activity is for pair work

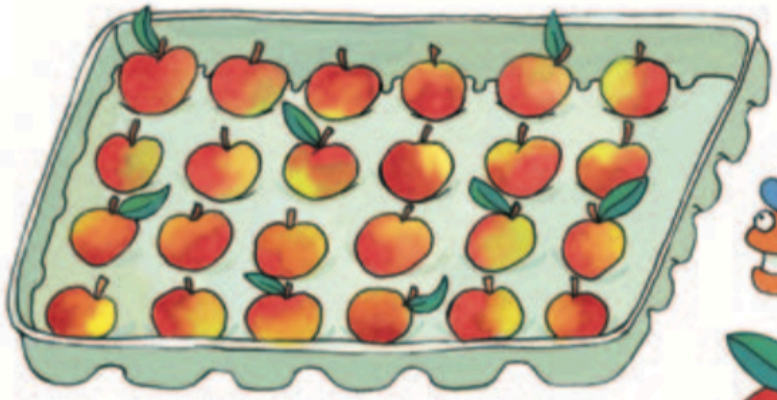
Put them into plastic sleeves (or laminate for extra durability)

Learners use dry-wipe markers to work on the sleeve.

Basic facts-Level 2-3

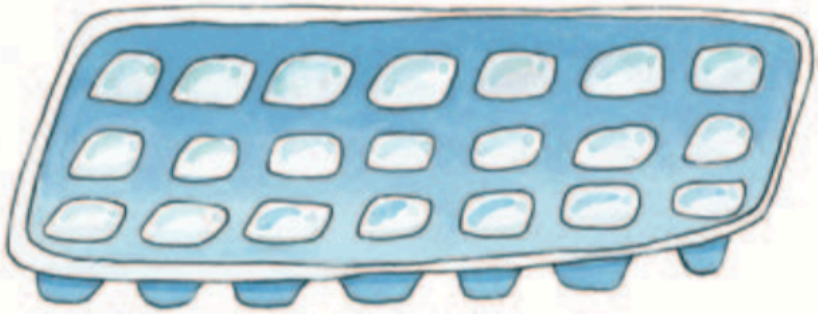
An Apple a Day

Activity One



1. How many apples are in this tray?
2. What different ways can you find to count them?
3. What would a tray holding 36 apples look like?

Activity Two



Lara worked out how many ice cubes were in this tray by going 7 x 3.

1. How else could she have counted the ice cubes?
2. Use the tray of ice cubes shown to work out the answers to:

a.	3 x 6	b.	2 x 7	c.	3 x 5
d.	7 x 1	e.	4 x 3	f.	3 x 3

Arrays 9

Source: http://www.nzmaths.co.nz/resource/apple-day?parent_node=

SANCP Tables 2, 4 & 8 Loop cards	**TABLES 2, 4 & 8 – 20 CARDS** **START** **I have 6** *Who has 6 x 4*	SANCP Tables 2, 4 & 8 Loop cards	**I have 24** *Who has 6 x 8*
SANCP Tables 2, 4 & 8 Loop cards	**I have 48** *Who has 5 x 4*	SANCP Tables 2, 4 & 8 Loop cards	**I have 20** *Who has 5 x 8*
SANCP Tables 2, 4 & 8 Loop cards	**I have 40** *Who has 4 x 4*	SANCP Tables 2, 4 & 8 Loop cards	**I have 16** *Who has 4 x 8*
SANCP Tables 2, 4 & 8 Loop cards	**I have 32** *Who has 7 x 4*	SANCP Tables 2, 4 & 8 Loop cards	**I have 28** *Who has 7 x 8*
SANCP Tables 2, 4 & 8 Loop cards	**I have 56** *Who has 9 x 2*	SANCP Tables 2, 4 & 8 Loop cards	**I have 18** *Who has 8 x 8*

SANCP Tables 2, 4 & 8 Loop cards	**I have 64** *Who has 9 x 4*	SANCP Tables 2, 4 & 8 Loop cards	**I have 36** *Who has 9 x 8*
SANCP Tables 2, 4 & 8 Loop cards	**I have 72** *Who has 10 x 8*	SANCP Tables 2, 4 & 8 Loop cards	**I have 80** *Who has 2 x 4*
SANCP Tables 2, 4 & 8 Loop cards	**I have 8** *Who has 7 x 2*	SANCP Tables 2, 4 & 8 Loop cards	I **have 14** *Who has 11 x 4*
SANCP Tables 2, 4 & 8 Loop cards	**I have 44** *Who has 11 x 8*	SANCP Tables 2, 4 & 8 Loop cards	**I have 88** *Who has 11 x 2*
SANCP Tables 2, 4 & 8 Loop cards	**I have 22** *Who has 5 x 2*	SANCP Tables 2, 4 & 8 Loop cards	**I have 10** *Who has 3 x 2*

SANCP Tables 3, 6 & 12 Loop cards	**TABLES 3, 6 & 12 – 20 CARDS** **START** **I have 66** *Who has 3 x 3*	SANCP Tables 3, 6 & 12 Loop cards	**I have 9** *Who has 3 x 6*
SANCP Tables 3, 6 & 12 Loop cards	**I have 18** *Who has 3 x 12*	SANCP Tables 3, 6 & 12 Loop cards	**I have 36** *Who has 5 x 3*
SANCP Tables 3, 6 & 12 Loop cards	**I have 15** *Who has 5 x 6*	SANCP Tables 3, 6 & 12 Loop cards	**I have 30** *Who has 5 x 12*
SANCP Tables 3, 6 & 12 Loop cards	**I have 60** *Who has 7 x 3*	SANCP Tables 3, 6 & 12 Loop cards	**I have 21** *Who has 7 x 6*
SANCP Tables 3, 6 & 12 Loop cards	**I have 42** *Who has 7 x 12*	SANCP Tables 3, 6 & 12 Loop cards	**I have 84** *Who has 8 x 3*

SANCP Tables 3, 6 & 12 Loop cards	**I have 24** *Who has 8 x 6*	SANCP Tables 3, 6 & 12 Loop cards	**I have 48** *Who has 8 x 12*
SANCP Tables 3, 6 & 12 Loop cards	**I have 96** *Who has 9 x 3*	SANCP Tables 3, 6 & 12 Loop cards	**I have 27** *Who has 9 x 6*
SANCP Tables 3, 6 & 12 Loop cards	**I have 54** *Who has 9 x 12*	SANCP Tables 3, 6 & 12 Loop cards	**I have 108** *Who has 2 x 3*
SANCP Tables 3, 6 & 12 Loop cards	**I have 6** *Who has 2 x 6*	SANCP Tables 3, 6 & 12 Loop cards	**I have 12** *Who has 10 x 12*
SANCP Tables 3, 6 & 12 Loop cards	**I have 120** *Who has 11 x 3*	SANCP Tables 3, 6 & 12 Loop cards	**I have 33** *Who has 11 x 6*

SANCP Tables Half/double up to 22	**START – 20 CARDS** **I have 19** *Who has double 6*	SANCP Tables Half/double up to 22	**I have 12** *Who has half of 10*
SANCP Tables Half/double up to 22	**I have 5** *Who has double 10*	SANCP Tables Half/double up to 22	**I have 20** *Who has half of 6*
SANCP Tables Half/double up to 22	**I have 3** *Who has double 4*	SANCP Tables Half/double up to 22	**I have 8** *Who has half of 12*
SANCP Tables Half/double up to 22	**I have 6** *Who has double 8*	SANCP Tables Half/double up to 22	**I have 16** *Who has half of 18*
SANCP Tables Half/double up to 22	**I have 9** *Who has double 5*	SANCP Tables Half/double up to 22	**I have 10** *Who has half of 14*

SANCP Tables Half/double up to 22	**I have 7** *Who has double 9*	SANCP Tables Half/double up to 22	**I have 18** *Who has half of 2*
SANCP Tables Half/double up to 22	**I have 1** *Who has double 7*	SANCP Tables Half/double up to 22	**I have 14** *Who has half of 4*
SANCP Tables Half/double up to 22	**I have 2** *Who has double 11*	SANCP Tables Half/double up to 22	**I have 22** *Who has double 6 + 1*
SANCP Tables Half/double up to 22	**I have 13** *Who has double 5 + 5*	SANCP Tables Half/double up to 22	**I have 15** *Who has double 7 + 3*
SANCP Tables Half/double up to 22	**I have 17** *Who has double 5 + 1*	SANCP Tables Half/double up to 22	**I have 11** *Who has double 9 + 1*

SANCP Tables Half/double up to 100	**START – 20 CARDS** **I have 19** *Who has double 10*	SANCP Tables Half/double up to 100	**I have 20** *Who has half of 24*
SANCP Tables Half/double up to 100	**I have 12** *Who has double 25*	SANCP Tables Half/double up to 100	**I have 50** *Who has half of 50*
SANCP Tables Half/double up to 100	**I have 25** *Who has double 3*	SANCP Tables Half/double up to 100	**I have 6** *Who has half of 16*
SANCP Tables Half/double up to 100	**I have 8** *Who has double 7*	SANCP Tables Half/double up to 100	**I have 14** *Who has half of 10*
SANCP Tables Half/double up to 100	**I have 5** *Who has double 5*	SANCP Tables Half/double up to 100	**I have 10** *Who has half of 14*

SANCP Tables Half/double up to 100	**I have 7** *Who has double 9*	SANCP Tables Half/double up to 100	**I have 18** *Who has half of 48*
SANCP Tables Half/double up to 100	**I have 24** *Who has double 8*	SANCP Tables Half/double up to 100	I have 16 *Who has half of 60*
SANCP Tables Half/double up to 100	**I have 30** *Who has double 11*	SANCP Tables Half/double up to 100	I have 22 *Who has double 15 + 6*
SANCP Tables Half/double up to 100	**I have 36** *Who has double 5 + 5*	SANCP Tables Half/double up to 100	**I have 15** *Who has double 20 + 8*
SANCP Tables Half/double up to 100	**I have 48** *Who has double 5 - 1*	SANCP Tables Half/double up to 100	I **have 9** *Who has double 10 - 1*

IP MULTIPLICATION BOARD GAME

BOARD ONE				
100	33	70	42	48
90	24	18	14	45
25	16	30	63	22
36	60	84	81	28
96	44	72	121	144

BOARD TWO				
32	22	56	84	45
18	13	96	60	44
40	72	10	48	49
90	39	108	52	36
55	12	26	100	99

IP Grid Method: 2 x 1 digit

Example

16 x 6 = 96

X	6
10	60
6	36
Answer →	96

X	
Answer →	

X	
Answer →	

X	
Answer →	

X	
Answer →	

X	
Answer →	

X	
Answer →	

X	
Answer →	

X	
Answer →	

X	
Answer →	

X	
Answer →	

X	
Answer →	

X	
Answer →	

X	
Answer →	

X	
Answer →	

X	
Answer →	

IP Grid Method: 2 x 2 digits

Example

16 x 26 = 416

X	10	6	Add up ...
20	200	120	320
6	60	36	96
Answer →			416

X			Add up ...
Answer →			

X			Add up ...
Answer →			

X			Add up ...
Answer →			

X			Add up ...
Answer →			

X			Add up ...
Answer →			

X			Add up ...
Answer →			

X			Add up ...
Answer →			

X			Add up ...
Answer →			

X			Add up ...
Answer →			

X			Add up ...
Answer →			

X			Add up ...
Answer →			

IP Grid Method 3 x 1 digit

Example

163 x 6 = 978

X	100	60	3
6	600	360	18
Answer →	978		

X			
Answer →			

X			
Answer →			

X			
Answer →			

X			
Answer →			

X			
Answer →			

X			
Answer →			

X			
Answer →			

X			
Answer →			

X			
Answer →			

X			
Answer →			

X			
Answer →			

X			
Answer →			

X			
Answer →			

X			
Answer →			

Example

163 x 16 = 2608

X	100	60	3	Add up …
10	1000	600	30	1630
6	600	360	18	978
Answer →				2608

X				Add up …
Answer →				

X				Add up …
Answer →				

X				Add up …
Answer →				

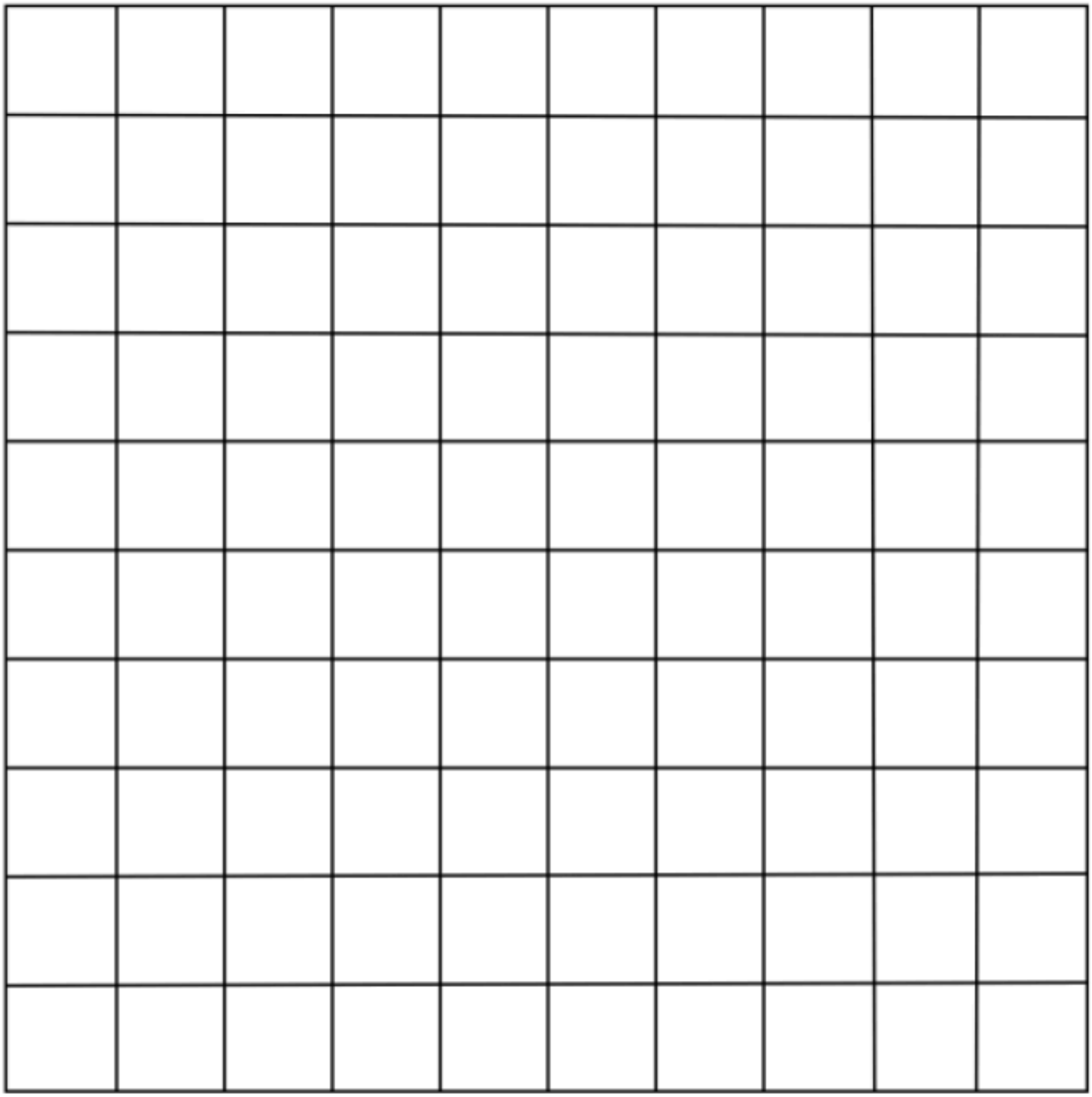
X				Add up …
Answer →				

X				Add up …
Answer →				

X				Add up …
Answer →				

X				Add up …
Answer →				

FP & IP HOW CLOSE TO 100



1. _____ x _____ = _____
2. _____ x _____ = _____
3. _____ x _____ = _____
4. _____ x _____ = _____
5. _____ x _____ = _____
6. _____ x _____ = _____
7. _____ x _____ = _____
8. _____ x _____ = _____
9. _____ x _____ = _____
10. _____ x _____ = _____

IP NUMBER PATTERN INVESTIGATIONS

Number Patterns Investigations 5 & 10

Count in fives - colour the block

Count in tens - put an X in the block

1									10
									20
									30
									40
									50
									60
									70
									80
									90
									100

Number Patterns Investigations 2, 4 & 8

Count in twos - colour the block

Count in fours - put an X in the block

Count in eights - put a line around the block

1									10
									20
									30
									40
									50
									60
									70
									80
									90
									100

Number Patterns Investigations 3, 6 & 12

Count in threes - colour the block

Count in sixes - put an X in the block

Count in twelves - put a line around the block

1									10
									20
									30
									40
									50
									60
									70
									80
									90
									100

IP NUMBER PATTERN INVESTIGATIONS continued

Find the patterns for the missing numbers and complete the grid

			12	15	18		
				20			
	10	15	20			31	
	12				33	36	39
	14						
8		24			53		59
			28	38			

Fill in the numbers for the blank squares only

101	102		104	105	106		108	109	110
	112		114		116	117	118		120
121		123		125	126	127		129	
	132			135			138		
141		143	144		146	147		149	
	152			155			158		160
161		163		165		167	168	169	170
			174		176		178		180
181	182			185	186	187		189	
	192				196		198		200
201		203	204			207		209	
211			214	215			218		

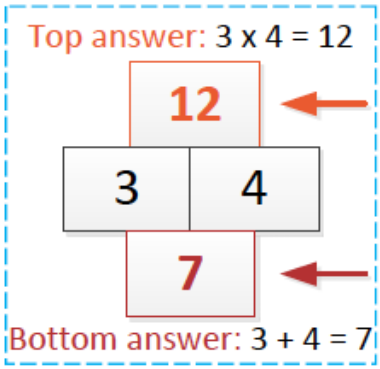
ADD & MULTIPLY PUZZLES

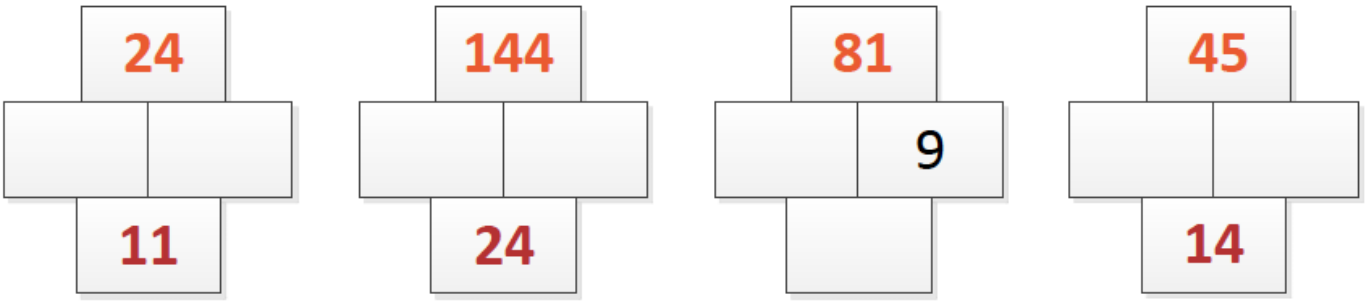
The 2 numbers in the middle

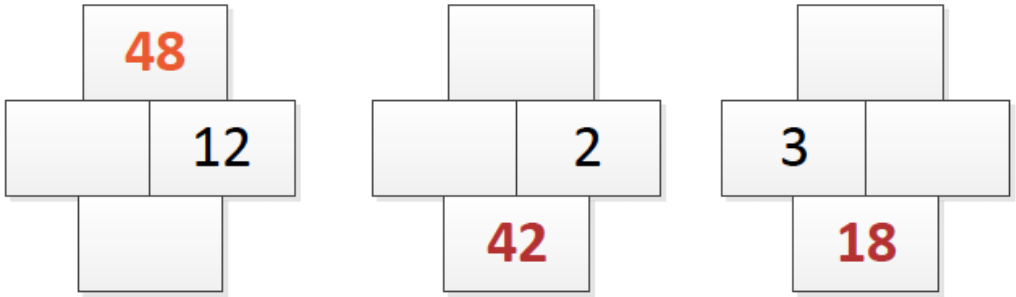
Add together to make the answer in the bottom box. In this example 3 + 4 = 7

Are multiplied to give the answer in the top box. In this example 3 x 4= 12

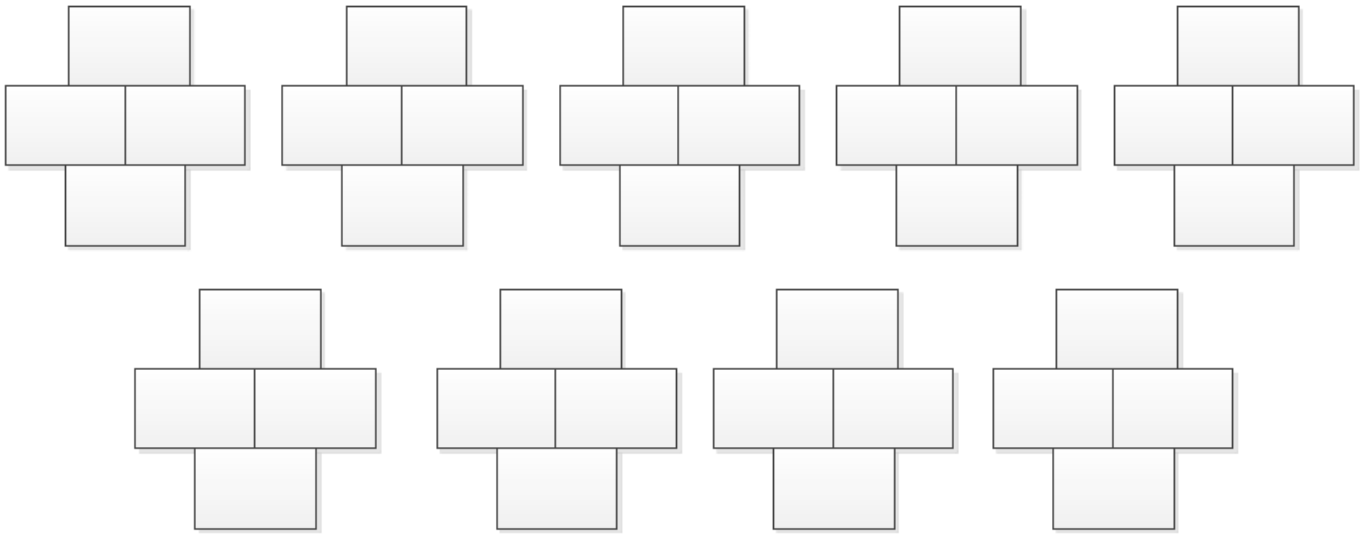
TRY THESE. The 1st 2 examples are to get you started. The next few will make you think a little more.







Now, make up some of your own.



MULTIPLICATION SQUARES

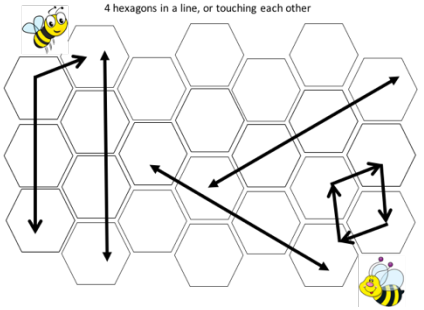
Multiply the numbers across the top with the numbers down the side. One example has been done e.g. 3 x 20 = 60. In the last one, you are given some answers and you need to work out the numbers for the row and column.

x	2	5	3
20			60
10			

x	4	5	2
25			
15			

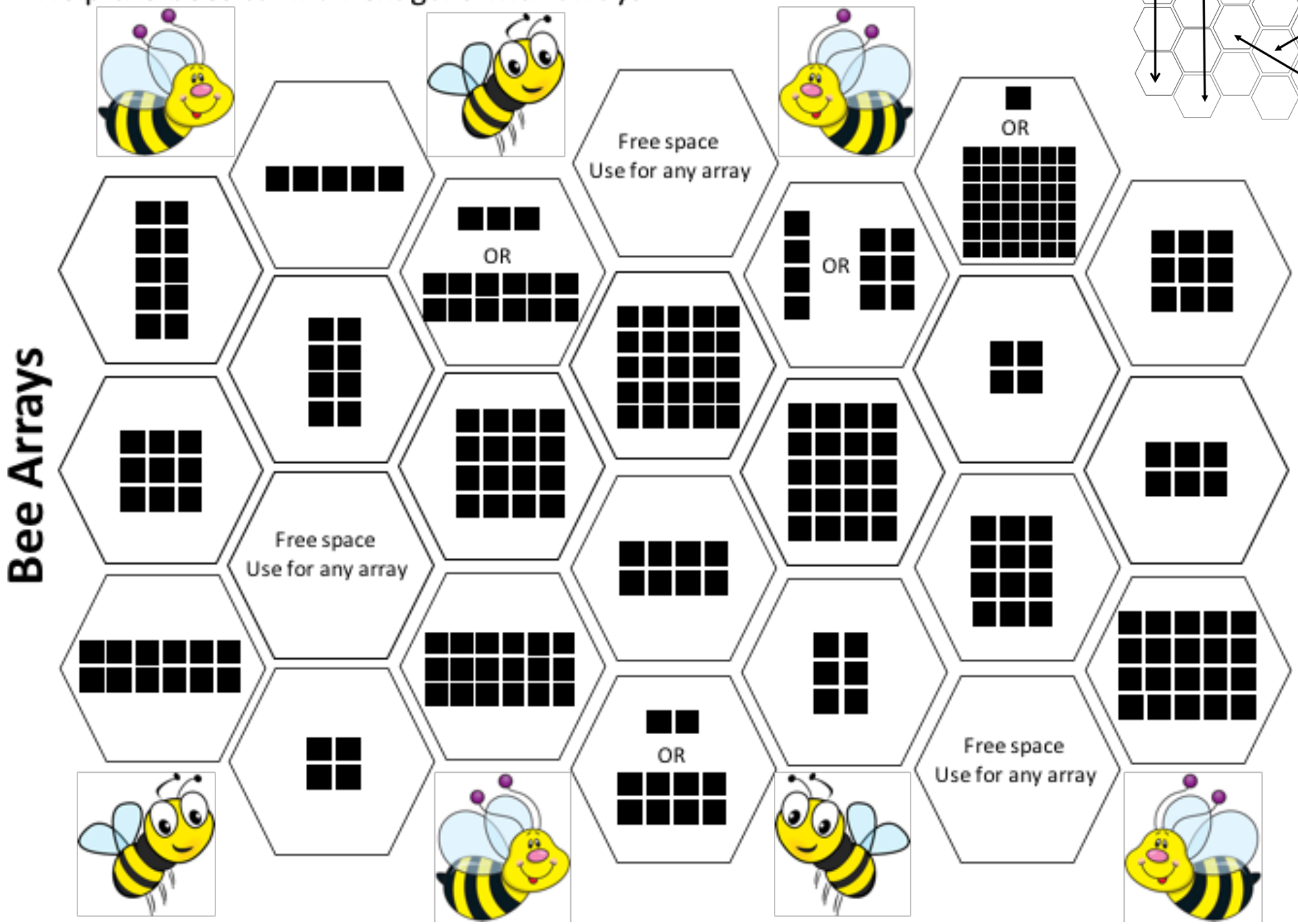
x	3		5
	60		
10		20	

x	4	5	2	3	5	10	0
5							
3		15					
1							
0							
2							0
10					50		



Help the bees to find hexagons with arrays

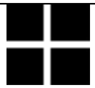
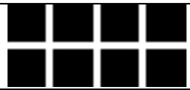
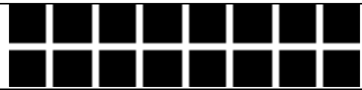
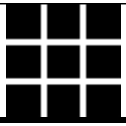
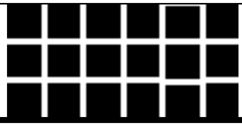
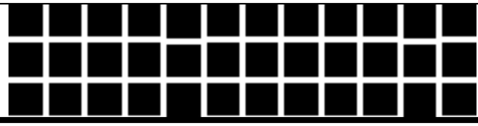
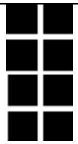
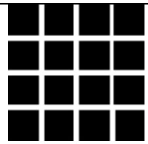
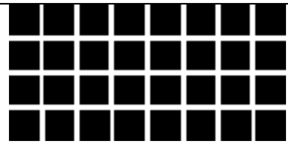
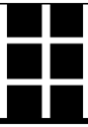
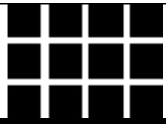
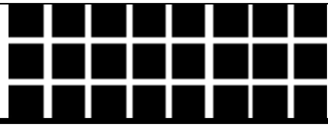
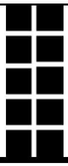
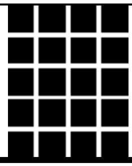
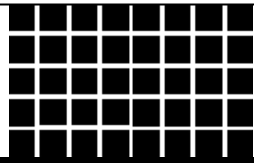
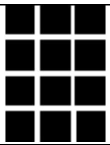
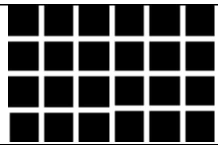
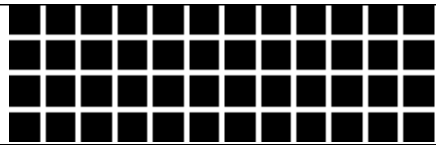
Bee Arrays



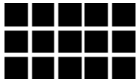
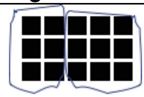
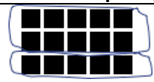
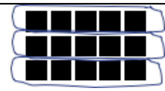
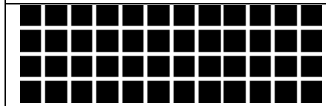
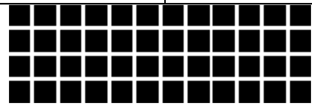
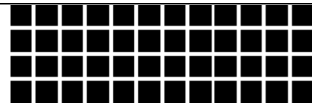
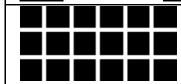
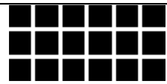
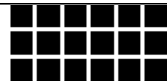
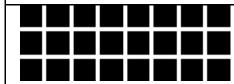
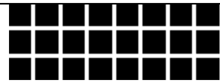
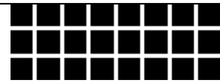
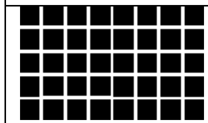
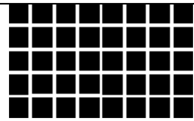
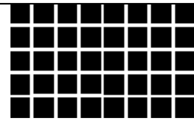
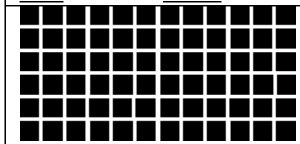
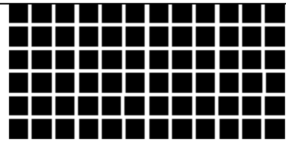
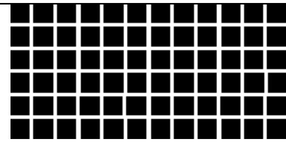
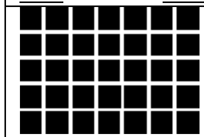
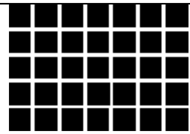
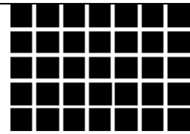
IP ARRAY HOORAY (1) - Match the multiplication fact to the array

Choose a multiplication fact to match the arrays. Then say how many altogether in each array.

3 x 3	4 x 4	3 x 12	4 x 6	2 x 8	5 x 4
4 x 2	2 x 2	4 x 3	4 x 8	5 x 2	3 x 2
3 x 4	3 x 6	2 x 4	4 x 12	3 x 8	5 x 8

Fact	How many?	Fact	How many?	Fact	How many?	What do you notice?
2 x 2 						
Fact	How many?	Fact	How many?	Fact	How many?	What do you notice?
						
Fact	How many?	Fact	How many?	Fact	How many?	What do you notice?
						
Fact	How many?	Fact	How many?	Fact	How many?	What do you notice?
						
Fact	How many?	Fact	How many?	Fact	How many?	What do you notice?
						
Fact	How many?	Fact	How many?	Fact	How many?	What do you notice?
						

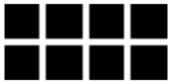
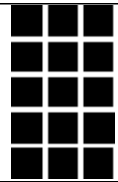
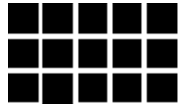
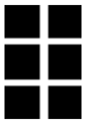
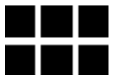
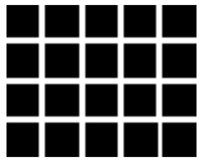
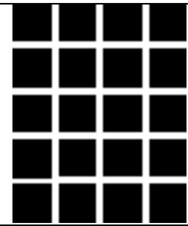
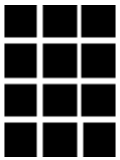
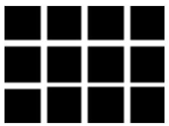
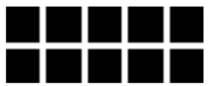
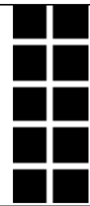
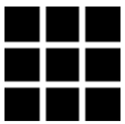
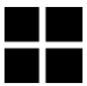
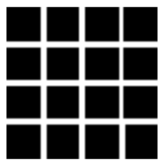
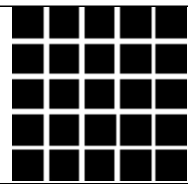
IP ARRAY HOORAY (2) - Use the arrays to write different types of facts.

This array shows…	Write a multiplication fact for this array	Write another multiplication fact for this array	Split the array into 2 smaller arrays and add them together. Use brackets to split up your sums.	Use the array to draw and write a division fact for this array
EXAMPLE  3 rows x 5 columns	3 x 5 = 15	5 x 3 = 15	 (3 x 2) + (3 x 3) 6 + 9 = 15  (2 x 5) + (1 x 5) 10 + 5 = 15	 15 ÷ 3 = 5
 ___ rows x ____ columns				
 ___ rows x ____ columns				
 ___ rows x ____ columns				
 ___ rows x ____ columns				
 ___ rows x ____ columns				
 ___ rows x ____ columns				

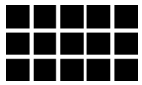
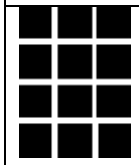
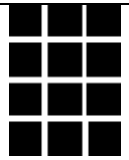
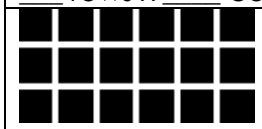
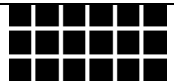
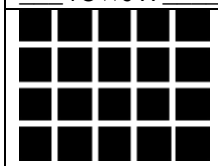
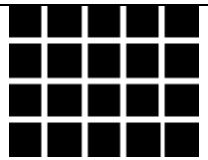
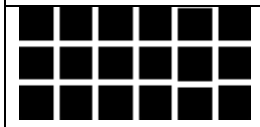
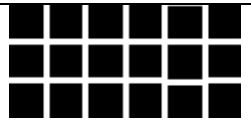
FP ARRAY HOORAY (1) - Match the multiplication fact to the array

Choose a multiplication fact to match the arrays. Then say how many altogether in each array.

4 x 3	4 x 2	3 x 4	2 x 2	4 x 5
5 x 3	3 x 2	5 x 4	2 x 4	5 x 2
2 x 5	4 x 4	2 x 3	5 x 5	3 x 5

Fact	How many?	Fact	How many?	Fact	How many?	Fact	How many?
							
Fact	How many?	Fact	How many?	Fact	How many?	Fact	How many?
							
Fact	How many?	Fact	How many?	Fact	How many?	Fact	How many?
							
Fact	How many?	Fact	How many?	Fact	How many?	Fact	How many?
							

FP ARRAY HOORAY (2) - Use the arrays to write different types of facts.

This array shows…	Write an addition fact for this array	Write a multiplication fact for this array	Write another multiplication fact for this array	Show how to split the array into equal parts
EXAMPLE  3 rows x 5 columns	3 + 3 + 3 + 3 + 3 = 15	3 x 5 = 15	5 x 3 = 15	 Split into 3 equal parts
 ___ rows x ____ columns				 Split into 4 equal parts
 ___ rows x ____ columns				 Split into 6 equal parts
 ___ rows x ____ columns				 Split into 4 equal parts
 ___ rows x ____ columns				 Split into 2 equal parts
 ___ rows x ____ columns				 Split into 2 equal parts
 ___ rows x ____ columns				 Split into 2 equal groups

IP 2, 4, 8 SCORECARDS

Look at the answer for each sum. If the answer is:

LESS THAN < 30 2 points	BETWEEN 31 and 79 1 point	MORE THAN > 80 3 points

NAME x SUM	SCORE	NAME x SUM	SCORE
TOTAL		TOTAL	

Look at the answer for each sum. If the answer is:

LESS THAN < 30 2 points	BETWEEN 31 and 79 1 point	MORE THAN > 80 3 points

NAME x SUM	SCORE	NAME x SUM	SCORE
TOTAL		TOTAL	

Look at the answer for each sum. If the answer is:

LESS THAN < 30 2 points	BETWEEN 31 and 79 1 point	MORE THAN > 80 3 points

NAME x SUM	SCORE	NAME x SUM	SCORE
TOTAL		TOTAL	

EXAMPLE COMPLETED SCORECARD

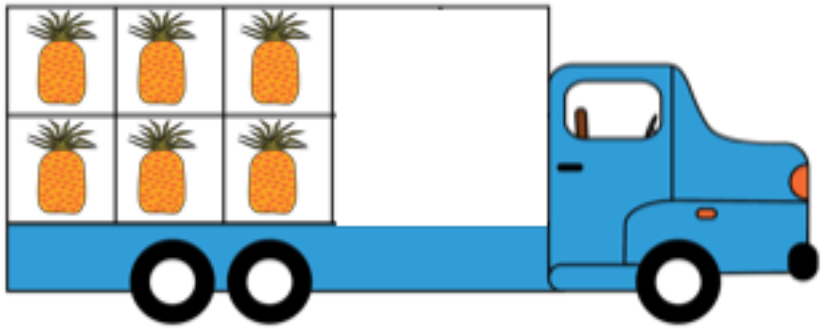
NAME Debbie **x SUM**	**SCORE**	**NAME** Thomas **x SUM**	**SCORE**
4 x 4 = 16	2	7 x 8 = 56	1
10 x 4 = 40	1	6 x 8 = 48	1
7 x 2 = 14	2	8 x 2 = 16	2
11 x 8 = 88	3	3 x 2 = 6	2
8 x 2 = 16	2	8 x 4 = 32	1
1 x 8 = 8	2	9 x 4 = 36	2
TOTAL	12	**TOTAL**	9

FIND THE ARRAYS IN THE PICTURE (1)

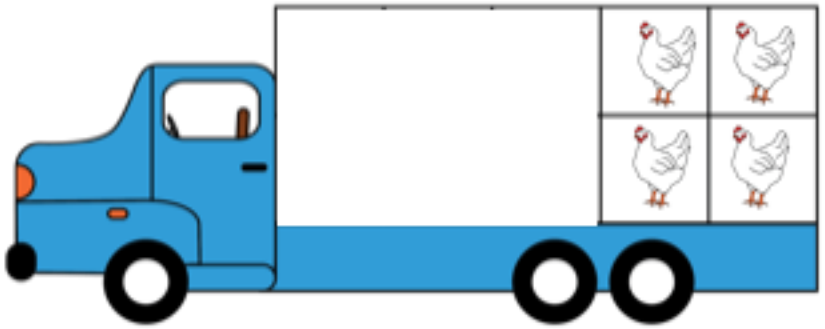
FILL IN THE GRIDS

EXAMPLE

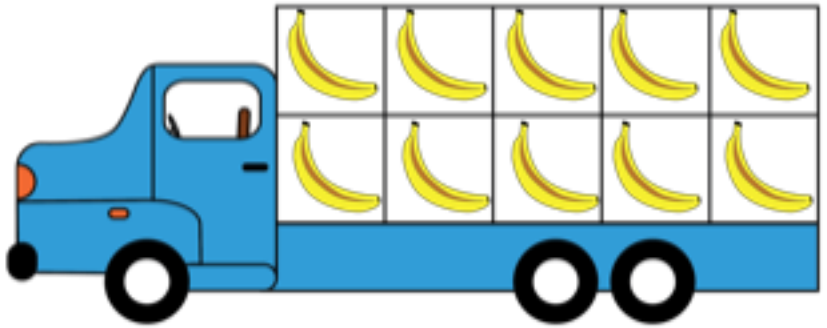
OBJECT	ROWS	COLUMNS	TOTAL
	2	2	4



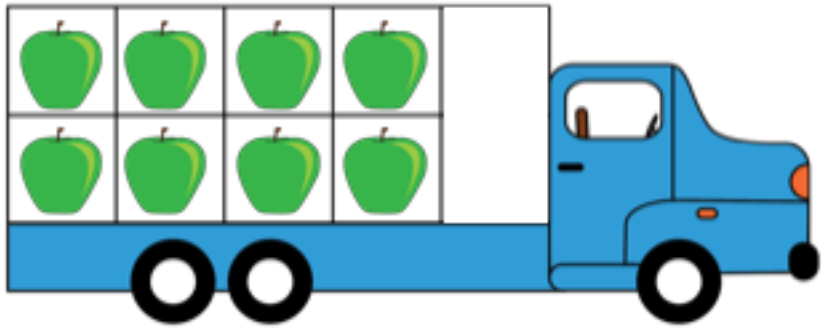
FRUIT 	ROWS	COLUMNS	TOTAL



ANIMAL 	ROWS	COLUMNS	TOTAL



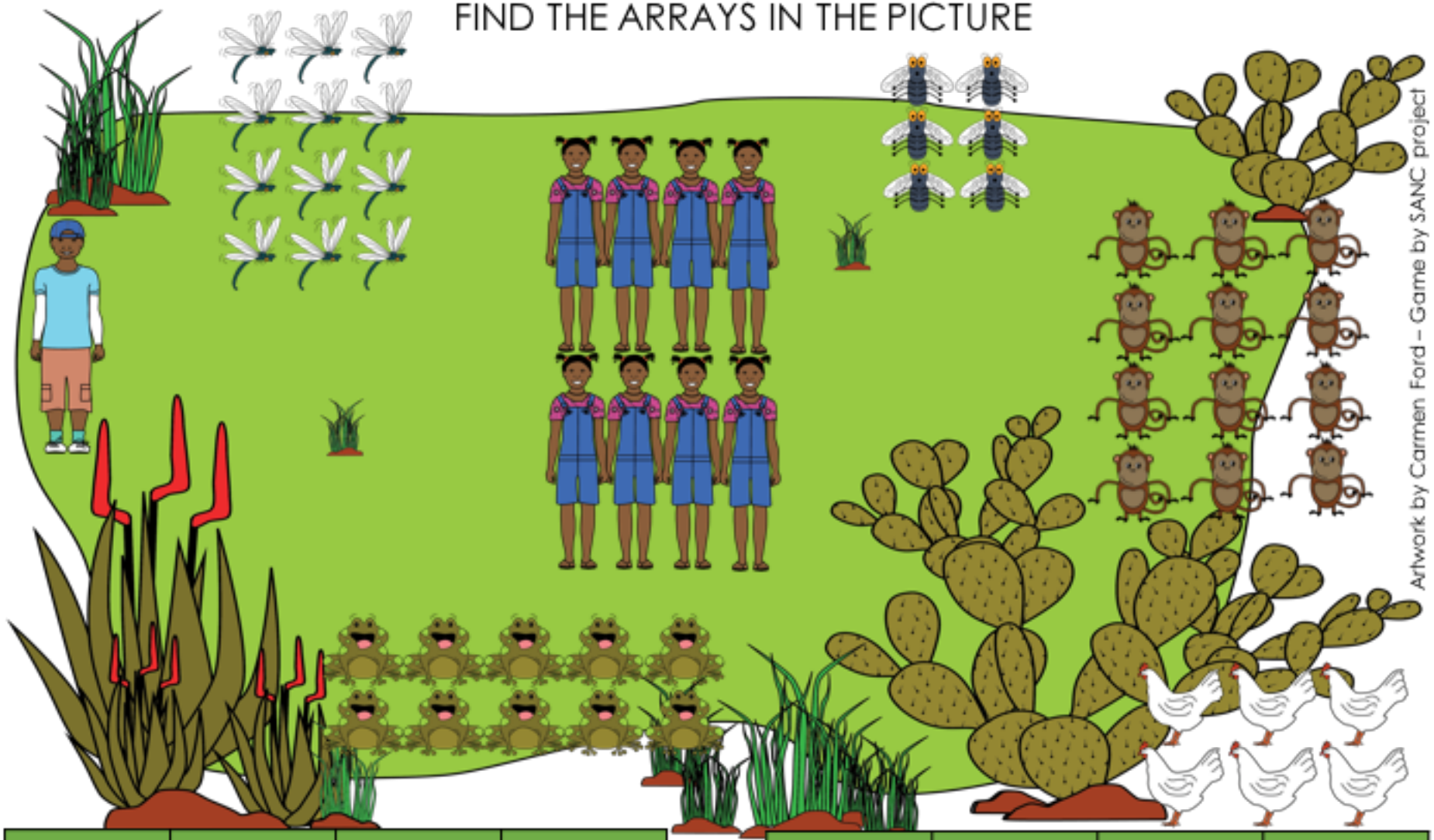
FRUIT 	ROWS	COLUMNS	TOTAL



FRUIT 	ROWS	COLUMNS	TOTAL

Artwork by Carmen Ford – Game by SANC project

FIND THE ARRAYS IN THE PICTURE



	ROWS	COLUMNS	TOTAL

FILL IN THE GRIDS

	ROWS	COLUMNS	TOTAL

FP SKIP COUNTING MAZES

HELP THE RABBIT TO FIND HIS CARROT. COUNT FORWARDS IN 2s UP TO 20

0	4	6	8	9	10	3	8	6
13	18	2	20	2	10	8	6	4
START	20	18	16	14	15	7	10	2
	2	18	17	14	1	14	7	0
	4	5	1	13	12	9	16	18
1	12	6	0	10	11	10	21	4
0	15	16	8	1	3	19	11	13
2	4	5	7	20	9	1	2	8

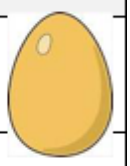
HELP THE MONKEY TO FIND HIS BANANA. COUNT FORWARDS IN 5s UP TO 50

8	4	6	8	9	10	3	8	6
6	18	2	20	2	11	6	5	20
5	10	6	18	16	14	40	10	8
START	24	15	17	14	35	10	45	50
	4	20	1	30	5	4	6	4
0	12	6	25	12	11	10	17	18
0	3	16	3	1	3	19	18	13
2	4	5	7	8	9	1	2	8

HELP THE MOUSE TO FIND HER CHEESE. COUNT FORWARDS in 10s UP TO 100

29	4	6	8	18	17	16	8	60	
25	23	10	9	19	11	15	5	10	
24	20	30	21	20	10	14	90	100	**END**
10	73	15	40	50	100	19	80	21	10
START	140	10	100	51	60	70	40	30	20
	130	120	110	85	32	12	120	130	10
	80	90	60	50	40	14	100	40	21
90	40	22	20	90	60	50	40	90	100

HELP THE CHICKEN TO FIND HER EGG. COUNT FORWARDS IN 3S UP TO 30

	4	6	8	9	16	14	12	10	0
	6	10	9	8	18	6	5	8	5
	14	13	11	15	20	21	10	6	4
	38	36	12	6	22	8	24	2	2
START	4	9	1	26	24	3	6	27	30
3	6	32	30	28	11	2	1	0	**END**
0	4	16	3	1	3	19	18	10	
2	4	5	7	8	9	1	2	8	