"Pushing for Progression" in number sense and fluency
Maths Club Development Programme
Phase II
2016

South African Numeracy Chair Project, Rhodes University

Authored by:
Mellony Graven
and Debbie Stott

Last updated: 11th May 2016

To cite this:

Graven, M., & Stott, D. (2016). *"Pushing for Progression" in number sense and fluency: Maths club development programme*. Grahamstown, South Africa: South African Numeracy Chair Project (Rhodes University).

Table of contents

Introduction

In Phase I of the project we focused on establishing after-school maths clubs for FP and IP learners in Grahamstown schools as a new initiative. Debbie Stott undertook her doctoral research in two of these clubs and also ran a number of other clubs. Other project team members and research students also ran clubs. Findings from this phase have been reported on by the Chair and show that the clubs provide enabling spaces for both recovery and extension of mathematical proficiency in learners as these spaces are free from several contextual constraints that teachers face in their classrooms. Our earlier clubs focused on working with mixed group of learners where we began with remediation work and then worked towards extending learners. In 2014 we piloted clubs that work with 'stronger learners' focused on consolidation of number knowledge and fluency and extension and challenge.

Local and national interest in our after school maths clubs is growing rapidly and Debbie Stott has run a number of teacher and conference workshops with teacher facilitators on how to start and run clubs[1]. This growing interest however comes with the challenge to engage more deeply with issues of sustainability and sphere of influence of our clubs and with facilitating change.

In Phase I publications we began exploring the issue of how our after school clubs have enabled 'agents of change' to emerge across four categories of participants, namely: learners, teachers/facilitators, teacher educators and researchers. SANC project team members, who both run clubs and work directly with teachers in the clubs, use the clubs as mini explorative spaces (or 'labs') to try out new activities and games, test theories, frameworks and assessments before sharing them with the teachers in the NICLE programme and with in-service teachers undertaking post-graduate mathematics courses (in which the SANC project is involved) at Rhodes University. Thus the clubs provide a safe space for building confidence and for trying new pedagogical approaches for the teachers and for ourselves.

While we continue to co-ordinate and run several after school mathematics clubs the focus of our club work from 2013 to 2015 has been on expanding the sphere of influence by encouraging many more teachers and educators across the Eastern Cape and beyond to start their own clubs. All sessions began with an introduction of the rationale, aims, ethos, and the role of mentors and thereafter participants experience various club activities for themselves as participators: that is sharing some fun games for use as mental maths starters and some longer activities. We had some wonderful feedback from the participants about our workshops. For example the following from ITEC:

> *"I enjoyed the workshop. It was informative and challenged some of my perspectives on teaching mathematics."*
>
> *"Excellent. The variety of approaches, ideas and inputs were truly inspiring. Really valuable."*

In Phase II of the project (2016-2020), our aim is to scale up the clubs beyond Grahamstown and the project team by offering a developmental programme to teachers in local education districts.

Rationale

Based on the research and development work that has been carried out in the clubs since 2011, we note that working with learners to focus specifically on the development of fluency and number sense over a short period of time has positive results. Thus we aim to builds on these successes with just one aspect of the club

[1] See Appendix A for details

research within the wider SANC project club programme with broader aims and outcomes. This programme aims to support teachers in setting up and running clubs of their own using a structured 15-week club programme. This programme is described below.

We have noted in schools where we run clubs, that many learners wish to join the club programme. Should teachers therefore wish more learners to be involved, they can offer two 15-week programmes a year with two different groups of learners by running the 15-week programme twice. Alternately, after the initial 15-week programme, teachers may wish to continue working with the same group of learners, extending them beyond what is covered by this programme. If this is the case, teachers may access our website and select from the broad range of activities there.

Overview of the development programme: Pushing for Progression (PfP)

South Africa currently sits with a crisis in primary education where learners are still bound by using concrete strategies to solve problems. Many learners are 'trapped' in using concrete one-to-one counting methods or dependence on algorithms without understanding. The result is an absence of flexibility and fluency with both numbers and operations.

The goal is to work with small groups of Grade 3, 4, 5 and 6 teachers in local DOE districts to set up clubs. The intention is to support these teachers to run clubs that focus on developing learners' increasingly efficient strategies in the 4 basic operations or what we term "Pushing for Progression". Teachers will be invited to attend a series of workshops which will be facilitated by the SANC project team. Each workshop will aim to provide the teachers with resources for assessment and club activities as well as an orientation to why it is important to focus on this progression. Further, in the workshops the team will work with teachers to understand how to progress the learners from concrete methods to more efficient ones.

Each teacher will be encouraged to start a club with up to 12 learners from their school / class over a 15-week period, with the goal of exploring the ideas presented in the workshops and to reflect on how these help them in progressing learners as well as how it may influence their own teaching.

"Pushing for progression": theoretical and conceptual ideas informing the development programme

A number of theoretical and conceptual ideas form the basis of the development programme. Each of these is described below and the links to how they influence the programme or how they will be used in the programme are made clear.

Mathematical proficiency and number sense

Working with Kilpatrick et al.'s (2001) strands of mathematical proficiency, the programme will focus on the two strands of procedural fluency and conceptual understanding, which are defined as:

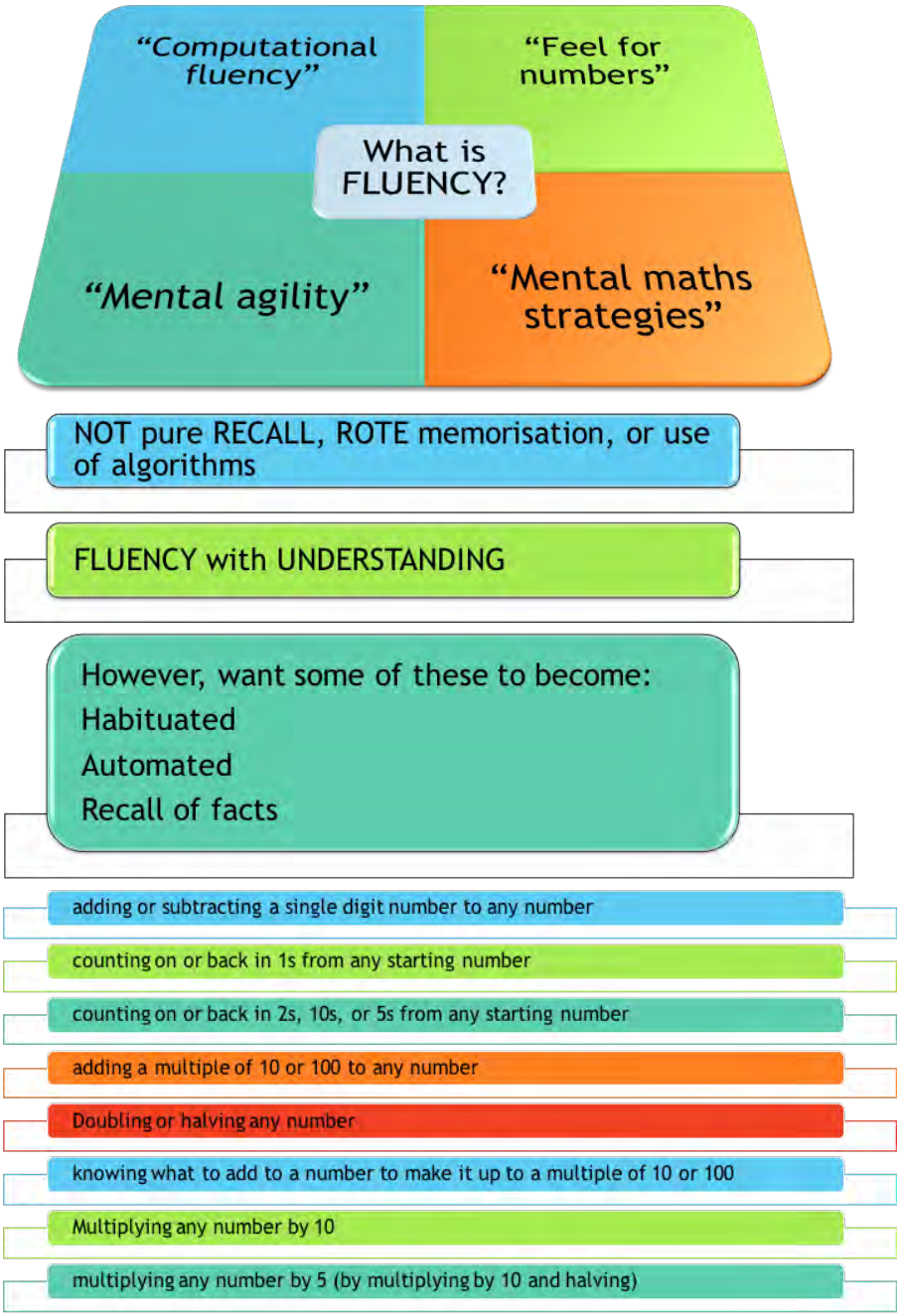
- conceptual understanding: comprehension of mathematical concepts, operations, and relations
- procedural fluency skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

These can be further expanded as:

- Conceptual understanding is the ability to use multiple representations, estimating, making connections and links and understanding properties of number systems (i.e. number sense).
- Procedural fluency is the ability to solve a problem without referring to tables and other aids, using efficient ways to add, subtract, multiply and divide mentally and on paper, understanding when it is appropriate to use procedures or not (as not all calculating situations are alike).

Number sense

Expanding on these definitions, a number sense approach can be useful to unpack the relationship between the two strands. In the context of the Foundation Phase the development of number sense includes the meaning of different kinds of numbers, the relationship between different kinds of numbers, and the effect of operating with numbers. In the Intermediate Phase this development of number sense and operational fluency should continue, with the number range, kinds of numbers, and calculation techniques all being extended. A child with number sense has the ability to work flexibly with numbers, observe patterns and relationships and make connections to what they already know, to make generalisations about patterns and processes. Number sense also includes a positive attitude and confidence (Anghileri, 2006)



Askew, M. (2012). *Transforming Primary Mathematics*. Abingdon: Routledge.

Importance for the programme

The programme aims to develop procedural fluency, conceptual understanding and number sense in the club learners as defined above.

Visualising progression

Another key idea is that of progression. A wide range of research points to the need for coherence and progression in the teaching of mathematics (Askew, Venkat, & Mathews, 2012; Schollar, 2008). However teachers are unlikely to identify useful resources or generate resources with carefully inlaid progression without a solid understanding of the level at which learners are operating and the various levels through which learners must progress in order for foundational numeracy proficiency to be sufficiently in place in order to progressively progress through the mathematics required in the IP grades. In this respect, across *a number* of research projects, we have found the work of Wright, Stafford, Stanger and Martland (2006) on delineating levels of mathematical progress in their early Learning Framework in Number (LFIN) to be particularly useful. We have used this framework not only for our analysis of learner levels of mathematical understanding in order to design learning activities but also for teacher development. Wright (2013) has argued that the interview tool from their mathematics assessment and recovery programme is useful for teacher development and understanding the developmental nature of numeracy learning. A number of researchers have however, found the LFIN difficult to use in practice. Additionally, the framework does not extend beyond Grade 3 to 4.

Our experiences of using Kilpatrick et al.'s (2001) strands of mathematical proficiency as a framework for analysing learner responses to assessment tasks has led us to develop a procedural fluency spectrum. When analysing an assessment, overlap of learner methods/responses as both procedural fluency and conceptual understanding often arise, especially when considering the dimensions of flexibility and efficiency in relation to procedural fluency. Following a procedure or method without evidence of understanding surely cannot be considered in the same way as adopting an appropriately chosen method flexibly and efficiently (Graven & Stott, 2012).

Our initial procedural fluency spectra for different assessment tasks ranged from restricted / constrained procedural fluency towards elaborated and fully flexible fluency. It became evident that as one moved to the upper end of the spectrum where flexibility and efficiency were high, conceptual understanding was increasingly intertwined with procedural fluency and the distinction between these strands became progressively blurred.

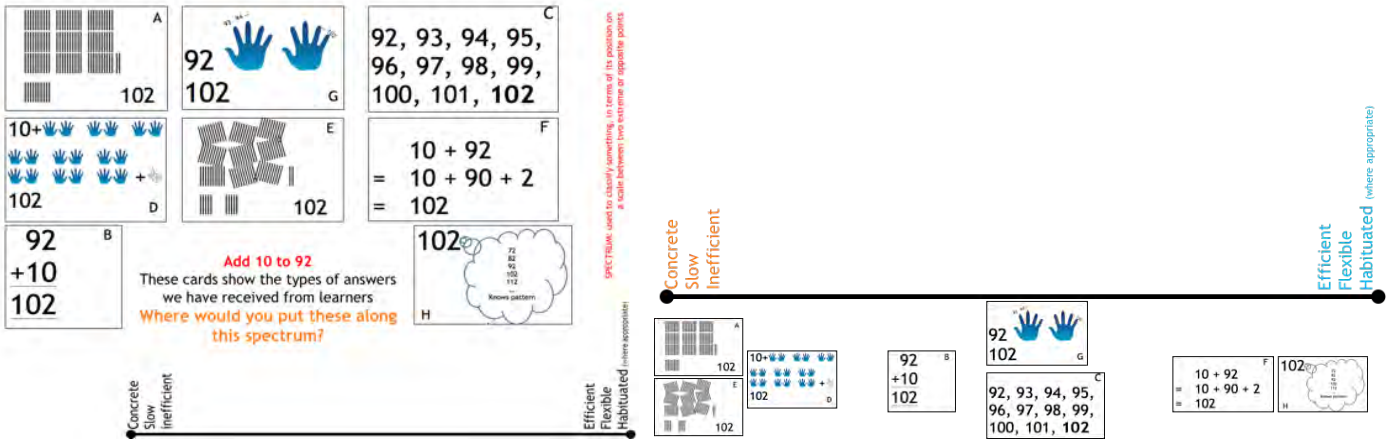


Figure 1: Example of a spectrum based on learner methods for solving the problem "add 10 to 92"

Subsequently, a number of research studies emanating from the SANC project have used spectra to analyse learner responses in different ways. Mofu (2013) for example adapted the procedural fluency spectrums to create a spectrum representing progression in early multiplication, whilst Young (2016) and Wasserman (2015) both developed spectra for understanding progression particularly in Early Arithmetic Strategies (Wright, Ellemor-Collins, & Tabor, 2012).

Importance for the programme

Spectra, which align with the progression framework will be provided for the teachers that highlight increasingly efficient strategies for a number of situations involving the 4 operations of addition, subtraction, multiplication and division.

Types of practice

Treffers (in van den Heuvel-Panhuizen, 2008) distinguishes between two forms of practice that are useful for mathematics teaching: *reproductive practice* and *productive practice* (p. 51). Reproductive practice can be thought of as a focus on automation of skills and memorisation of basic facts for numbers up to 20. Reproductive practice can involve the practice of both context-based tasks and raw problems. Productive practice on the other hand can be thought of as more "indirect and problem-linked" (p.51), where the tasks are more open, solutions and answers allow for differentiation and which require the learner to show more initiative in solving the problem.

Askew (2002) also advocated that learners can be encouraged to engage in different types of practice to develop fluency: guided practice when learners still need the help of a more experienced other to keep them on track; over-practice where learners practice the same thing every day and finally deliberate practice where what learners practice is made harder by setting a time limit or by working with bigger numbers. The core of deliberate practice seems to be continually stretching the learner to just beyond his or her current abilities (Askew, 2002; Pegg, 2010).

Guided practice and over-practice can be linked to the idea of reproductive practice whilst deliberate practice can be thought of as aligning with productive practice. For the purposes of this programme, we will use the terms reproductive practice and productive practice.

Importance for the programme

During the programme, reproductive practice for the learners will take place through the playing of fun dice and card games, other club games such as Fizz Pop and by independent activity provided by the Tailored Independent Activity (TIA) books. Based on the results of the initial baseline assessments, teachers will be able to provide a book or series of TIA books to the learner. Productive practice will be encouraged via math talk between the teachers and learners in the club sessions by sharing ideas and methods for solving problems. Puzzle type activities will also be used for productive practice to encourage the learners to be creative in how they approach solving the puzzles.

"Pushing for progression": development programme

Aims

- learn more about the development of FP and IP learner's early number skills along a learning pathway
- identify when children are learning securely along this pathway through effective assessment and focused teaching
- learn more about the mathematics involved in early number skills and where this can underpin future mathematical knowledge and understanding

Participants

The aim is to invite 10 FP or IP teachers in a district who will each start a club with up to 12 learners at their schools. They will run one club per week for approximately 1 hour.

Timescale

This is at minimum a 16-week programme, which equates to approximately two school terms (taking into account public holidays). The first workshop session will take place one week before the programme starts to give teachers a complete overview of the programme and set them up to run the first three clubs. Teachers will then attend another workshop which will focus in the next 6 club sessions. A third workshop will support the teachers to the final 6 sessions. The workshops will take place as per the plan below.

Teacher Resources

Pedagogic	Activity-based	Manipulative resources			Assessment
Facilitators Handbook including spectra 15-week programme plan Tailored Independent Activity (TIA) books (1 set per learner)	Activities and games for each club session	Playing cards White 1-6 dice 1-12 dice 1-20 dice Place value dice pack	6 25 6 6 6	packs dice dice dice packs	Pre and post assessment tasks Marking schedules and profiling forms based on spectra to determine learner progression

Learner Resources

Playing cards	1 pack
White 1-6 dice	2 dice
1-12 dice	1 dice
1-20 dice	1 dice
TIA books	As required

16-week development programme schedule

Weekly timetable	Week 0	Weeks 1 to 3	Week 3	Weeks 4 to 9	Week 9	Weeks 10 to 15
Event(s)	Workshop One	Run 3 weekly club sessions	Workshop Two	Run 6 weekly club sessions	Workshop Three	Run 6 weekly club sessions
Overview	Orientation Why progression? Introduce spectra Assessments and profiling In-depth look at the the programme for 1st 3 clubs	1st session: administer assessment and profile learners	In-depth look at the the programme for next 6 clubs with a focus on addition and subtraction	On-going informal assessment and profiling of learner progress	In-depth look at the the programme for next 6 clubs with a focus on multiplication and division	On-going informal assessment and profiling of learner progress In 15th session: re-administer assessment and re-profile learners
Resources	Handbook including and spectra 15-week programme plan Assessment tasks, marking and profiling forms BEETLE game scorecards SANC Project Dice and Card games booklet Dice and cards for facilitators		Addition and subtraction games and activities for next 6 clubs Dice and cards for learners TIA packs for learners		Multiplication and division games and activities for next 6 clubs Multiplication game and dice	

References

Anghileri, J. (2006). *Teaching number sense* (2nd ed.). London: Continuum International Publishing Group.

Askew, M. (2002). Time to face facts. *Teach Primary*, *162*(2), 43–45.

Askew, M., Venkat, H., & Mathews, C. (2012). Coherence and consistency in South African Primary Mathematics lessons. In T. Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 27–34). Taipei, Taiwan: PME.

Graven, M., & Stott, D. (2012). Conceptualising procedural fluency as a spectrum of proficiency. In S. Nieuwoudt, D. Laubscher, & H. Dreyer (Eds.), *Proceedings of 18th Annual National Congress of the Association for Mathematical Education of South Africa (AMESA)* (pp. 146–156). Potchefstroom: North-West University.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington DC: National Academy Press.

Mofu, Z. A. (2013). *An investigation of a mathematics recovery programme for multiplicative reasoning to a group of learners in the South African context: a case study*. Rhodes University, South Africa.

Pegg, J. (2010). Promoting the acquisition of higher-order skills and understandings in primary and secondary mathematics. In *Teaching Mathematics? Make it count: What research tells us about effective teaching and learning of mathematics* (pp. 35–38). Melbourne: ACER Press.

Schollar, E. (2008). *Final Report: Short version The Primary Mathematics Research Project 2004-2007 - Towards eveidence-based educational development in South Africa*. Johannesburg.

van den Heuvel-Panhuizen, M. (2008). *Children learn mathematics: A learning-teaching trajectory with intermediate attainment targets for calculation with whole numbers in primary school*. (M. van den Heuvel-Panhuizen, Ed.). Rotterdam: Sense Publishers.

Wasserman, A. (2015). *Investigating a mathematics recovery program for assessment and intervention with groups of grade 4 learners*. Rhodes University, Grahamstown.

Wright, R. J. (2013). Assessing early numeracy: Significance, trends, nomenclature, context, key topics, learning framework and assessment tasks. *South African Journal of Childhood Education*, *3*(2), 21–40.

Wright, R. J., Ellemor-Collins, D., & Tabor, P. D. (2012). *Developing number knowledge: Assessment, teaching & intervention with 7-11-year olds*. Los Angeles: Sage Publications.

Wright, R. J., Martland, J., & Stafford, A. K. (2006). *Early numeracy: assessment for teaching and intervention*. London: Sage Publications Ltd.

Young, C. (2016). *Adaptation of the Mathematics Recovery programme to facilitate progression in the early arithmetic strategies of Grade 2 learners in Zambia*. Rhodes University, Grahamstown.

Appendix A - Previous workshops

- We ran a workshop entitled "Mathematics games for Foundation Phase learners" in August in Port Elizabeth as part of National Science Week for 80 learners from 2 pm – 4 pm.
- **Introductory Maths Club Workshop for 240 teachers in the Port Elizabeth District**. June 2014. Presented by Debbie and supported by Varonique.
- **Introductory Maths Club Workshop for 23 Eastern Cape District Subject Advisors** in Port Elizabeth. February 2014. This workshop was organized by Zanale Mofu who is now the Eastern Cape Mathematics Curriculum Advisor for Foundation Phase and one of our graduated Masters students.
- **Grahamstown Child Welfare Maths Club Workshop for 7 staff and volunteers** from three Child Welfare Community Programmes being, Ikhaya Losizo (Cluster Foster Homes for children found in need of care); Nompumelelo Pre-School and the Sun City After-Care Programme.
- October 2013, **ITEC in East London** asked us to run a 'Starting a maths club' workshop for **12** of their staff and lecturers from the Education Department at Fort Hare. This workshop was arranged by someone who had attended a workshop at the SARAECE conference in September 2013.
- September 2013: Debbie, Nombelelu Koliti and Yaleka Mase (from Samuel Ntsiko school in Grahamstown) ran a games with cards and dice at the **SARAECE** regional conference in East London for about **40 people**. It was at this conference that Barbara Valentine from ITEC expressed an interest in using the club model in her organisation.
- June 2013: Debbie and a team of educators ran a games with cards and dice at the **AMESA** conference in Cape Town for about **50 teachers**. The games featured are those we use often in clubs, but we are also hoping that people will try them in classrooms too!
- On 21st May 2013, we ran our **first public after-maths club introductory** workshop in Grahamstown. We advertised the workshop in our local newspaper, Grocotts as a way of opening up the idea to the broader Grahamstown community. The ad was accompanied by a lovely maths club article[2] written by Grocotts reporter Hancu Louw. **12 people from 3 organisations** attended the workshop: educators from Alexandria Primary and Fikizolo School and facilitators from the Lebone Centre. Prof Graven gave an introduction to the workshop. Debbie Stott facilitated the remainder of the workshop with assistance from Varonique Sias.

  All sessions begin with an introduction of the rationale, aims, ethos, and the role of mentors and thereafter participants experience various club activities for themselves as participators: that is sharing some fun games for use as mental maths starters and some longer activities. We had some wonderful feedback from the participants about our workshops. For example the following from ITEC:

  "I enjoyed the workshop. It was informative and challenged some of my perspectives on teaching mathematics."

  "Excellent. The variety of approaches, ideas and inputs were truly inspiring. Really valuable."

[2] http://www.ru.ac.za/media/rhodesuniversity/content/sanc/documents/Grocotts_30th_April_2013-Club_Invite.pdf

Appendix B – 4 operations assessment instrument

SA NUMERACY CHAIR PROJECT

SANC PROJECT

4 OPERATIONS ASSESSMENT

MATHEMATICS - CLUBS

Learner Information				30 minutes		Date	
Surname				First Name			
Grade		Gender		M ☐	F ☐	Age	
Club Leader				Club Venue			
SCORE OUT OF 20							

Question 1: Calculate			
1.1	3 + 4 =	1.2	8 + 6 =
1.3	23 + 18 =	1.4	55 + 67 =
1.5	104 + 97 =		

Page score

Adapted from Brombacher & Associates confidential test based on the EGMA (RTI) test. Page 1

Question 2: Calculate			
2.1	8 – 2 =	2.2	12 – 5 =
2.3	23 – 18 =	2.4	467 – 43 =
2.5	305 – 97 =		

Page score

Question 3: Calculate			
3.1	2 x 4 =	3.2	5 x 3 =
3.3	12 x 4 =	3.4	24 x 6 =
3.5	120 x 15 =		

Page score

Question 4: Calculate			
4.1	6 ÷ 3 =	4.2	18 ÷ 2 =
4.3	24 ÷ 3 =	4.4	75 ÷ 3 =
4.5	120 ÷ 15 =		

Page score

Appendix C – 4 operations marking and profiling forms

Addition and subtraction assessment

Club name			No. of learners	Note 1		Notes			
Date of assessment			Grade						

Spectrum

	Constrained methods	Less constrained	Semi fluent methods	Flexible fluency
	Inefficient (I)	*Somewhere in between (IE)*		*Efficient (E)*
	Use of fingers, tally marks, circles, drawings of any kind	Breaking down into place value, using some kind of expanded notation	Another strategy such as splitting, working with a friendly number	Use of known addition and subtraction facts, appropriate use of algorithms for 2 and 3 digit problems

Assessment Profile

Questions	Answers	Constrained methods		Less constrained methods		Semi fluent methods		Flexible fluency		Overall	
		Wrong answer	Correct answer	Wrong answer	Correct answer	Wrong answer	Correct answer	Wrong answer	Correct answer	No. of CORRECT answers	Predominant strategy used (I, IE or E)
Addition Questions											
A. Q1											
3 + 4	7	𝍸 𝍸 ||	𝍸 𝍸 𝍸 | Note 2					|||	𝍸 𝍸	31 Note 4	I Note 5
Totals		12	16					3	15		
A. Q2			Note 3								
8 + 6	14										
Totals											
A. Q3											
23 + 18	41										
Totals											
A. Q4											
55 + 67	122										
Totals											
A. Q5											
104 + 97	201										
Totals											
Subtraction Questions											
S. Q1											
8 - 2	6										
Totals											
S. Q2											
12 - 5	7										
Totals											
S. Q3											
23 - 18	5										
Totals											
S. Q4											
467 - 43	424										
Totals											
S. Q5											
305 - 97	208										
Totals											

NOTE:	**Learners using fingers to calculate answers**
	Observe learners as they are writing the assessment. Note learner intials on a blank 4 operations script under the relevant question number
	Then allocate a tally mark to the Constrained Methods column for that learner

Multiplication and division assessment

Club name			No. of learners	Note 1		Notes			
Date of assessment			Grade						

Spectrum

	Constrained methods	Less constrained	Semi fluent methods	Flexible fluency
	Inefficient (I)	*Somewhere in between (IE)*		*Efficient (E)*
	Use of fingers, tally marks, circles, drawings of any kind	Skip counting, repeated addition	Arrays, breaking down into expanded notation	Use of known multiplication and division facts, appropriate use of algorithms for 2 and 3 digit problems

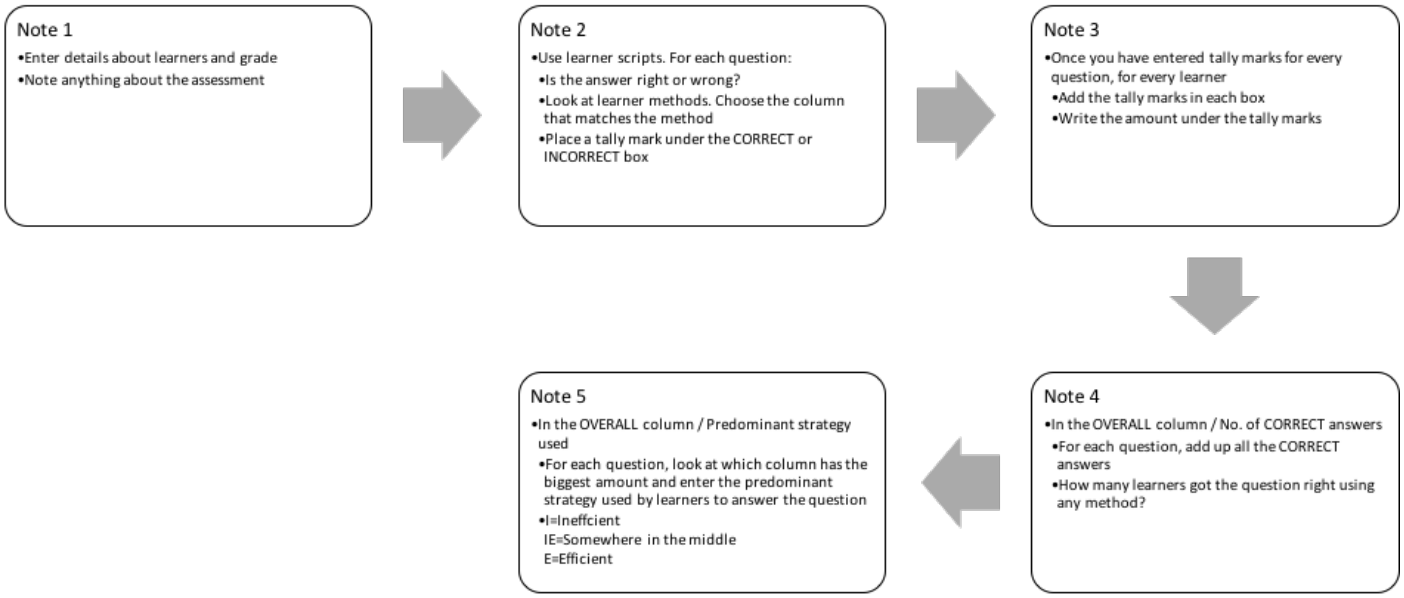
Assessment Profile

Questions	Answers	Constrained methods		Less constrained methods		Semi fluent methods		Flexible fluency		Overall	
		Wrong answer	Correct answer	Wrong answer	Correct answer	Wrong answer	Correct answer	Wrong answer	Correct answer	No. of correct answers	Predominant strategy used (I, IE or E)
Multiplication Questions											
M. Q1											
2 x 4	8	7	9					3	15	31	I
Totals		12	16					3	15	Note 4	Note 5
M. Q2											
5 x 3	15										
Totals											
M. Q3											
12 x 4	48										
Totals											
M. Q4											
24 x 6	144										
Totals											
M. Q5											
120 x 15	1800										
Totals											
Division Questions											
D. Q1											
6 ÷ 3	2										
Totals											
D. Q2											
18 ÷ 2	9										
Totals											
D. Q3											
24 ÷ 3	8										
Totals											
D. Q4											
75 ÷ 3	25										
Totals											
D. Q5											
120 ÷ 15	8										
Totals											

Note 2

Note 3

NOTE:	Learners using fingers to calculate answers
	Observe learners as they are writing the assessment. Note learner intials on a blank 4 operations script under the relevant question number
	Then allocate a tally mark to the Constrained Methods column for that learner


Note 1
•Enter details about learners and grade
•Note anything about the assessment
Note 2
•Use learner scripts. For each question:
•Is the answer right or wrong?
•Look at learner methods. Choose the column that matches the method
•Place a tally mark under the CORRECT or INCORRECT box
Note 3
•Once you have entered tally marks for every question, for every learner
•Add the tally marks in each box
•Write the amount under the tally marks
Note 4
•In the OVERALL column / No. of CORRECT answers
•For each question, add up all the CORRECT answers
•How many learners got the question right using any method?
Note 5
•In the OVERALL column / Predominant strategy used
•For each question, look at which column has the biggest amount and enter the predominant strategy used by learners to answer the question
•I=Ineffcient
IE=Somewhere in the middle
E=Efficient