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NICLE PRESENTATION ON

DEVELOPING FLUENCY IN MULTIPLICATION AND DIVISION IN PRIMARY SCHOOL LEARNERS

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MATHEMATICS CURRICULUM PLANNER: MOFU Z.A

OUTLINE OF THE PRESENTATION

- Problem with multiplication
- Mathematical proficiency
- What is multiplication?
- What is multiplicative reasoning?
- Maths Recovery Program
- Video of pre Interviews for group A and B, and reflection
- Activities on multiplication
- Video of post Interviews and reflections for group A and B, and reflection
- Overall discussions on the impact of MR programme

Problem with multiplication

- I found that my learners struggle with multiplication. Multiplicative teaching is introduced in the early grades at the beginning of Grade 1 as repeated addition as indicated in CAPS and continues up to upper grades. At Grade 3 learners should have developed more efficient strategies for calculations, which would indicate that they are progressing across the grades. The multiplication tables are stressed in Grade 3.
- It is evident that some learners lack the ability to make the shift from concrete counting based strategies to more abstract strategies, i.e. learners cannot find the answer to a multiplication problem without using concrete objects (either counting with fingers or using tallies or small circles). Learners seem to lack multiplicative reasoning strategies.

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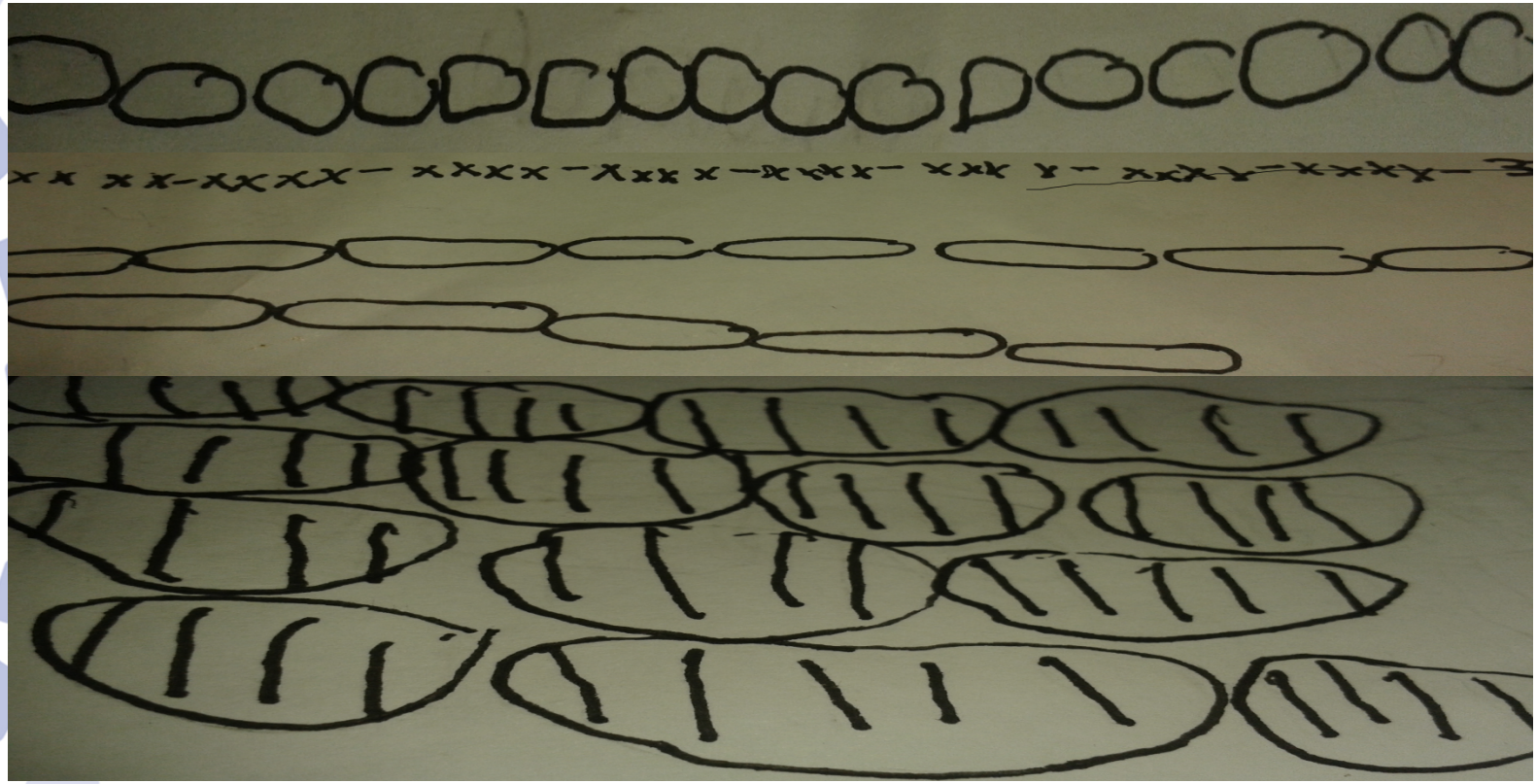

MATHEMATICAL PROFICIENCY

- **Mathematical proficiency provides a way to think about mathematics learning in that it encompasses the keys of knowing and doing mathematics.**
  - ***Conceptual understanding: an integrated and functional grasp of mathematical ideas;***
  - ***Procedural fluency: a knowledge of procedures,***
  - ***Strategic competence: the ability to formulate mathematical problems, represent them and solve them;***
  - ***Adaptive reasoning: the ability to think logically and to justify and prove the correctness of mathematical procedures;***
  - ***Productive deposition: the ability to see sense in mathematics and encourage learners to believe that they are capable of learning it***

What is multiplication?

- Some researchers consider multiplication as a faster way of doing repeated addition while others say that repeated addition is an implicit, unconscious and primitive intuitive model for multiplication (Clark & Kamii, 1996).
- (Anghileri, 1989) indicated that addition forms the basis of multiplication, addition theory processes support the learner to transfer from counting meaning to the cardinal meaning.
- Clark and Kamii (1996) used Piaget's 1987 work as a point of reference which shows that multiplication is not just a faster way of doing addition but is an operation that requires higher-order multiplicative thinking that children construct out of their ability to think additively.

What is multiplication? cont.

- Piaget differentiated addition from multiplication in that addition is the construction of number which is accomplished by the repeated addition of ones, whereas multiplication is a more complex operation that is constructed out of addition at a higher level of abstraction
- The level of abstraction shows the developmental trajectory of learners being able to solve problems from a concrete level of using manipulative to an entirely abstract level where the learner uses verbal arithmetic.

What is multiplicative reasoning

- Multiplicative reasoning is characterised by a capacity to work flexibly and efficiently with an extended range of numbers
- It requires the ability to recognise and use strategies to solve a range of problems involving multiplication or division (Mulligan & Mitchelmore, 1997).
- The learner must have the means to communicate multiplication effectively in a variety of ways for example, words, diagrams and symbolic expressions.
- Teachers need to have mental image of a developmental trajectory along which they could expect children to develop and to understand the nature of multiplicative thinking in order to support children along the path of gradual sophisticated multiplicative reasoning (Wright et al., 2006).

What is multiplicative reasoning? Cont.

- Anghileri (1989) described how learners should develop multiplicative reasoning in that they progress in stages from unitary counting or counting by ones, skip counting and repeated addition by understanding multiplication facts and their application.
- Mulligan (2002) highlighted that learners move from one-to-one counting, additive composition, many-to-one counting, multiplicative relations, and operating on the operators.
- Mulligan (2002) showed multiplicative reasoning as a mathematical structure which is described as spatial organization of objects such as arrays and squares, and that these are ways of promoting multiplicative reasoning where the whole and equal groups are reinforced by visual images.

MATHEMATICS RECOVERY PROGRAMME (MR)

- Wright et al. (2006) have developed (MR) programme framework of intervention as a learning pathway in an effort to increase learner achievement in number concepts
- The MR programme has been applied to a multitude of situations and contexts (Wright et al., 2006)
- MR includes assessment interviews (including tasks and schedules), a teaching framework and teaching resources and a learning progression model for early number learning.
- The use of a framework is to enable profiling of student's current knowledge and levels that indicate numeracy development of learner's knowledge (Wright et at., 2006), for a learner to be able to develop multiplicative reasoning he must go through different stages of cognitive development.

MR Programme: Learning Framework in Number

- Level 1: Initial Group, a learner uses perceptual thinking to establish the numerosity of collections of equal groups when items are visible and counts by ones not in multiples
- Level 2: Perceptual Counting in Multiples, a learner uses multiplicative counting strategies to count visible items in equal groups that involve counting in multiples and counting strategies but still relies on visible items.
- Level 3: Figurative Composite Grouping, a learner uses multiplicative counting strategies to count items in equal groups in cases where the individual items are not visible.
- Level 4: Repeated Abstract Composite Grouping, the learner counts composite units in repeated addition or subtraction, the learner is simultaneously aware of both the composite and unitary aspects.
- Level 5: Multiplication as Operations, a learner can regard both the number in each group as a composite unit, and can immediately recall many basic facts for multiplication and division. able to use a known fact to work out an unknown fact, 3 x 6 = 18 , 18 ÷ 3.

Video interviews (AND B)

- **GROUP A on multiplicative reasoning**
- **Recording**

The codes indicated how the child responded and the way they gave answers and are shown in below.

Table : Assessment schedule codes (Wright et al. (2006)

✓✓	correct with certitude	??	needs time to think
✓	correct	x✓	initially incorrect and correct
? ✓	needs time to think and correct	Rev	assessor revisit an item
∧	omission of a number in FNWS	IDK	I don't know
Red	teacher prompt		

Example for Learner : Assessment schedule

STRATEGIES	Perceptual counting by ones	Visible and count in multiples													Multiplicative strategies where items are screened				Abstract composite and unitary aspect	Coordinate two composite units								
TASK QUESTIONS		1a	1b	2a	2b	2c	2d	3a	3b	3c	3d	4a	4b	4c	4di	4dii	5a	5b	5c	5d	5e	ADVANCED MULTIPLICATION								
																						1a	1b	1c	1d	2a	2b	2c	2d	3
PRE-ASSESSMENT RESPONSES																														
POST ASSESSMENT RESPONSES																														

Example for Learner : Assessment schedule

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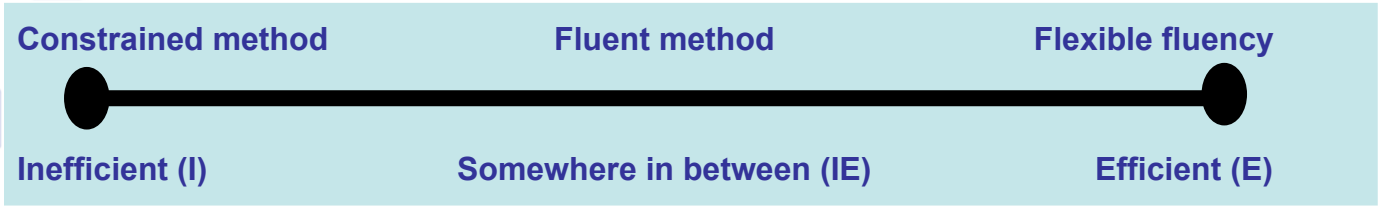
																						1a	1b	1c	1d	2a	2b	2c	2d	3
PRE-ASSESSMENT RESPONSES		✓	✓	^	✓✓	✓✓	✓✓	✓	✓	✓	✓	✓✓	x✓	??	Rev✓	?✓	✓	??	Rev??	?✓	??	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
POST ASSESSMENT RESPONSES		✓	✓	✓	✓	✓	✓	✓✓		✓	✓✓	✓✓	✓✓	✓✓	✓	✓	✓✓	✓✓	✓✓	✓✓	✓✓	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK

LEARNERS MULTIPLICATIVE PROFICIENCY

- **GROUP B**
- When using the same Wright et al interview tasks, Graven & Stott (2013b) explained that although the oral interview instruments and scripts show the methods the learners used to solve a task, coding responses simply as correct or incorrect, they developed an efficiency spectrum for procedural fluency that ranged from restricted / constrained procedural fluency towards elaborated and fully flexible fluency.
- I adapted Graven & Stott's (2013b) spectrum for procedural fluency into multiplicative spectrums for each task in the interview to help me understand learner progress. This progress would be evident when learners moved to the middle or upper end of the spectrum. Figure 3.6 below shows my adapted spectrum of multiplicative proficiency.

Figure : Spectrum of multiplicative proficiency for Constrained, Fluent and Flexible fluency



(Adapted from: Graven & Stott, 2013b)

Assessment schedule on multiplicative proficiency

TASK INVOLVING EQUAL GROUPS OF VISIBLE ITEMS						
	Solve the task count one by one (I)		Solve the task counting by one and some counting by multiples (IE)		Solve the task by counting using multiples of 3's, 4's and 5's (E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER						

TASK INVOLVING FNWS OF MULTIPLES						
	Count in 2's, 5's. 10' and 3's omitting some numbers in all the multiples of 10's and 3's (I)		Count in 2's, 5's. 10' and 3's but only omitting multiples of 3 (IE)		Count in 2's, 5's. 10' and 3's Fluently (E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER						

Assessment schedule on multiplicative proficiency

TASK INVOLVING VISIBLE ITEMS ARRANGED IN ARRAYS						
	Solve the task count one by one (I)		Solve the task counting by one and some counting by multiples (IE)		Solve the task by counting using multiples of 3's, 4's and 5's (E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER						

TASK INVOLVING EQUAL GROUPS OF VISIBLE ITEMS						
	Solve the task count one by one (I)		Solve the task counting by one and some counting by multiples (IE)		Solve the task by counting using multiples of 3's, 4's and 5's (E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER						

Assessment schedule on multiplicative proficiency *cont,*

TASK INVOLVING EQUAL GROUPS OF VISIBLE ITEMS						
	Solve the task count one by one (I)		Solve the task counting by one and some counting by multiples (IE)		Solve the task by counting using multiples of 3's, 4's and 5's(E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER						

TASK INVOLVING SCREENED ITEMS						
	Counting using fingers to keep track of groups and count (I)		The learner is able to count the counters after having shared (IE)		Solve the task by counting using multiples or using addition or subtraction for quotient division with an array (E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER						

ADVANCED MULTIPLICATION AND DIVISION TASK PRESENTED VERBALLY WIHOUT VISIBLE OR SCREENED ITEMS						
	Exhibit knowledge of composite units (I)		Exhibit knowledge of communicative principle of multiplication (IE)		Exhibit knowledge of inverse relationship between multiplication and division and multiplication facts to derive division facts (E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER						

Learner Assessment schedule on mathematical proficiency

FORMING EQUAL GROUPS						
	Counting by ones and confirm the number in groups by counting in ones (I)		Counting in ones and confirm and confirm counting in groups (IE)		Counting in groups and confirms in group counting (E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER	I					E

Key findings of five learners

Multiplicative reasoning

Table: Learners overall progress in LFIN levels over time from pre-assessment to post-assessment

	Learner A		Learner B		Learner C		Learner D		Learner E	
	PRE	POST	PRE	POST	PRE	POST	PRE	POST	PRE	POST
LEVELS	2	3	2	3	1	3	3	4	4	5

Given the relatively short intervention in this study, four sessions over five weeks, progress made by learners from level one to another level was one of the most important results for both myself as researcher (and teacher) and the learners themselves. The data showed that in the pre-assessment, some learners were counting in ones (positioning them at level 1) and relying on using constrained methods. After the intervention, the post assessment learners were able to count in equal groups and use more efficient and fluent methods to solve the tasks., all learners progressed at least one level. Learner C progressed from level 1 to level 3 in the short time, which represents a significant shift in her multiplicative reasoning.

LEARNER'S MULTIPLICATIVE PROFICIENCY

	I Constrained	IE Fluent	E Flexible fluency
LEARNER A			
PRE	5	1	1
POST	2	0	5
LEARNER B			
PRE	4	3	0
POST	2	0	5
LEARNER C			
PRE	6	1	0
POST	2	2	3
LEARNER D			
PRE	3	1	3
POST	0	1	6
LEARNER E			
PRE	1	2	4
POST	0	0	7

Learner progress in multiplicative proficiency emerges. Shifts in learner responses over time are evident. In the pre-assessment Learners B, C was split between using constrained methods and flexible methods, with Learner C using mostly constrained method. Learner A was split in all the methods but with more of constrained method. Learner D was split using all the methods but least of fluency. Learner E was split between using constrained and flexible methods but more with flexible fluency. Most learners seemed to rely most on using constrained methods as compared to fluently and flexible fluency. Learners B and C did not use any flexible fluency methods at all.

LEARNER'S MULTIPLICATIVE PROFICIENCY

- **A summary of learners' proficiency levels over time contributed to the potential value of the MR programme.**
- **From the levels data and methods data we note that all learners progressed in both aspects indicating the closeness of the relationship between these aspects.**
- **For example, Learner C (as in the levels), made good progress in methods; i.e. from a ratio of incidences of constrained to incidences of flexible fluency of 6 : 0 in the pre-assessment to 2 : 3 in the post assessment.**

I believe that there is evidence that the MR programme can be effectively implemented in the South African context to a group of learners,

Role play on interviews

THANK YOU **BAIE DANKIE** **ENKOSI**