# Using a transdisciplinary framework to examine mathematics classroom talk taking place in and through a second language

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This paper proposes a transdisciplinary framework to allow for a multifocal exploration of classroom talk practices. It draws on data from a broader study of talk in South African Grade 4 mathematics classrooms where the language of teaching and learning (English) was the home language for neither the teachers nor their students. Lesson transcript data from one teacher's lessons on fractions are used to demonstrate how working with three strands of conceptual insight from the disciplines of psychology, sociology and linguistics conduces to a potentially richer understanding of a teacher's use of classroom talk in mediating her students' mathematical understanding. By drawing on elements of Vygotsky's sociocultural psychology, we make visible in the lesson data the ways in which this teacher used the 'everyday' in trying to navigate her students' towards more 'scientific' conceptualizations of unit fractions. By then taking up aspects of Bernstein's sociological work, we articulate, and make visible, how societal circumstances impinge on students' access to exploratory mathematical discourse needed for epistemological access to abstract and generalized mathematical concepts. Finally, through Halliday's work on the power of particular linguistic registers for meaning-making, we highlight challenges in learning mathematics in and through a second language and reveal the constraints placed on students' opportunity to maximally exploit the distinct forms of meaning contained within the mathematics register.

**Keywords:** Transdisciplinarity; Mathematics classroom talk; Spontaneous and scientific concepts; Recognition and realization rules; Mathematics register

#### 1 Introduction

We argue in this paper for the use of a transdisciplinary framework to scrutinize aspects of mathematics classroom talk. Our purpose is to illustrate how, by examining such talk through a *multifocal* conceptual lens, transdisciplinarity allowed us to build up a more holistic image than might otherwise have been possible. Although this is a conceptually- rather than empirically-driven paper, we import data from a broader study of mathematics classroom talk into our discussion as empirical exemplification of the ideas we have taken from the disciplines of psychology, sociology, and linguistics, represented – respectively – by Vygotsky, Bernstein and Halliday. It is the work of these three luminaries that constitutes our sources of conceptual and theoretical insight.

Given the common threads running through Vygotsky's, Bernstein's and Halliday's work, namely emphasis on the centrality of language in mediating learning and on the influence of sociocultural, sociopolitical, and sociohistorical factors on learning outcomes, their work coheres well. We see their work as having particular cogency for analyses of post-colonial educational circumstances, where a majority of learners are learning in a colonial language not spoken at home. The following question guides our discussion: 'In what ways might a transdisciplinary approach

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potentially enrich insights into challenges facing a primary school mathematics teacher in getting her students to engage in classroom talk on fractions in and through a language (English) in which they are not yet fluent?'

Before sharing aspects of our 'dialogue across disciplines' (after Martin 2011) in exploring how one Grade 4 mathematics teacher used classroom talk to mediate (or not) her students' understanding of fractions, we engage in a generic consideration of transdisciplinarity.

## 2 Transdisciplinarity's potential 'across', 'between' *and* 'beyond' established disciplinary boundaries

Piaget introduced the term 'transdisciplinarity' in 1970. He described transdisciplinarity as a "higher stage succeeding interdisciplinary relationships ... within a total system without any firm boundaries between disciplines" (1972, p. 138). Nicolescu (2010) suggested that, though the full meaning of the Latin prefix 'trans-' includes 'beyond', as well as 'across' and 'between'; "the intellectual climate [in 1970] was not yet prepared for receiving the shock of contemplating the possibility of a space of knowledge *beyond* the disciplines" [italics added] (p. 20). This, he postulated, is what led Piaget to confine his initial construct of 'transdisciplinarity' to the idea of working 'across' and 'between' disciplines (Nicolescu 2010, p. 20). Bernstein (2015) noted that "conditions for beginning transdisciplinary work in earnest did not fall into place for at least two more decades", but that, from then onwards, there has been increasing transcendence of traditional disciplinary boundaries.

Roughly consonant with this 'time lapse', Lerman (2001) noted the widening range of intellectual resources from which mathematics education was drawing. Simon (2009) subsequently observed that "currently there are more theories of learning in use in mathematics education research than ever before" (p. 477). Such diversification is partly a consequence of mathematics education having a 'horizontal' knowledge structure; one in which "a range of languages [of description] have to be managed, each having its own procedures" (Bernstein 1999, p. 164). Commenting on adherence to particular conceptual standpoints, Simon (2009) rued "an unproductive divide that sometimes is a part of discourse in our [mathematics education] field," arguing instead for "coordinating the results of work from different theories" (p. 479, 481). Stating his own preference for a '*trans*disciplinary' approach' when drawing on different intellectual resources, Halliday argued that transdisciplinarity might avoid the sort of "a little bit of that, a little bit of the other" situation that could then lead to dilution and/or fragmentation of the individual strengths of contributing disciplines (Halliday and Burns 2006, p. 114-115).

While inter- or trans-disciplinarity might entail some fragmentation and possible "loss of disciplinarity" (Martin 2011, p. 35), this, we argue, could be offset by its *synergistic* potential. Writing of such potential, Martin noted:

Conversation is fostered by having a problem with which both disciplines are concerned, the ability to trespass on each other's domain by providing complementary perspectives on comparable phenomena, and possession of a discursive technology which can *make visible* things the other discipline wants to know [italics added]. (2011, p. 37)

The 'problem' under consideration in this paper concerns barriers likely to be encountered in contexts where mathematics teachers are obliged to conduct classroom discussion in and through a second language in which students are not yet fully proficient (as is the case for the students and teachers in the broader study from which the

present article is taken). Such barriers are inevitably amplified by virtue of mathematical language's inherently multi-semiotic nature (Schleppegrell 2007; O'Halloran 2011) whereby being able to talk' mathematics calls for considerable "semiotic integration" (Lemke 2002, p. 22).

Challenges in expressing mathematical reasoning are widely implicated as contributing to students' lack of engagement and consequent mathematical underperformance. Transdisciplinarity, we argue, offers a potentially helpful mechanism for illuminating different facets of the linguistic challenges involved. As Hasan (2005) observed, dialogue across disciplines is best facilitated where there is "a reciprocity of concern" (p. 50). In her exploration of links between semiotic mediation, language and society, and the respective contributions of Vygotsky, Halliday and Bernstein, Hasan noted that "any story which has for its theme the conditions of human existence is bound to remain incomplete within the bounds of one discipline" (2005, p. 156). Figure 1 below illustrates the possibilities for dialogue across, between *and* beyond the discipline boundaries of psychology, sociology and linguistics in pursuit of fuller, more complete insights into aspects of mathematics classroom talk. For this figure we have borrowed from Martin (2008) his graphic style of using speech boxes to represent different dialogue 'participants'.



Fig.1 Addressing common interests through dialogue across disciplines

There has been growing recognition of discussion, dialogue, and talk as essential mechanisms for meaning making in classrooms (see Calcagni and Lago, in press). It is not within the scope of this paper to expand on this proliferation of interest, beyond noting that research provides compelling evidence of the cognitive value of getting students 'thinking together' orally. Mercer and Littleton (2007) described this "interthinking", process as representing "sociocultural theory in action" (p. 6). Lerman (2000) was amongst those to first note an increasingly collective, socio-cultural orientation in mathematics education research literature. He noted, too, an increased interest in the ways in which "consciousness is constituted through discourse" (Lerman 2001, p. 88).

To help frame our focus on mathematics classroom talk, we have used a lens metaphor. Lerman (1998) made an analogy between a camera's zoom lens function and a researcher's need to zoom in and zoom out on micro-/macro-aspects of a research site. Our metaphorical lens is of a more elementary sort: that of a simple convex lens functioning to converge parallel rays of conceptual light onto a particular focal point: the discursive practices observed in a Grade 4 mathematics classroom. Through these multiple insight sources - and notwithstanding Lerman's warning that "incompatibilities lurk in incautious complementarities" (2001 p. 88) - we are able, we argue, to bring our observations more sharply into analytical focus. Figure 2, below, illustrates our analogy.



Fig. 2 A convergence of different conceptual light sources in examining mathematics classroom talk

To exploit the multiple light sources analogy of our lens metaphor, we close this sub-section with a fitting comment from Halliday (2008). Noting its paradoxical advantages, he described 'complementarity' as that which "turns 'either/or' into 'both/and'. Light is either particle or wave; it can't be both – but it is" (Halliday 2008, p. 36).

In the next subsection we sketch details on the research site from which we take the classroom talk episodes used to animate our arguments on the value of transdisciplinarity in looking at classroom talk in second language learning contexts. It is one of the sites used in the broader study. In the three sub-sections thereafter we demonstrate how each of our conceptual light sources helped to make visible different facets of classroom talk, together enabling richer understanding of the complexity of challenges facing the learners and the teacher in our study. The context of the classroom talk extracts used to illuminate discussion around transdisciplinarity The broader case study involved non-participant observation of Grade 4 mathematics lessons over four teaching weeks. The lessons were both audio- and video-recorded and subsequently transcribed verbatim. Included in the broader study were fourteen 50-minute Grade 4 mathematics lessons taught by Ms M (pseudonyms are used throughout). Ms M's inclusion in the broader study constituted an opportunity sample (Robertson, 2017). Ms M had represented her school as a participant in the South African Numeracy Chair Project, a project which merges research with professional development and which is led by the second author (see Pausigere and Graven 2014; Graven and Coles 2017). Ms M taught at a township school. In the South African context the word 'township' refers to the apartheid-era 

urban residential areas designated for 'non-whites'. South Africa, despite being well into its third decade of postapartheid governance, remains a deeply divided society, and significant geographical separation along racial lines persists even today. Ms M taught two Grade 4 mathematics classes, Class A and Class B (32 and 31 students respectively). All the children were, as was Ms M, native speakers of isiXhosa (the second largest of South Africa's indigenous language groups, and the *lingua franca* for the community surrounding the school). The school's chosen language of teaching and learning policy was 'straight-for-English'. Students' only regular contact with English, however, was the classroom, and, despite having been taught in English from Grade 1, many of Ms M's Grade 4s had achieved only limited proficiency in English. Peer-to-peer exchanges observed in the course of the broader study were exclusively in isiXhosa. During the observation period students' verbal contributions in English were overwhelmingly of a one or two word order, often given in chorus, and only ever in direct response to promptings from Ms M. No student-initiated use of English was observed.

'Straight-for-English' represents a subtractive model of bilingualism insofar as use of languages other than English is discouraged, thereby constraining any role students' native languages could play as tools for mediating learning (Robertson and Graven, submitted paper). Subtractive bilingualism poses a considerable risk relative to students' overall epistemological access to the discourse of the classroom. Research-based evidence cited by South Africa's Department of Basic Education (2010) shows positive correlations between learning in mother tongue and scholastic achievement, but, as Setati (2008) noted, so great is the perceived power of English that a majority of black South African parents explicitly choose it as the medium for their children's education. Having themselves been educated under apartheid, they associate teaching through an indigenous language with inferior education (Nomlomo 2006). Related to this, many black teachers express discomfort about bringing in an indigenous language for mediating breakdowns in students' understanding (hence, for example, Probyn's reference (2009) to teachers' *smuggling* the vernacular' into their classrooms [italics added]).

The topic most taught across Ms M's fourteen observed lessons was fractions. We thus use her fraction lessons for our exemplification of talk practices across a topic. We have selected five classroom-talk episodes from two lessons on fractions on one day: three episodes from the Class A lesson; two from the Class B lesson. Choosing episodes from lessons taught on the same topic to two Grade 4 classes on one day enables us to note some subsequent adaptations Ms M made when teaching 'the lesson' to Class B later in the day. We have chosen the five episodes because they together illuminate, and enable us to animate, our discussion around the merits of transdisciplinarity. While other episodes from other fraction lessons could similarly have been used, we found taking these episodes from a single day's teaching allowed for coherent explanation that did not require lengthy discussion of Ms M's fraction lessons across the full observation period. Table 1 outlines the lesson activity sequences, together with the particular focal aspects relative to our three disciplinary sources of conceptual insight.

	LESSON ACTIVITY SEQUENCE	FOCAL ASPECT FOR CONCEPTUAL ILLUMINATION
Class A	Fractions as equal parts of a wholeIdentifying relative sizes of unit fractionsIdentifying and naming fraction parts(denominator/numerator)Worksheet: fractions of shaded circlesIdentifying parts of a whole $(\frac{1}{2} \frac{2}{7} \text{ etc.})$	<ul> <li>Moving towards more scientific forms of mathematical representation (Vygotsky)</li> <li>Recognition and realization rules (Bernstein)</li> <li>Moving towards a specialized mathematics register (Halliday)</li> </ul>
Class B	<b>Fractions as equal parts of a whole</b> Counting in halves Counting in quarters Identifying parts of a whole Identifying fraction symbols Identifying relative sizes of unit fractions Worksheet: fractions of shaded circles	<ul> <li>Using the everyday to access the scientific (Vygotsky)</li> <li>Recognition and realization rules (Bernstein)</li> </ul>

### 4 Vygotsky as a conceptual source for focusing on episodes of mathematics classroom talk

Though Vygotsky's work is "inherently cross disciplinary" (Wertsch 1985, p. 230), he is perhaps best known for his work in developmental psychology in relation to which he highlighted the "sociocultural roots of consciousness" (Lerman 2001, p. 90). Vygotsky's core premise in relation to teaching and learning is that all learning is socially mediated and historically and culturally situated (1930). Our focus here is on his ideas firstly, around links between the use of language as a socio-cultural tool and children's cognitive development; and secondly, the transition from 'everyday' to 'scientific' conceptualizations of [mathematical] meaning.

Vygotsky was ahead of many of his contemporaries in characterizing language as an integral aspect of culture. He was amongst the first also to draw attention to the links between the *micro-aspects* of individual cognitive development and the *macro-context* of wider socio-historical and socio-cultural teaching and learning settings. Mathematics research using a Vygotskian sociocultural frame of reference takes account, therefore, of how a particular issue appears to be "constituted in its relations to the wider macro-situation and the micro-situations [making clear] the links between structure and agency and between culture, history and power and students' learning of mathematics" (Lerman 2001, p. 90). Internationally, Lee, Park and Ginsberg (2016) note significant difference in

mathematical performance across different socio-economic groups. In terms of the local setting for our paper, as noted, South Africa's apartheid past created deep divisions along racial and linguistic lines, which played out also in socio-economic terms whereby it is mainly the country's Black people who continue to occupy the country's lower socio-economic strata. Not only is Mathematics performance in South African schools poor compared to other countries, it also has among the largest performance gaps across its different socio-economic strata (Graven 2014).

Vygotsky emphasized that inextricable links exist between linguistic interaction and the development of thought (sense-making), and that sociocultural circumstances act also as major shaping forces. It is the human capacity to produce meaning through the use of signs and symbols that enables functioning on both a social "*between* people (*interpsychological*)" level, and an individual "*inside* the child (*intrapsychological*)" level (Vygotsky 1930, p. 48). Social interaction and dialogue in a cultural context makes sense-making possible (Renshaw and Brown 2007). Through their participation in cultural activity, children gradually gain individual access to their society's "sense-making resources" (Mercer and Littleton 2007, p. 13). As noted earlier, however, many South African students' opportunities to maximize their linguistic participation are compromised by language policy choices and decisions.

South Africa is perhaps an extreme example of a multilingual society in which a single, non-indigenous language dominates. English is South Africa's main language of teaching and learning. While census data indicates that it is the home language of less than 10% of the population, South Africa's Department of Basic Education data reports that, by Grade 4, 79.1% of students are officially learning in and through English (2010, p. 16). Effectively, then, by Grade 4, a great majority of South African students have only limited access to their native language as a resource for mathematical meaning-making in their classrooms. This, we argue, is a key contributory factor to the country's low levels of mathematics achievement. We note, however, that demographic shifts globally have seen increasing numbers of students elsewhere having to engage with mathematics in and through a second language, thus South Africa is no exception relative to challenges at the mathematics/language interface.

Vygotsky's work highlighted the developmental value of being able to conceptualise things in 'scientific' (or 'academic') ways rather than in 'everyday (or spontaneous) ways. Children's spontaneous concepts, Vygotsky argued, provide "the necessary, *but not sufficient* [italics added], conditions for progress toward more powerful forms of thinking" (Renshaw and Brown 2007, p. 533). It is in a child's zone of proximal development (ZPD), according to Vygotsky, that the "child's empirically rich but disorganised spontaneous concepts "meet" the systematicity and logic of adult reasoning [and] as a result of such a "meeting," the weaknesses of spontaneous reasoning are compensated by the strengths of scientific logic" (Kozulin in Vygotsky 1986, p. xxxv).

For Vygotsky, all higher mental functions are "products of mediated activity" (Kozulin in Vygotsky 1986, pp. xxivxxv). Bruner referred to Vygotsky's idea of mediation as a "loan of consciousness" (1986, p. 76). It is through the skilled making of such 'loans' that mathematics teachers mediate between their students' existing levels of mathematical conceptualization and other, more specialized, ways of thinking and talking about mathematics. Development of academic (or scientific) concepts rests upon deliberate and systematic cooperation between teacher and students, and upon students' growing linguistic consciousness and proficiency. Language provides the symbolic means to "direct … control … and channel" thinking (Vygotsky 1994, p. 47) in increasingly logical and disciplineappropriate ways. Lerman (2014) shared the following imaginary anecdote to elaborate upon Vygotsky's work on the distinction between spontaneous and scientific concepts and to exemplify the way language use can change and develop in terms of its degree of semantic precision:

If I am in the playground as a little child and somebody says, "I am going to share my chocolate with you," and breaks the chocolate into two pieces, it is fair enough for me to say, "Your half is bigger than my half." Not fair to say that in the classroom – in the mathematics classroom. It is only a half if it is exactly the same size as the other piece. (p. 13)

In the first of our classroom talk episodes we show Ms M mediating the semantic and symbolic precision with which her Grade 4s' conceptualized 'half' as a fraction of a whole.

 Table 2 Classroom Talk (Class A): Moving towards more scientific forms of mathematical representation

STUDENT(S)	Ms M
	Let me take an easy fraction. [ <i>Teacher draws on the chalkboard a rectangle</i>
	divided into 2 equal parts with one part shaded.]
	WITH .
	201
	What is the denominator of this fraction: in other words the number at the
	bottom of this fraction? What is the denominator going to be? Lonwabo – you
	are not listening. You are doing your own thing. Put that pencil down. Anathi?
Half, ma'am.	
	Half. How do I write half? Come and write half for me. I don't know how to
	write it. Come and write it for me. I don't know how to write half. Quickly. I
[Child somes up to front ]	don't know how to write half. Ms M doesn't know how to write half.
	Okay Write it
[Child writes 'half'.]	Okay. which.
b $t$	
IUMI	
	Okay Now I want to see numbers. You've written the word 'half' I want to
	see now numbers – a fraction that is going to show a half. Let me not help him
	He must do his own thing. Then you're going to tell me when he's finished.
[Child writes '2'.]	
I. I.	
Nall	
2	
5	
[Chorus ] No [Some children	
[Chorus.] NO. [Some chuaren laugh ]	
01	What is this?

STUDENT(S)	Ms M
[Chorus.] A whole number.	
	A whole number.
[Child thinks awhile, and then adds in the numerator 1 and the separation line to make it '½'.]	
	[ <i>Ms M nominates students to tell her what each of the two numbers in a fraction signifies.</i> ] Zuki – that one is a half. So – this two ( <i>touching the two in the fraction ½ on the board</i> ) means what? This two means what? This two means what, Phumla?
Two parts.	
	Two parts. It means this shape or whatever, is divided into how many parts? ( <i>points to the rectangle shape on the chalkboard</i> )
[Chorus.] Two parts.	
	Two parts. It means this shape or whatever, is divided into how many parts?
[Chorus.] Two parts.	
	Two parts. Are these ( <i>pointing to the rectangular picture</i> ) two parts?
[Chorus.] Yes.	
	Is this (pointing to the rectangular picture) two parts?
[Chorus.] Yes. [Chorus crescendos!]	
	Is this two parts?
No! Yes! [Mixed responses.]	
	Is this two parts?
[Chorus.] Yes.	
	Nyaniso, is this two parts?
Yes.	
	Yes. Do we count shaded only or unshaded only, or both shaded and unshaded?
[Chorus.] Both.	
	Both. Because if we counted only shaded, we are going to write something like this [ <i>Pointing to the 2</i> ] which is not true. So, that fraction is divided into two equal parts. Rock and roll, hmm? [ <i>A number of children laugh</i> .] And then, how many are shaded?
One.	
	One. That is one, but we say two, but from the two parts, only one. [ <i>Teacher draws another rectangle, divided into 7 equal parts</i> .] Okay. Into how many parts is that one divided? Zikhona?

The struggle the students appear to have had in unpacking the concept of 'half' in this episode is the more remarkable given that the episode comes near the end of Class A's lesson. Preceding this, Ms M had had them answer questions about dividing a cake into equal parts and the relative sizes of different unit fractions (principally 'one quarter' and 'one-eighth'). They had re-visited the meanings of the terms 'numerator' and 'denominator', and they had tackled a worksheet task requiring that they correctly identify various fractions of a whole (e.g. two-fifths; three-tenths; five-eighths).

In response to Ms M's question: "What did you find challenging about this [worksheet task]?" some children indicated they had found it easy, one that it was "a little bit hard", another that she had struggled to identify 'two-sevenths'. In responding to this Ms M went back to simpler terrain, hence the episode's opening comment: "Let me take an easy fraction." We note that the children's struggle to connect this "easy fraction" to its symbolic

representation may have been eased had students had the opportunity to engage with the concept and the structure of its symbolic representation in isiXhosa. The second of our classroom talk episodes (below) is Ms M's lesson opening for Class B which followed immediately after the Class A lesson. We surmise that Class A's struggle with halves was what influenced Ms M's decision to begin Class B's lesson with the mental arithmetic task of counting in halves.

Table 3 Classroom Talk (Class B): Using the everyday to access the scientific

STUDENT(S)	Ms M
	Now, I want you – you are going to count in halves. What is the next number after
	half? Half, Tumelo?
Ten.	
	No! Half?
Quarter.	
	Half, Abongile?
One.	
	A half, and then after the half, what number follows?
[Chorus] One.	
	Think of a loaf of bread, a half, and then one, and then from that half I add one half
	again.
One and a half.	
	One and a half! One and a half. After one and a half, Pumele?
Two	

We see from the first of the learners' responses, they do not understand the task. Once, however, Ms M 'loaned' them the everyday bread image, they continued the counting in halves mostly without difficulty. Linking this turning point to the socio-cultural (and socio-economic), we note that, given the less than affluent home circumstances of Ms M's students, bread (particularly a *half* loaf, which is all many families can afford to purchase at any one time) is a common feature of their daily lives.

Vygotsky's ideas have helped make visible some of the ways in which Ms M used the empirical richness of the 'everyday' to help mediate her students' passage towards a more 'scientific' conceptualization of 'half'. His work has been described as lying at the "intersection of psychology and linguistics" (Renshaw 2004, p. 1), a placement that sits well with our transdisciplinary lens metaphor. In our next sub-section we focus on our second source of conceptual light: Bernstein's sociologically-oriented ideas about links between language and learning. Bernstein was a boy of ten when Vygotsky died. He explained that while he had never attempted to integrate his work with that of Vygotsky, from early on he had been "deeply influenced" by Vygotsky's ideas around the links between thought and language: "It set up a buzzing in my head," he wrote, "which I can still hear" (1995, p. 400).

## 5 Bernstein as a conceptual source for focusing on episodes of mathematics classroom talk

Bernstein's work further illuminates how external, socio-economic, socio-cultural, and socio-political factors influence the nature and quality of students' linguistic behaviours and their subsequent realization of academic potential. Bernstein's interest lay in understanding how societies reproduce themselves: how existing power structures within a society tend to remain in place, so enabling the maintenance of powerful groups' positions of dominance. He cautioned that "symbolic 'tools' [read language, primarily] are never neutral; intrinsic to their

construction are social classifications, stratifications, distributions and modes of recontextualising" (Bernstein, 1993, p. xvii). His work provides "a theoretically informed approach to the awkward question of the intractability of unequal schooling outcomes" (Hoadley and Muller 2010, p. 69). In drawing on his ideas here, we focus on two aspects: his sociolinguistic work on speech codes and his explication of recognition and realization rules.

Bernstein's early work was around the dialectical role language played in the reproduction of social and educational inequalities. He described language as "one of the most important means of initiating, synthesising, and *reinforcing* ways of thinking, feeling and behaviour which are functionally related to the social group" (1959, p. 312). His research led him to the view that a significant contributor to differentials in educational achievement was students' access to, and use of, 'formal' (as opposed to 'public') language (Bernstein 1959). He subsequently re-labeled these 'elaborated' and 'restricted' codes. Bernstein saw these different speech codes as arising from different forms of social relationship, and from differences deriving from whether a child's parents worked in 'mental' or 'manual' occupational categories (Bernstein 1971). He identified context as being central to understanding how these different codes, or patterns, of speech emerge and play out. A restricted code is essentially an oral language, one embedded in context ("particularistic"), and one in which much of the sub-text is implicit, whereas an elaborated code is generally more contextually disembedded and explicit ("universalistic") (Bernstein 1973, p. 70). These codes, linked as they are both to social experience and to context, conduce to what Hasan called "variant forms of consciousness" (1995, p. 188), or, as Diaz termed them, "meaning matrices" (cited in Hoadley and Muller 2010, p. 70) which then influence the ways in which individuals make sense of experiences. As Bernstein (1971) explained:

Particular forms of social relation act selectively upon what is said, when it is said, and how it is said. [These can then] generate very different speech systems or codes ... [and thereby] create for their speakers different orders of relevance and relation. The experience of the speaker may then be transformed by what is made significant or relevant by different speech systems. (p. 144)

As Lerman (2001) noted, Bernstein's linguistic code theories helped explain aspects of "differential opportunity" for mathematics learning (p. 94). What children come to school with from their primary sites of socialization inevitably influences the extent to which they are able to resonate with the 'consciousness' or 'meaning matrix' required of them in the mathematics classroom. Many children from working class backgrounds may struggle to recognise that classroom discourse is – perforce – different from the particularistic, context-embeddedness of their everyday discourse (see, e.g. Cooper 1998; Hoadley 2007). "Social biases," Bernstein argued, "lie deep within the very structure of the educational system's processes of transmission and acquisition and their social assumptions" (2000, p. xix). We note that while Bernstein's work on speech codes was with native English speakers, Ms M's students are second language English speakers. In the following lesson episode, we see Ms M using everyday, context-embedded discourse around her students' food preferences to initiate discussion around fractions as parts of a whole.

 Table 4 Classroom Talk (Class A): Recognition and realization rules (1)

STUDENT(S)	Ms M
	Lonwabo, we have done fractions for quite a long, long, long time – last term.
	I'm not going to draw anything on the chalkboard. But, if I'm saying to you
	that there's this cake, do you like cake?

STUDENT(S)	Ms M
Yes ma'am.	
	Who doesn't like cake? Who doesn't ~
[Child indicates that he doesn't	
- like cake.]	
	You don't like it?
No, ma'am.	
	You like -? What do you like?
Pizza, Ma'am.	
· · · · · ·	Which one do you like – pizza or cake?
Pizza, Ma'am.	
· · · · · ·	Pizza? [Teacher asks another child.]
Fish.	
	You don't like pizza? Because I'm saying pizza. Pizza or cake, which one do
	you like? Pizza or cake, or none of them?
Pie.	
	Eh! She likes pie.
Pizza, ma'am.	
	Pizza?
Both.	
	Both? If you can be given, and then you are asked to take one, which one would you choose? Because you can't take both. There's pizza. There's cake. Which one would ~?
Pizza.	
	~ you choose, because you can't take both?
Cake.	
	Cake. Nyaniso?
Pizza, ma'am.	
	Pizza. Phumla?
None.	
	You don't know what you like?
KFC [laughter]	
	This one likes chicken! You like the KFC one? You don't like Steers' ones, or Nandos' ones? [ <i>KFC</i> , <i>Steers and Nandos are fast-food chicken outlets</i> ]

Much of Bernstein's work focused on the ways in which "power relationships created outside the school penetrate the organisation, distribution and evaluation of knowledge through the social context" (1972, p. 217), thereby affording differing learning opportunities to students from different social groups. The way educational knowledge is structured (its organisation, distribution (transmission and acquisition), and evaluation) acts as "an agency of social control" (1973, p. 65).

The ways in which knowledge is classified and framed plays out in pedagogic discourse and practice to create the recognition and realization rules (Bernstein 2000). Classification determines the orientation required within a particular learning context (what is, or will be, *recognized* as legitimate). Framing determines how this particular orientation is to be *realized* (the means whereby something is considered successfully achieved). We notice in the lesson episode above that those of Ms M's students who offered 'fish' and 'chicken' had not recognized that in talking about fractions of *a whole*, Ms M was wanting them to think in terms of dividing a single whole regular-

shaped object into *equal* parts, something the everyday image of a circular cake, a pizza or a pie lent itself to; chicken and fish much less so.

As Bernstein (2000) explained, having the recognition rule enables acquirers to appreciate what sorts of things are required: "the special features which distinguish the context" (p. 17). This often 'invisible' aspect of schooling is something with which students from working class backgrounds may struggle. The recognition rule is but a first step, however. The realisation rule needs also to be invoked for the requisite (legitimate) text to be produced: "recognition rules regulate what meanings are relevant ... realisation rules regulate how the meanings are to be put together to create the legitimate text" (Bernstein 2000, p. 18). The most centrally important mechanism in a teaching/ learning context whereby both the recognition and the realisation rules may be realised rests with the evaluation process (both formal and informal), and the explicitness thereof (Jorgensen 2013). In the next lesson episode, we note an interesting paradox whereby the children are *recognising* what Ms M is asking of them (their initial responses are mathematically correct), but they frustrate her because, in terms of *her* evaluation criteria, the students do not *realise* it in the 'form' she deems correct.

	How many quarters make one?
[Chorus.] Four.	
	How many quarters make one?
[Chorus.] Four.	
	How many quarters make one?
[Chorus.] Four. Three.	
	How many quarters make one?
[Chorus.] Four	
	Phew! How many quarters – nobody has said – how many quarters make one?
[Chorus.] Four. Three. Four. [Loud, elongated enunciation of 'four'.] Four.	
	Four? [ <i>Teacher draws a square on the chalkboard, dividing it diagonally into four sections.</i> ] How many parts are those?
[Chorus.] Four.	
	They make what?
[Several different chorused offerings.] They make two. They make one. They make one. They make two.	
Eventually one student offers Ms M the 'legitimate text' she is seeking ] Four quarters.	
	Four quarters make one, yes.

 Table 5 Classroom Talk (Class B): Recognition and realization rules (2)

We notice how Ms M's apparent non-acceptance of their *mathematically correct* response bewildered some students who then began to suspect the reason for her repeatedly asking the question was that they had answered incorrectly. This pushed them to guessing, proffering seemingly nonsensical answers ('three' 'one' 'two'). What they had not realized here was that what was wrong (for Ms M) was the *form* of their answers. 'Four' was not sufficient of an answer in Ms M's eyes. She wanted that full phrase: 'Four *quarters* (makes one)'; but she had not made this requirement explicit at the outset.

The aspects of Bernstein's work we have drawn on here have helped articulate, and so make visible some of the ways in which differential access to what is deemed mathematically appropriate discourse may act to impede epistemological access. Bernstein's work, certainly his early speech code work, lies at the intersection of sociology and linguistics. In our final sub-section of this overview we explore the third of our conceptual light sources: Halliday's pioneering work in linguistics. Halliday was for a time a colleague of Bernstein's at the University of London. It is Halliday that Bernstein credited with helping him "to think about linguistics in sociological terms and sociology in linguistic terms" (1995, p. 396).

### 6 Halliday as a conceptual source for focusing on episodes of mathematics classroom talk

Halliday is perhaps the arch trans-disciplinarian. He has throughout opted for a multi-faceted approach in the development of his theories about language, noting that:

Weak boundaries have always been characteristic of my approach ... I have never really thriven in a discipline-based structure of knowledge ... To the extent that I favoured any one angle it was the social: language as the creature and creator of human society, as expounded by my teacher J. R. Firth and by my friend and colleague Basil Bernstein. (2002, p. 1; pp. 6-7)

Halliday's views on the links between language, learning, and social context strongly parallel those of Vygotsky:

When children learn language they are not simply engaging in one type of learning among many; rather, they are learning the foundations of learning itself ... the distinctive characteristic of human learning is that it is a process of making meaning - a semiotic process. (Halliday 1993, p. 93)

Halliday's systemic functional linguistics theory provides not simply a theory about language, but also a set of tools with which to examine and describe language practices in their actual contexts (Eggins 2004). Listing the many ways in which Halliday's theory is used, Eggins observed that "underlying all [of] these ... is a common focus on the analysis of authentic products of interaction (texts), considered in relation to the cultural and social context in which they are negotiated" (2004, p. 2). These points, together with the frequent links that Halliday makes in his work between language and learning, underscore the value of drawing on his ideas in analysing linguistic interactions taking place in mathematics classrooms.

Halliday's work emphasizes the systemic aspects of language and its use in natural contexts. Aligning his thinking with that of Vygotsky and Bernstein, for Halliday, meaning and meaning-making is "a social and cultural phenomenon" (Halliday and Matthiessen 2004, p. 133). His interest lies in how language functions as a social semiotic resource from which language users learn to actively select, and make choices, in order to express and

exchange meaning (Halliday 1993). In drawing on the work of Halliday here, we focus on his work around language 'register' and the idea that skilful language use involves making the kinds of choices that maximize the potential for making meaning in a given context, in this instance the mathematics classroom.

Halliday's presentation at the 1974 UNESCO Symposium: *Interactions between linguistics and mathematical education* appears to mark the first time he spoke of 'register' in specific relation to mathematics. In his keynote address Halliday explained that while "every language embodies some mathematical meanings in its semantic structure", not every language is necessarily "sufficient ... [in terms of serving] the needs of mathematics education", particularly at post-primary levels of study (1974, p. 65). To serve specialized functions languages need to add new registers. In so doing, they expand their functional variation.

Halliday defined 'register' as "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (1974, p. 65). A mathematics register, therefore, is "the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes" (Halliday 1974, p. 65).

Gibbons (2003, after Halliday) wrote of a 'mode continuum' whereby students learn "to use language for a range of purposes and in a range of cultural and situational contexts" (p. 250) in order to transition from commonsense ways of expressing their thinking towards increasingly more specialist expressions of their thoughts. A 'scientific' and specialized – rather than an 'everyday' – mathematics register ultimately constitutes a more powerful and precise linguistic and conceptual tool for making meaning of the mathematical ideas encountered in the classroom context (Schleppegrell 2010).

Halliday's articulation of the "notion of a mathematical register helps us understand the ways that language constructs mathematical knowledge in different ways than it constructs other academic subjects" (Schleppegrell 2007, p. 140). The fact that such construction is more often than not *multimodal* (not just language, but symbols and images) makes for semantic complexity, as does the fact that the spoken language of mathematics may "bear little resemblance to its written form" (Pirie 1997, p. 229). Pirie provided the following example of how a mathematical meaning may be expressed along a continuum ranging from everyday language to the equivalent symbolic representation.

T-LL- (	England	~ ~ f	a agent		_ <u>r</u>	·	A	:1		~ ~ ~ ~ ~ ~	
I able o	Example	еог	a con	inuum.	OI.	mau	nemai	icai	meaning	exr	ression
1	2	• • •			· ·					•···	

Common 'everyda spoken forms →	ay'	Mathematically expressed spoken/ written form →	Symbolic representation
Four times three	Four threes are	The product of three and four is	4x3=12
is twelve.	twelve.	twelve.	

(Adapted from Pirie 1997, p. 229)

Any text, Halliday and Matthiessen (2004) note, "is the product of ongoing selection in a very large network of systems — a *system network* ...the *system* ... [being] the potential that lies behind the *text*" [emphasis in the original] (p. 33). Here, Maton's concepts of 'semantic gravity' and 'semantic density' (2011) are helpful. Ms M's

transitions across the mode continuum from 'commonsense' to more 'scientific' mathematical meanings in Table 7, below, illustrate her attempt to decrease semantic gravity by moving away from the concrete (the cake), and to increase semantic density by packing more discipline specific meaning into the language used (symbolic representations of fractions, mathematical terms such as 'denominator' 'numerator') so as to better exploit the sorts of meaning potential offered by the mathematics register. We saw this in Ms M's talk around the meaning of 'half' and its symbol in the first episode shared. Table 7's text is taken from Ms M's same (Class A) lesson.

 Table 7 Classroom Talk (Class A): Examples of Ms M's transitioning students towards using more semantically dense mathematics text

Commonsense mathematical meaning-making	Specialized mathematical meaning-making		
(greater semantic gravity)	(greater semantic density)		
One fourth	One quarter; <sup>1</sup> / <sub>4</sub>		
Would you rather have a quarter of a cake or an eighth	Show me here [fraction chart] which one is a quarter.		
of a cake?	Which one is an eighth? Then we can agree. $\frac{1}{4}$ ; $\frac{1}{8}$		
What is going on about all those fractions?	What is the same? What is common?		
Any number that's on top.	Numerator		
What about the number[s] at the bottom?	Denominator		

As Halliday and Matthiessen (2004) note, language has the capacity to "expand more or less indefinitely [to meet] the functions that language serves in human lives" (p. 24). From within any linguistic system for making meaning the choices that are made will determine the extent to which the full potential of a particular system is being realised. Ideally, language users make linguistic choices based on their understanding (consciously or otherwise) of their intended purpose and audience. In the lesson episodes shared in this paper, the fact that the text contains words such as 'one eighth', 'denominator', 'numerator', indicates that this was a lesson on fractions. But in the same way that this text suggests the context, so too can the broader context pre-empt the text that is *possible*. Ms M pointed her students towards using discipline-appropriate terms. She directed proceedings in ways she saw as consistent with the ways in which fractions lessons 'are normally' construed: she constructed her own fractions lesson through her choice of text, and she chose to do it that way because of her perceptions about what constituted an appropriate fractions lesson. The extent to which her students were able to make meaning of the lesson, however, was in large part a function of their limited proficiency in English. As Ms M herself remarked, "All these learners here at school are Xhosa-speaking learners, and maths is done in English. And maths also has its own language. ... I think that is a main problem." This, in turn, imposed constraints upon the extent to which they were able to fully realise the potential offered by the mathematics register.

## 7 Concluding remarks

Ms M's ability to develop and sustain more exploratory forms of classroom talk was clearly constrained by conceptual backlogs in students' mathematical understanding (as can be seen, for example, when she needed to recap the symbolic meaning of ½). More importantly for this paper, however, we argue that it was constrained by her students learning mathematics in and through a language in which they had limited proficiency. Graven (2014) noted that South Africa's apartheid history "systematically disempowered people politically, economically, socially

and educationally (p. 1048). The limited access Ms M's students had to their first language we see as core to their ongoing educational disempowerment. It compromised opportunities of using it to talk their way towards potentially enhanced mathematical understandings. While not denying the value of students developing proficiency in English as their second language, we assert that had the students' first language been legitimized as an *additional* linguistic resource, they would almost certainly have been better placed to engage more fully with the mathematical meanings they encountered.

In this paper we have shown how using a transdisciplinary framework to analyse mathematics classroom talk facilitates a richer, more multi-faceted investigation of such talk. As a conceptual paper, it draws on excerpts of one teacher's fraction teaching on one day to exemplify how working across, between and beyond the discipline boundaries of psychology, sociology and linguistics can produce a richer overview of classroom talk practices. While Vygotsky, Bernstein and Halliday each in turn emphasized the role of language and socio-cultural-historical factors in mediating learning, each provided a different gaze that - through our conceptual lens (Figure 2) converged to refract and project a multifaceted picture of classroom talk on fractions. Independently of the synergistic potential of their common interest in language as a mediating factor in learning outcomes, each theorist additionally provided - from their respective parent disciplines - different tools and terminology that allowed for a more nuanced articulation of certain ideas, so contributing to the visibility of different aspects of the teacher's use of talk in teaching fractions. Vygotsky's spontaneous and scientific concepts shed light on the way Ms M struggled to move her learners from the 'everyday' towards 'scientific' mathematical concepts. When, however, she invoked everyday imagery to support the concepts, the students made some progress. Bernstein's concepts of recognition and realization rules illuminated the way in which Ms M's evaluative rules undermined acknowledgement of learners' recognition and realization of the mathematics she was asking. Finally, in terms of Halliday's ideas around language as a system for making particular and specific kinds of meaning (in this case, the meanings made possible by a specialized mathematics register), Ms M's classroom talk extracts show that her students were a long way off realizing the full potential of meaning-making offered by the system network.

It is not possible within the confines of a single paper to do justice to the extensive bodies of work each of our three theorists represent. We trust, however, that we have provided insight into some of the possibilities transdisciplinarity provides for fuelling, and thus illuminating, dialogue across, between and beyond discipline boundaries in pursuit of deeper understanding, and thus potential remediation, of a shared concern: concern in this instance around facilitating students' opportunities to maximally exploit the potential inherent in their linguistic repertoires in the service of mathematical meaning-making.

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