Exploring Frameworks for Identifying Learning Dispositions: the Story of Saki

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This paper investigates how one sampled learner's mathematical learning dispositions evolved within the context of his participation in a weekly after school mathematics club over a one year period. The study is informed by Kilpatrick, Swafford & Findell's (2001) definition of a productive disposition (the fifth strand of mathematical proficiency) and Carr & Claxton's (2002) three indicators of key learning dispositions. This paper analyses the shifting nature of Saki's responses to an instrument focused on learning dispositions and points to ways of extending dispositional definitions and frameworks. We use Saki's learning story to illuminate the way in which restricted mathematical learning dispositions, particularly in terms of sense making and resourcefulness, can impede mathematical proficiency progress and thus require increased attention.

Introduction

This paper draws on a broader research project in which the first author, as part of the South African Numeracy Chair (SANC) project and as part of his masters research investigated the evolving learning dispositions of three learners participating in an after school mathematics club run by the second author. Data collection involved video recordings of club sessions, numerous individually administered learner interviews (including task based interviews) and written numeracy assessments. All learner interviews were conducted by the second author, as the club facilitator, audio recorded and later transcribed. Additionally the first author's research was supervised by the second author and insights from the broader SANCP research into learner dispositions informs this research.

It is beyond the scope of this paper to elaborate on all three case study learners' dispositions and so we have chosen to share the story of only one learner, Saki, in order to illuminate ways in which dispositional definitions and frameworks enable us to explore dispositions of learners as well as to unpack the limitations of these definitions.

In this paper we thus summarize and discuss the responses of Saki, to the productive disposition instrument (PD Instrument) developed by Graven (2012). In this paper we focus on and compare Saki's written responses to this orally administered written questionnaire in 2012 and his oral interview responses in 2013. Our work is guided by a socio-cultural theory of learning where learning involves developing dispositions and ways of being (Gresalfi & Cobb, 2006). Additionally our work assumes that learning to learn can be actively and deliberately supported by developing productive learning habits (Claxton & Carr, 2004). Our analysis of Saki's responses theoretically draws from Kilpatrick et al.'s (2001, p.131) definition of productive disposition as the fifth of five interrelated strands of mathemtical proficiency, i.e. 'the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner,' and Carr and Claxton's (2002) three dimensions of learning disposition (viz. resilience; reciprocity; playfulness).

Contextualizing the study

Across numerous national, regional and international studies (e.g. the Annual National Assessments (DBE, 2013); SACMEQ (see Spaull, 2011) and TIMSS (see Reddy, 2006) South African primary mathematics learners fare poorly. Fleisch's (2008) book, Primary Education in Crisis: Why South African schoolchildren underachieve in reading and mathematics explores influencing factors on South African learner performance such as parental education, books in the home, language in the home, opportunity for ECD care, families without parents as well as the effects of child labour. He argues however that even while the quantum of poverty in SA is experienced in other countries, the dependency of the poor and their profound disempowerment is perhaps greater than for the "poverty stricken peasantry of our neighbours of the north" (p. 59). This perhaps is a factor in why our performance in international comparative mathematics assessments such as TIMSS (Reddy, 2006) has us performing below other developing countries with less wealth. Thus Graven (2014, 9) argues 'more research is needed to examine ways in which dependent poverty and dependent passive learning dispositions might impact on mathematical learning'.

The results of the 2003 TIMSS study place SA 50th of 50 countries but furthermore foregrounded that had the largest variation in scores where the average scores of SA learners in African schools were almost half of that of historically white schools. Furthermore the average scores of these African schools has decreased from TIMSS 1999 to TIMSS 2003. Finding possible ways forward in the South African mathematics education crisis in ways that addresses the inequality and inequity of performance of our schools previously disadvantaged under apartheid is a key aim of the South African Numeracy Chair Project. This project began in 2011 and works with a large community of primary mathematics teachers and post-graduate researchers to explore ways forward to the many challenges faced. One innovation which was piloted in 2011 and introduced in 2012 is that of setting up after school mathematics clubs for Grade 3 and Grade 4 learners. This is discussed below as one such club formed the empirical field for the research on which this paper is based.

After school mathematics clubs

The after school weekly maths club that forms the empirical field for this study involved 6 learners participating in this club for about an hour. A central objective for the club was to develop learner sense-making in numeracy, 'shifting learner dispositions from being passive learners to becoming active participators' (Graven & Stott, 2012). Thus the clubs are intended to be a hub of increased sense-making where learners mathematically engage with content and with one another. This club is part of a broader after school club program which runs clubs across a range of schools and development centres in the broader Grahamstown area. The club program is expected to create an environment that is less structured than 'traditional' classrooms and that can offer opportunities and more affordances for the creation of active engagement, negotiation and participation for learners. Additionally the smaller numbers of learners in clubs (between 6-15 learners) enables increased individualized learner attention where activities can be directly tailored for where learners are in their numeracy development rather than dictated to by the grade level they are in at school. At the same time the clubs are not intended to overshadow in any way the normal and structured school curricula or program but clubs come in to 'provide more freedom to focus on the deliberate construction of positive participatory mathematical identities, at the expense of covering the range of skills and knowledge required to 'get through' the curriculum' (Graven & Stott, 2012, p. 96). The combination of the first author's role as researcher and the second author's role as facilitator enabled a strong working relationship for investigating learner dispositions. Three key research questions provide the focus for this paper:

1. What is the nature of Saki's mathematical learning disposition? How might this disposition evolve within the context of his participation in a weekly after school mathematics club over time?
2. What adaptations/elaborations of existing dispositional instruments are required to better access and assess learner dispositions?
3. What are the implications of this analysis for adapting/extending Kilpatrick, Swafford and Findell's (2001) definition of productive disposition?

Theoretical Framing

The study is underpinned by a socio-cultural perspective of learning. An emergent body of literature (see Schoenfeld, 1992; Schoenfeld & Kilpatrick, 2008; Lerman, 2000) conceives of mathematics learning as an inherently social activity, an essentially constructive activity. The definition of mathematical/numeracy proficiency that guides our research is that of Kilpatrick et al.'s five interrelated strands, namely: procedural fluency, conceptual understanding, adaptive reasoning, strategic competence and productive disposition. Productive disposition, as they define it,

> refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics. (p.131).

In relation to the fifth strand Kilpatrick et al. (2001, p.131) note that developing a productive disposition requires 'frequent opportunity to make sense of mathematics, to recognise the benefit of perseverance and to experience the rewards of sense making in mathematics.' They argue that productive disposition develops when other strands develop. For example, as students build strategic competence in solving non-routine problems, their attitudes and beliefs about themselves as mathematics learners become more positive. The more mathematical concepts they understand, the more sensible mathematics becomes. This coheres with the South African Curriculum and Assessment Statements for Foundation and Intermediate Phase (DBE, 2011a, 2011b) which also connects sense making and conceptual understanding and says that mathematics is a creative part of human activity and that learners should develop a deep conceptual understanding in order to make sense of mathematics.

Researching dispositions and identifying a gap in research literature

Schoenfeld (1992, 348), mentions five aspects of cognition drawn from a range of literature important for mathematical problem solving, these include: 'the knowledge base; problem solving strategies; monitoring and control, beliefs and effects, and practice'. While these can be linked to some of Kilpatrick et al.'s (2001, 131) strands of mathematical proficiency there is no mention of learning disposition although 'beliefs and affects' does connect with the part of Kipatricke et al.'s productive disposition definition that says 'to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner'. Carr & Claxton (2002, pp. 9-10) drawing on Carr's earlier work differentiate between capabilities and learning dispositions as follows:

> Crudely, we might say that this real-life 'learning power' (Claxton, 1999a) consist of two interrelated facets: capabilities and dispositions. Capabilities are the skills, strategies and abilities which learning requires: what you might think of as the 'toolkit'

> of learning. To be a good learner you have to be able. But if such capabilities are necessary, they are not of themselves sufficient. One has to be disposed to learn, ready and willing to take learning opportunities, as well as able.

Despite increasing attention being paid to learning dispositions (in general) over the past two decades however Graven (2012) asserts that there is little mathematics education research that elaborates on the nature of the relationship between learners' knowledge base (procedural fluency and conceptual understanding), problem solving strategies (adaptive reasoning and strategic competence) and learner dispositions. Such an understanding might be used to support the design of rich learning opportunities across the strands of proficiency. Thus, there is a gap in the maths education research on this strand (Graven, 2012; Graven, Hewana & Stott, 2013).

Gresalfi's (2009, p.327) view is that 'although this work has identified areas commendable of further enquiry, it has not stimulated the scrutiny of how classroom practice could boost learners learning practice and motivation.' She further argues that 'the literature does not help to explain why classroom practice does not impact all learners the same way or which aspects of classroom practice serve to support the development of various dispositions towards learning among learners who are members of the same classroom.' Gresalfi and Cobb (2006, p.329) assert 'thus learning is a process of developing dispositions; that is, ways of being in the world that involves ideas about perspectives on, and engagement with information that can be seen both in moments of interaction and in more enduring patterns over time.' Thomas & Brown (2007, p.8) noted:

> Dispositions involve 'attitude or comportment' toward the world, generated through a set of practices which can be seen to be interconnected in a general way…. dispositions are not descriptions of events or practices; they are the mechanisms that engender those events or practices. In short, dispositions capture not only to what one knows but how he or she knows it; and not only the skills one has acquired, but how those skills are leveraged.

Thus according to a range of literature developing a productive disposition requires frequent opportunity to make sense of mathematics, to recognise the benefit of perseverance and to experience the rewards of sense making in mathematics.

Bringing together Kilpatrick et al.'s 2001 indicators of Productive Disposition with Carr & Claxton's (2002) three dimensions of 'Resilience, Playfulness and Reciprocity'

Within the broader SANC project work on dispositions (e.g. Graven, Hewana & Stott, 2013) we have collaboratively worked towards combining the work of Carr and Claxton (2002) with the notion of productive dispositions and designed rubrics and observational grids that pull these together for the purposes of analysis. Below we explain the similarities and differences of Kilpatrick et al. (2001) and Carr and Claxton's (2002) key dispositional indicators as these together provide the framework for analysis of Saki's evolving disposition.

Table 2. Cross mapping dispositional indicators within definitions

Kilpatrick et al.'s (2001) indicators of a productive disposition	**Carr & Claxton's (2002) 3 dimensions of disposition**
Tendency to see sense in maths	Links to 'resourcefulness' – conceptual/explorative understanding
Perceive it as both useful and worthwhile	Not connected – no equivalent in Carr and Claxton's three dimensions
Believe steady effort pays off	Links to resilience
See oneself as effective and doer of maths	Links to some extent to resourcefulness however the notion of self-efficacy is not directly addressed in Carr and Claxton
No indication of willingness to engage with others as an indicator of a productive disposition	Reciprocity – willingness to engage with others

From the above 'reciprocity' (the third dimension of Carr and Claxton) is notably absent from Kilpatrick et al.'s (2001) definition. Conversely there is no link to the notion of seeing mathematics as useful and worthwhile in Carr and Claxton's three key learning dispositions - maybe because their suggested grid is not subject specific but developed from the early childhood learning context. However a positive affective relationship towards an area of learning like mathematics or even towards learning in general could be a useful fourth dimension or added indicator.

Resourcefulness links directly with sense making, conceptual understanding, adaptive reasoning, strategy (strategic competence etc.) and independence of learning in seeing that one can figure it out drawing on one's own thinking.

While actively seeking help might be considered resourceful it is not placed here as it goes against this sense of resourcefulness in terms of one's own ability. Additionally 'enjoyment or passion' - enthusiasm/creativity both are missing (Graven & Schafer, 2014). For the purposes of this research we have included this aspect as a dimension of strength to resilience, playfulness and reciprocity (when indicators are present).

Research Methodology

A qualitative research design with a case study approach was used for the broader study from which this paper emerges. A wide range of data collection tools informed the learner stories of the broader research. This range of data allowed for both triangulation of data and 'thick description' (Cohen, Manion & Morrison, 2005). However for the purposes of this paper we focus only on data gathered from one instrument used first as an orally administered but written response questionnaire and secondly as an interview.

The questionnaire involved several "complete the sentence" items. One involved locating oneself on a spectrum of learner performance (from 1-9), others involved describing Mpho and Sam who were explained to be weak and strong at maths respectively. A final question asked learners about what they do if they do not know an answer. Thus the five items discussed here include:

1. Maths is….
2. Effective scale (Range 1-9)
3. Describe an effective learner: Sam is…
4. Do you love maths or are you scared of maths?
5. What do you do if you don't know an answer in a maths class?

The instrument is taken from Graven (2012). Interviews were fully transcribed. Permission was obtained from both the centre where the club took place as well as from parents of learners. Learners were given the right to withdraw at any time and it was made clear to learners and parents that participation was entirely voluntary and that learners could withdraw at any time. Thematic content analysis was conducted on learner repsonses.

Findings and Discussions

When categorising learner responses to the instrument the following legend was used for indicators: Kilpatrick et al. (2001) = (K); Carr & Claxton (2002) = (C&C); Emergent Categories = (E).

Table 6. Summary of Saki's responses to the learning disposition instrument.

Indicator	*Questionnaire instrument item*	May 2012	May 2013
Effective learner and doer of mathematics (K)	*Scale 1-9 (Q2)*	9	9
Seeing mathematics as useful and worthwhile (K)	*Maths is: (Q1)*	Die beste (*the best*)	goed om te leer (*good to learn*) want dit help (*because it helps* it *makes you clever)*
Sense making (K) resourcefulness (E) (which includes what Carr & Claxton call playfulness (C&C)	*Maths is: (Q1)*		
	What do you do if you don't know the answer in maths class? (Q6)	Ek vra die juffvrou om te help (*I ask the teacher to help*)	vra die juffrou, (*ask the teacher*) tel op my hande (*count on my hands*) , tel op die tel-kaart (*count on the counting card*)
Steady effort (K) resilience (C&C)	*Describe an effective learner of mathematics (Q4)*		
	What do you do if you don't know the answer in maths class? (Q6)	Ek vra die juffvrou om te help (*I ask the teacher to help*)	tel op my hande (*count on my hands*) , tel op die telkaart (*count on the counting card*)
Reciprocity (C&C)	*No question directly related to reciprocity although some other learners indicated aspects of this when answering what they did when they did not know an answer. For example: 'I discuss it with my friend.'*		
Compliant behaviour (E)	*Describe an effective learner of mathematics (Q4)*		luister na die juvrou *(listens to the teacher*) hy doen goed want hy wen

Indicator	*Questionnaire instrument item*	May 2012	May 2013
			want hy luister (*he does well because he wins because he listens*).
	What do you do if you don't know the answer in maths class? (Q6)	Ek vra die juffvrou om te help (*I ask the teacher to help*)	vra die juffrou, (*ask the teacher*) tel op my hande (*count on my hands*) , tel op die telkaart (*count on the counting card*)
Enjoyment /affective relationship (with maths) (E)	*Do you love maths or are you scared of maths? (Q5)*	ek hou wiskunde (*I like maths*)	lief want dis goed en lekker om die tel.it te doen, hou om te tel (*love it because its good and nice to count and do it and I like to count*)
Language repertoire/able to describe mathematical activities (E)	*Maths is: (Q1):*		tel; minus,as gelyke (*count, minus and equals*)

From the above table we note the absences as much as the presences. Saki across both years clearly sees himself as an 'effective learner and doer of mathematics' (allocates himself at the top of the performance spectrum, at 9) and sees mathematics as worthwhile ('the best' and 'it makes you clever'). Additionally he 'loves maths' – although this is because he loves counting – 'I love to count'. Of interest, a love of mathematics is not included within Kilpatrick et al.'s (2001) definition and nor is a love of learning noted as a key dispositional dimension for Carr and Claxton (2002).

We see some shift in terms of his teacher dependence when he doesn't know an answer (in 2012) towards still asking the teacher but also having counting on fingers or on a 100 counting chart available to him as an alternative strategy (in 2013). In terms of his description of an effective learner of Mathematics (i.e. Sam is…) he moves from being unwilling or unable to describe him in 2012 toward describing him as a compliant learner who 'listens to the teacher'. While few teachers would argue against learner's compliance and listening, our new curriculum puts emphasis on sense making. Additionally individual learner agency, steady effort, resilience and so on are noticeably absent from Saki's shift. Additionally while Saki sees himself as good at mathematics his performance in other assessments does not match this self-evaluation. Thus perhaps it needs to be noted that various aspects of Kilpatrick et al.'s (2001) productive disposition should all be present for optimal learning. Thus in the absence of sense making confidence in one's mathematical ability can be problematic. For Saki since he eventually arrives at answers correctly through slow one-to-one finger counting for calculation (even in Grade 4) he sees himself as good at maths and enjoys it. He puts in 'steady effort' (using one-to-one finger counting), is compliant and listens to the teacher. However these two aspects of his disposition do not enable mathematical proficiency as his sense of mathematics as being worthwhile needs to be connected to the other strands of conceptual understanding, strategic competence, procedural fluency and adaptive reasoning. In this respect Saki's disposition is restricted.

In response to the latter two research questions, Saki's data points to some limitations of the instrument itself, which are explored elsewhere (e.g. Graven, Hewana & Stott, 2013; Graven & Heyd-Metzuyanim, 2014). Particularly that the instrument focuses on what learners say about mathematics or being good/bad at mathematics (or what they say they do – as in the last question) rather than what they actually do (which is better gleaned from observation) is a key limitation. Indeed we gathered much richer data from observing Saki across club sessions.

Saki's data also points to an absence of the notion of learner independence as an important dispositional trait. Compliance coupled with learner agency and an ability to make progress even when a teacher is not present perhaps need to be considered. Compliance and agency are not either-or but perhaps need to be considered and developed together. Observational data pointed to Saki's progress in mathematical proficiency being held back by his poor conceptual understanding and low levels of procedural fluency even while he displayed hard work and steady effort. He also showed a strong positive attitude towards the subject saying that he loved it. He was almost always first to arrive in the club and across all club learners had completed the highest number of homework pages each week. However his effort (and his love of maths) tended to foreground the method of one to one counting (with his fingers) often irrespective of how small or large the numbers in the calculation. As a result his performance progress was however slower than others in the club. His absence of sense-making – the aspect of Kilpatrick et al's (2001) definition of productive disposition - is thus essential for progress and this sense making clearly links to conceptual understanding as this is defined in sense making terms. Saki and the other case study learners, while not generalisable, have provided us with powerful illuminatory vignettes which carry the theoretical insights of this study forward into our future teaching and work in mathematics education.

Concluding remarks

It has been beyond the scope of this paper to explore evolving dispositions in detail or to review the data obtained from more than one learner. However we hope that we have illuminated both the strengths and limitations of both our instrument used to gather dispositions as well as the definitional frameworks of Kilpatrick et al. (2001) and Carr and Claxton (2002). A much larger study would enable elaboration of a much wider range of indicators perhaps with some specificity to South African learners.

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Mathematical Literacy: Are We Making Any Headway?

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Student enrolment trends in Mathematical Literacy and Mathematics since the inception of the NSC Mathematical Literacy is a focus of this paper. We found that enrolments for Mathematics largely lagged behind those for Mathematical Literacy. While the statistics was not fixed, we found a greater emphasis on trying to prove the uptake and success in Mathematics than Mathematical Literacy. We also explored and developed the critique of Mathematical Literacy assessment standards and compared those standards against an external education authority that was the reference for a benchmarking process by the Department Of Basic Education in 2013. We determined that the South African programme compared unfavourably against that of New South Wales. We also showed that the external body included algebraic modelling and financial mathematics which seemed to align better with our Mathematics curriculum than Mathematical Literacy.

Lastly, we analysed the environment of Mathematical Literacy tasks and the cognitive loading of those tasks, and proposed, by way of illustration, ways in which similar tasks could be improved. These illustrative tasks we called *authentic tasks*.

Our conclusion is that Mathematical Literacy in its present form offers a weakened version of a mathematically based subject for "making sense" in the world and should be improved.

Introduction

Since the introduction of Mathematical Literacy (ML) in the Further Education and Training (FET) curriculum in 2006, all Further Education and Training (FET) students were required to take either Mathematics or ML as a subject. The Mathematics/ML divide replaced the system whereby students could chose to do Mathematics or not, and if they chose to do Mathematics, whether they could do Mathematics on the so-called Higher Grade or Standard Grade. This move was listed in government policy documents as an attempt to provide every student taking ML with the mathematical tools needed to make sense of, participate in and contribute to the twenty-first century world, a world characterised by numbers, numerically based arguments and data representation. Such competencies include the ability to reason, make decisions, solve problems, manage resources, interpret information, schedule events and use and apply technology to name but a few (NCS ML assessment guides, p.13).

This definition concurs with, among others, Schleicher (1999, p.39):

> Mathematical Literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well informed judgements, and to engage in mathematics in ways that meets the needs of that individual's current and future life as a constructive, concerned and reflective citizen.

The emphasis in the definitions is on "making sense" of the world through the use of mathematical tools, and included in the list of tools are skills such as "ability to reason" and " solve problems", two key features which act, to some extent as markers for the quality of the ML that is delivered. The use of "world" is understood to mean the world outside the classroom (although the latter provides the setting where ML takes place). To match up these two settings, researchers have suggested the use of *authentic contexts* to act as a bridge (Madison, 2014).

Mathematical literacy is not the same as mathematics, nor is it an alternative to mathematics – ML and Mathematics are two different subjects at the FET phase. In the words of Steen "Mathematics is abstract and Platonic, offering absolute truths about relations among ideal objects" (Steen, 2001, p.1). Madison (2004, p.12) posits that the reason why these two subjects were separated could well been that those advocating for the teaching of mathematics feared that "teaching contextualised mathematics will water down the mathematics, that fewer students will learn the formal mathematics for science and engineering". Is this the rationale for the National Curriculum Statement Mathematical Literacy? The trend whereby students are divided along academic grounds according to whether they take ML or Mathematics is clearly observed in South Africa, where more students are opting to study ML than Mathematics at the FET phase, are as a resulted counted out of the major science, engineering, certain business and medical fields as possible careers.

The divide between ML and mathematics, understood in the traditional sense as composed of content such as algebra, geometry, trigonometry and calculus, favours a strong academic framing for mathematics and weaker academic framing for ML. This divide has implications such as access to certain fields of study at post-secondary schooling but also has the uncomfortable effect of dividing students along academic status lines in school (to all involved). ML does not enjoy the headline attraction of mathematics; in contrast it is marginalised. At best it is seen as a "safer" option to attain credits to enable students to success at the very significant high stakes examinations, notwithstanding its limited options beyond schooling.

This study

The challenge for ML resides in the question: Do the current ML teaching and assessments practices do justice in meeting the goals of ML as described by the NCS? If not, should the stakeholders in mathematics education be concerned? Although much of this study is concerned with the migration of students from Mathematics to ML, a word of caution is necessary here: ML can and must be seen as a subject in its own right. Consequently, critiquing ML has relevance outside of the debate of how and why students have ended up taking that subject. This study considers the implications from both angles but favours and emphasises ML as the subject of study. We seek to demonstrate that more students at the FET phase are opting to study ML, partly because it is perceived to be an easier option to get a mathematical subject pass. Many do the subject because they have no need to utilise mathematics subsequent to school and are not attracted by the promise of the worthwhileness or beauty of mathematics. In addition, the study also seeks to show that the teaching and assessment of ML uses non–authentic tasks which demand very low level cognitive skills, which is contrary to what the NCS states. Given also that students who study ML cannot enrol in quantitative disciplines such as science, technology, engineering, and mathematics (STEM) we are left wondering what are the future implications of so many learners taking ML in the mathematics education landscape in South Africa are designed to be? This will be addressed at the end.

To consider these issues we therefore, in this study, i) survey the trends in student enrolments for ML and Mathematics, both at a national, provincial and local (school) level (using the Western Cape as an example), ii) revisit the assessment standards against other established education bodies, and, iii) consider the notion of authentic environments, the place it has in a ML classroom and ML assessments, against the current reality, as reflected in NSC ML assessments. We analyse statements made (or not made) by academics and government officials in relation to these trends and ask some probing questions as a consequence. In particular, we posit whether ML in its present form can or should continue.

Enrolments in Mathematics and Mathematical Literacy

At national level, the trend has been for ML to increase and Mathematics to decrease (Table 1). Although Mathematics shows an increase (from 2012 to 2013 it was 15 469) it is the comparatively more dramatic increase of ML that is the concern of this paper (33 715). More so, 85 126 more students were enrolled for ML compared to Mathematics. While we must applaud the fact that in the new curriculum everyone is engaged in a mathematically based subject, there is some evidence that the quality of the ML assessment is of a reduced quality and the implications of this has far reaching consequences, especially for the candidates.

Table 1. Enrolments of students in ML and Mathematics between 2009 and 2013

	2009	2010	2011	2012	2013
Mathematical Literacy	284 174	288 370	281 613	297 074	330 789
Mathematics	296 164	270 598	229 371	230 194	245 663

Source: Technical report 2013 Department of Basic Education, p.21

At Western Cape provincial level, the strategies for Mathematics and Science document is illustrative in a number of ways. If Mathematics is the foil to ML then intentions about Mathematics need to be read both in its own right and for signs about the provincial or national intentions for ML. As we see stated in this document the province is intent on improving both the enrolment figures for Mathematics as well as the results in the examinations, especially for grade 12. Is there a parallel strategy for ML? Although there is acknowledgement that the behaviour of one may directly influence that of the other (enrolment is an obvious example but trends in assessment may be another):

> An analysis of the enrolment trends over the last 3 years indicates a downturn in the numbers taking Mathematics, both as an absolute value (2 412 fewer since 2008) and as a percentage of the total enrolment. *The reduction in numbers is linked to increases in numbers of those taking Mathematical Literacy*. (Western Cape Education Department Strategy, p.1, *my emphasis*).

Admittedly, this is document dealing with strategies for Mathematics and Science, two key subjects for admission to certain mathematics and science based tertiary programmes. Nonetheless, the document does provide useful insights, albeit obliquely, into ML in the province. Table 3 offers such an opportunity.

Table 2. Overall mathematics enrolment in the Western Cape Education Department between 2008 and 2011.

Overall Mathematics enrolment				
	Grade 10	**Grade 11**	**Grade 12**	**Total**
2008	**35 036**	**29 418**	**18 668**	**83 122**
2009	**29 818**	**27 122**	**18 778**	**75 718**
2010	**29 513**	**22 965**	**16 922**	**69 400**
2011	**29 893**	**23 233**	**13 375**	**66 501**

Source: Western Cape Education Department, 2013

In line with the decrease in Mathematics enrolment, one must assume, as the report states, an increase in ML enrolment. So as enrolment in mathematics declines from 35,036 in grade 10 in 2008, to 27,122 in grade 11 in 2009, allowing for failures and drop outs we are looking at a figure of 7914, some of which must end up in the grade 11 ML classes. Again from grade 11 in 2009 to grade 12 in 2010 the drop in Mathematics is 10 200. This figure contains failures in grade 11, dropouts and those migrating to ML. While the concern for the drop in Mathematics is huge and valid, the increase in ML, given what kind of assessments learners doing ML are exposed to, and the implications thereof, is surely of equal concern. As the Mathematics enrolment decreases from 18 668 in 2008 to 13 375 in 2011, this has to be taken in conjunction with the pass rate figures (notwithstanding that those achieving over 30% but less than 40% are considered having passed). Increases in the number who pass ML leads to dubious joy: it illustrates that more are passing ML than mathematics, both in number and as a percentage and it illustrates that more learners will be drawn to ML if it means a pass mark on a NSC certificate (ML) and not a fail (Mathematics). Such simple analysis is unfortunately born out at the local school level: In a Western Cape school, the principal reported that, on account that all his pupils have failed mathematics in grade 12 in 2007, the school decided that they would not offer “pure” mathematics the following year. Similarly, the principal of another school stated that the school did not have any mathematics pupils in 2014 (as reported in the newspaper Cape Times, 29 July 2014, p.3). In declaring its aim to increase the number of learners taking Mathematics and to improve the results in Mathematics in the province, the impression is strongly created that the departmental mathematical goal is geared towards Mathematics not ML. Is this an indirect acknowledgement that ML, certainly in its current form, is not desirable from the provincial point of view? No parallel document at present exists for improving ML.

In July, 2014 the Minister of Basic Education told parliament that there were 327 schools “at which no Grade 12 pupils has registered to write matric maths” (Cape Times, 29 July, 2014, front page). The newspaper article continues with a direct quotation from the minister: “This does not imply that the school did not offer mathematics, but rather there were no learners who registered for mathematics” (p.3).

Furthermore, the paper continued, the Department (of Basic Education) were unable to “indicate precisely” (their direct quotation from the minister’s response) why pupils were choosing maths literacy rather than mathematics. Five years into the new NSC Mathematics/ML divide, this is quite an extraordinary admission.

Assessments in Mathematical Literacy

Benchmarking of ML was done for the first time in 2013. The verdict was that “the paper (2013 ML) was considered to assess critical thinking and was deemed comparable to General Mathematics offered by the Board of Studies New South Wales (NSW) in Australia” (Technical Report, Department of Basic Education (DBE), p.44-5).

The present reality of ML assessments is that emphasis have been placed on level 1 and 2 of the assessment taxonomy (knowing; and, apply routine procedure in familiar context) which together can account for 60% of the final mark, while level 3 (apply multi-step procedure in a variety of context) and level 4 (reasoning and reflecting) type questions are minimally represented. Where they are, they are often reduced in complexity (thus cancelling them as a complex problem solving type) due to the ‘multi-scaffolding” that is present, resulting in a multi- procedural series of steps, often with additional information about the steps to take provided. (Venkat, Graven, Lampen & Nalube, 2009, p.50)

Venkat et al. (2009), present a cogent argument that the shortcomings in the structure of the ML assessment taxonomy preclude the development of reasoning and problem solving competencies in the ML assessment. These, they say, lead to the kinds of questions found in ML assessments, questions which place a high emphasis on routine procedures. They follow four threads of mathematical development in the taxonomy to determine how complexity is conceptualised and suggest that there is a “tendency towards procedural orientations to progression” and “the notion that the degree of “immediacy” of information availability and/or “explication” of the required mathematical tools provides a sub-thread contributing to mathematical progression” (Venkat et al., 2009, p.50)

The General Mathematics NSW Australia pre-2014, on the other hand, contains sections not covered by ML (NCS, 2003), such as algebraic modelling and financial mathematics (covered in greater depth) (NSW, Australia, 2001). A brief overview of a sample of questions, taken from NWS Board of Studies Specimen Paper (copyrighted) indicated that their standard may be higher: although on the face of it some questions looked similar in standard, the absence of sub-scaffolding in the NSW questions placed the questions on a higher scale. Also there were questions which dealt with cognitive areas that were not tested in the ML question papers, such as algebraic modelling. Learners have to reason *without any cues* in the NSW papers, thus adding to the cognitive load.

The ML questions are typical of the style and intention of the question papers, as also pointed out in explicit detail five years ago by Venkat et al. (2009): questions in the main direct the candidate to what needs to be done in order to successfully answer them. Examples abound where formula are provided in the question and candidates are instructed to “use the formula” to answer the question. In some cases the formula has been manipulated (by, for example, changing the subject *beforehand*), requiring the candidate to substitute the facts from the information provided, without indicating that the formula is understood. The candidate is also prevented, on account of such direct instructions, to seek alternative solutions. The questions tend to be closed. The general approach in the two ML examination papers appears to be to set questions of a decent standard (in the main) but then break these down into sub-sets and guide (or direct) candidates towards solutions, including providing formulas at appropriate times, with details about what information to input. This excessive *dumbing down* of the ML assessment appears to be geared towards the group of learners with the least chance of understanding what ML (the proper version) is about, given their circumstances. And given that teaching tends to follow assessment, the options for improving the quality of the ML assessment appear few.

It is perhaps in that vein that the benchmarking process revealed some gaps in our assessments: learners did not get “adequate opportunity to demonstrate critical skills” in the assessments. (Department of Basic Education, Technical report, p.45)

Alarmingly, therefore, in spite of strong evidence that ML is not delivering to its original curriculum policy intentions, and despite the many career doors at tertiary institutions that are closed to the student who completes grade 12 with ML not Mathematics, consistent trends seem to indicate that ML enrollment in schools continues to increase. The corollary to that is that Mathematics enrollment is reducing, or where it shows an increase, in not increasing at the same rate. This is highly worrisome to many in education and broadly in society, especially those concerned with growing our numbers in the engineering, science, technology, certain business and the medical fields. In the following we describe ML learning environments, and we give example of a level question that can be used in ML high level (levels 3 & 4) authentic assessments.

ML authentic learning environments

The National Curriculum Statement (NCS) (2011, p.8) identifies authentic real-life contexts as one of the tenets of ML. The question that follows from this is: Are students exposed to authentic assessment questions/activities during the high stakes ML examination and/or tests during the FET phase? In this section we consider: what should an authentic learning environment for ML entail? Secondly, what learning theory supports authentic learning environments?

In this study we use the term ‘authentic learning environments’ to refer to an ambient that “provides a context that reflects the way knowledge and skills will be used in real life” (Gulikers, Bastiaens & Martens, 2005, p. 509). Establishing a connection between classroom and the contemporary world will both enhance student learning, and keep students abreast on whatever is taking place in their immediate environments (Madison, 2014). In this study we call these types of tasks: *authentic tasks*. Palm (2002, p.7) posits that:

“Authentic task” refers to one in which the situations described in the task compares favourably with a real-life situation outside the world of school mathematics. In addition, the task situation is truthfully described and the conditions under solving the task takes place in the real situation are simulated with some reasonable comparison in the school situation.

In ML learning environments, students solve mathematical activities that are embedded in authentic tasks. The main sources of the authentic sources are varieties of media articles, and contexts in non-school environments, for example, policy documents (Vos, 2011, Mhakure, 2014). It is important to note that if media articles and policy documents are used, it then it has to be assumed that these authentic learning environments change rapidly, thus necessitating the exploration of ways to develop characteristics of *adaptive expertise* (Madison, 2014). In this study we argue for the teaching of ML in authentic learning environments. If this is acceptable, then authentic assessments should typically include authentic tasks where students are expected to “demonstrate the same (kind of) competencies, combinations of knowledge, skills, attitudes, that they need to apply in criterion situations” in real-life. (Gulikers et al., 2004b, p.5).

Cognitive apprenticeship as theory that support ML learning environments

The cognitive apprenticeship theory – is foregrounded within the broader social constructivist paradigm. Cognitive apprenticeship has roots in and is strongly influenced by traditional apprenticeship model, which most of us are aware of, where learning takes place as novices and experts interact socially emphasising teaching skills in the context of their use (Collins, 2006; Dennen, 2006; Wang & Bonk, 2001). The difference though between the two apprenticeships is that "cognitive apprenticeship emphasizes the solving of real world problems under expert guidance that fosters cognitive and metacognitive skills and processes*"* (Wang & Bonk, 2001, p.132) whereas traditional apprenticeship focus on the completion of tasks in the psychomotor domain where the apprentice "owns the problem" of moving on to the acquisition of the next skill (Berryman, 1991). Central to the cognitive apprenticeship theory in formal education is that students learn to become practitioners – not simply learning about the ML practice but by engaging with real world authentic contexts (Dennen, 2006). Thus, the cognitive apprenticeship, as an alternative to other conventional approaches to formal education and training, aims to "produce graduates with equal thinking and performance capabilities" (Bockarie, 2002, p.48). We find this theory of learning quite useful in understanding of how the teaching of ML using authentic tasks could help students in acquiring lifelong mathematical skills.

An example of a ML authentic assessment questions

This particular example which shows how ML can be assessed (or taught) uses an excerpt from the Cape Times newspaper (4th March 2009). The mathematical content skills required in the NCS (2011) fall under the topics *"Patterns, relationship and representations"* and *"Measurements".* The specific learning outcomes from the two topics are "compound growth and other non-linear relationships" and "measuring length, conversions, and calculating area" respectively. The mathematics of the excerpt is aligned to the key five tenets of ML, that is, ML involves: the use of elementary mathematical content, authentic real-life contexts, solving familiar and unfamiliar problems, decision making and communication, and the use of integrated content and/or skills in problem solving (NCS, 2011). In the NCS (2011, p.8) it states "learners must be exposed to real accounts containing complex and "messy" figures rather than contrived and constructed replicas containing only clean and rounded figures". What is also intriguing about this excerpt and was highlighted by Gulikers et al. (2005, p.510) is that it deals with issues "from real life outside mathematics itself that has occurred or that might well happen"

The extract below appeared in the newspaper Cape Times on 4 March 2009. There are some errors and inaccuracies in the data given in the extract. The following questions will guide your investigation into these errors.

Development land poser in Cape Town as city size expected to double by 2030.

Reporter - Anel Powell.

The City of Cape Town, estimated to be almost 35 000 hectares (ha) in size in 2007, will double by 2030. This means it is expanding by 650 ha annually, increasing by what is the equivalent of two rugby fields in size every day.

Using the latest available aerial photography, taken in 2007, the city's urban growth monitoring project has analysed the city's urban development since 1945.

The city in 2007 was seven times larger than it was in 1945. Since 1977, the city has been growing at a fairly constant 2 percent until 1988, with the rate declining to 1.5 percent thereafter. City and population growth were similar until 1988, when the population growth outstripped the city's growth.

Questions

1. This question refers statements *"The City of Cape Town, estimated to be almost 35 000 hectares (ha) in size in 2007, will double by 2030. This means it is expanding by 650 ha annually, increasing by what is the equivalent of two rugby fields in size every day".*

Do a calculation to correct the statement "by what is the equivalent of ***two*** rugby fields in size per day".

(*Hint: a rugby field measures 50m by 100m*)

2. This question refers to the statement *"The city in 2007 was seven times larger than it was in 1945. Since 1977, the city has been growing at a fairly constant 2 percent until 1988, with the rate declining to 1.5 percent thereafter".*

Use the data in the statement to find the size of the city in 1977.

Figure 1. An example of a high level ML question

This example typically shows the type of data that learners in ML could be exposed to as critical citizens. Earlier in the paper we raised the question whether FET students are exposed to these types of questions during the teaching and assessment of ML. The answer to this question appears to be negative. There is doubt that the NCS is clear on the type of learner who should graduate with certain skills in quantitative reasoning. Our analysis of the assessments questions show that learners are not sufficiently developed in order to cope with the higher cognitive level authentic tasks that we propose here. The question which should be posed to ML stakeholders, especially, those involved in teacher education, is: What can be done to the teaching and learning of ML so the learners can acquire the higher cognitive skills they require in order that they can function in careers and fields where those generic skills are vital. They also need them, in line with the general impetus for creating a mathematical subject for all: to function as critical citizens in a democratic society?

Conclusion

In this paper we surveyed trends in student enrolments in Mathematical Literacy and Mathematics since the inception of the NSC Mathematical Literacy. Our objective was to determine whether there were signs of migration by students, schools and provinces from Mathematics to ML. We also wanted to determine what the official Education Department (national and provincial) position was, via reports and public statements, with regard to these trends. We found evidence that there were declining enrolments for Mathematics across the board and that, as a parallel movement there were increases in enrolment for ML. The picture is not static, because one provincial department was shown to be working hard at changing the pattern and was actively supporting schools that had shown a degree of success in Mathematics, in order for them to increase their enrolment and future success. We questioned whether this emphasis on improving Mathematics, in absence of statements about ML did not in effect follow an international trend whereby ML and courses like those are essentially marginalised. Given the huge numbers of students who do ML we want to raise the question about the long term implications allowing this trend to continue, especially in light of the critique of ML, which was our next focus.

We critiqued the current assessment standards and show that the critique at the onset of ML still applies today. In light of the benchmarking process the DBE undertook for ML in 2013, we interrogated the standards of the New South Wales, one of the bodies DBE chose for its benchmarking process, against the current standards of ML and found the latter to be wanting in certain respects. We found that there was no algebraic modelling in ML and that the financial mathematics resembled the DBE Mathematics better than ML. We also found that the standard of the questions was higher than ML.

Lastly, we assessed the contexts created in ML against a notion of authentic environments and illustrate through an example the possibilities which exist for bringing the classroom context closer to a real life situation to give effect to the motivation that ML is designed to "make sense" of real world contexts. We found that some of the contexts that are used in the teaching of ML do not mirror the way the ML content is experienced in everyday lives as evidenced by media excepts.

Implications

What are the long term implications of huge numbers of students migrating from mathematics (as a learning subject) to ML (as a learning subject) at the FET phase? What are the implications for the individuals (career choices, life choices), education departments (admission into certain disciplines) and the country as a whole (reduced numbers in specific fields in our economy and the implications for our place in the world)? In light of the critique about ML in its own right, as a subject of choice for those who genuinely have chosen not to pursue careers which require Mathematics (notwithstanding the concern about a lack of numbers doing mathematics) that concern takes on a different flavour: what should such students be doing and why? We have no doubt that the sense of the purpose for introducing the subject ML as part of the NCS is valid and just; our analysis shows that the implementation, as outlined against the assessments, falls far short of delivering the kind of subject that is promised in the policy statement for ML. By drastically lowering the assessment standards, by "multi-scaffolding" tasks which may go some way to being "authentic" and of a higher level, and thereby watering down their cognitive demand, by perpetuating a pattern which lends credence to the notion that ML does not benchmark favourably with more established versions elsewhere, and more pertinently, does not prepare students for roles in society outside of certain disciplines and careers, we do those who take ML as a subject a great injustice. We also do ourselves, in all respects, a great disservice.

Recommendations

There is no doubt that ML has become the refuge of all who fear Mathematics. As such the fate of ML is bound up with that of Mathematics. As a first win-win, the crisis surrounding Mathematics needs to be sorted out as a matter of urgency. Fortunately, stakeholders are engaged in this process in an on-going basis, but more can always be done. In parallel, because ML is a subject in its own right, ML needs to be raised in terms of quality and attraction for the right reasons. Benchmarking must be followed by change to the ML curriculum where this is found to be needed. The use of authentic environments is a suggestion that can be explored more fully and assessments must follow classroom based methods as closely as possible, and less the other way around. The use of multi-scaffolding should be downgraded in order that a greater differentiation can be introduced into the ML assessments. Problem solving as a concept, which does not necessarily require the use of numbers or data, can and should be explored, in line with the NSW, a benchmarking ally of standing, and other education departments elsewhere. The use of algebraic modelling should be introduced into the ML curriculum.

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