Identifying Stages of Numeracy Proficiency to Enable Remediation of Foundational Knowledge Using the Learning Framework in Number

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The Annual National Assessments (ANAs) implemented in all South African government schools since 2011 in primary Grades 1-6 and Grade 9 point consistently across the three years for which results are available to learners operating far below their grade level in numeracy and mathematics. While a key aim of the ANAs is to provide system wide and teacher specific information on how to pin point several challenges the results are so poor for the majority of learners that the assessments fail to identify for teachers the numeracy development level at which their learners are at and thus fail to provide useful information for informing remediation interventions. This paper reports on each of our use of a numeracy assessment and recovery framework from the Maths Recovery Programme (by Wright and colleagues) as a tool for assessing learner levels of numeracy proficiency across learners in four foundation and intermediate phase after school mathematics clubs in the Eastern Cape. The findings across these studies point to the usefulness of this tool for planning subsequent interventions. The paper illuminates, through examples of data gathered across each of our research projects, the usefulness of identifying stages of numeracy development across their different research foci.

Introduction

Mathematics Education in South Africa is widely acknowledged to be 'in crisis' (for example Fleisch, 2008) and increasingly attention is diverted from only addressing the problem in the Further Education and Training band (FET) to addressing it in the early foundation years of learning. The Foundations for Learning Campaign (Department of Education, 2008) was introduced by the DBE in 2008 in order to bring a specific focus to improving reading, writing and numeracy in South African learners. One feature of this campaign has been the introduction of systemic assessments in the form of the Annual National Assessments (ANAs) written in Numeracy/ Mathematics and Language in Grades 1-6 and Grade 9 in all government schools.

The results of these assessments over the past three years confirm that the majority of learners do not have basic numeracy skills and that with each progressive year of schooling more and more learners lag behind meeting the basic requirements for their grade level (Schollar, 2008). The data given below constructed from the 2013 ANA report (Department of Basic Education, 2013) shows these results:

Table 1: 2012 and 2103 National ANA results for grade 3 and 4

	2012 learners achieving less than 50%	2012 Learners average	2013 Learners achieving less than 50%	2013 Learners average
Grade 3 National	36.3%	41.2%	59.1%	53.1%
Grade 4 National	26.3%	37%	27.1%	36.8%

We can thus conclude from the above table that the majority of South African Grade 3 learners in 2012 had not developed foundational number sense before entering the intermediate phase (IP) and that while the figures had improved somewhat in 2013, still almost half the learners had not achieved what the Department of Basic Education terms 'acceptable achievement >/50%' or what we term basic foundational number sense required for enabling progressive learning in the intermediate phase. We note also in the table the large drop in results from Grade 3 to Grade 4 and argue that this is likely the result of learners having to learn intermediate phase content without having the requisite foundational mathematical knowledge of the foundation phase.

Given that the above data points to the majority of intermediate phase learners not having the grade level competence of the grade in which they are studying points to a problem with the opportunity to learn and the validity of the assessments that they participate in. So for example a Grade 4 learner asked to solve 243 x 59 in class or in an ANA cannot participate if they are still at the level of drawing three groups of 9 in order to find 3 x 9. In this sense their performance on this ANA question would tell the teacher little about the level of multiplicative reasoning that the learner does have and whether remediation should begin with focusing on grade 1, 2 or 3 work.

A wide range of research points to the need for coherence and progression in the teaching of mathematics (Askew, Venkat, & Mathews, 2012; Schollar, 2008). However teachers are unlikely to identify useful resources or generate resources with carefully inlaid progression without a solid understanding of the level at which learners are operating and the various levels through which learners must progress in order for foundational numeracy proficiency to be sufficiently in place in order to progressively progress through the mathematics required in the IP grades.

In this respect, across each of our research projects, we have found the work of Wright, Stafford, Stanger and Martland (2006) on delineating levels of mathematical progress in their early Learning Framework in Number (LFIN) to be particularly useful. We have used this framework not only for our analysis of learner levels of mathematical understanding in order to design learning activities but also for teacher development. Wright (2013) has argued that the interview tool from their mathematics assessment and recovery programme is useful for teacher development and understanding the developmental nature of numeracy learning. Wright et al.'s (2006) Maths Recovery (MR) programme is gaining popularity both internationally (it has been used in Australia, New Zealand, UK, USA, Canada) and in South Africa (see for example Weitz (2012), Mofu, (2013), Ndongeni (2013) and Stott (2014)) and has thus been tested across multiple contexts.

In this paper we discuss the ways in which our four research projects, in the context of primary after school mathematics clubs, drew usefully on Wright et al.'s framework in order to illuminate the usefulness of this framework as both an analytical and a developmental tool for informing teaching practice.

The empirical field of our research

The South African Numeracy Chair Project (SANCP) is tasked with researching sustainable ways forward to the many challenges faced in primary mathematics education in South Africa. As part of this project one development initiative that we piloted in 2011 and rolled out in 2012, was that of after school mathematics clubs. Within the SANC project, the clubs serve two purposes: firstly, they are a place where we can directly influence what happens with learners and secondly, they provide us with an empirical research field in which we can interact directly with the learners and thus be insiders to the learning process.

These clubs are conceptualised as informal learning spaces focused on developing a supportive learning community where learners can develop their mathematical proficiency, make sense of their mathematics and where they can engage and participate actively in mathematical activities. Individual, pair and small group interactions with mentors are the dominant practices with few whole class interactions. The clubs were intentionally designed to contrast more formal aspects observed in the classrooms of the SANC project participating schools (Graven & Stott, 2012; Graven, 2011). Of note is the promotion of learner-centred practices in clubs are also promoted in the official curriculum documents (Department of Basic Education, 2011) and which Hoadley (2012) notes are absent in South African classrooms.

All four authors are part of the SANCP. All ran a club in 2012 / 2013 and conducted research in their clubs drawing on Wright, Martland and Stafford's (2006) assessment interview instrument in order to assess learner levels of conceptual understanding as part of their broader club research. Details of the research are available in their theses and other publications (see for example (Graven, 2012; Mofu, 2013; Ndongeni, 2013; Stott, 2014; Weitz, 2012).

Theoretical framework, methodology and analytic tools

Across all of our research we have taken a broad socio-cultural perspective in relation to interpreting learner understanding and progression. This assumes that learning is an active construction of knowledge through social interactions with others.

Wright et al. (2006) state that they are "strongly constructivist" (p. 7). Their work is based on the principles that learning mathematics is an active process, each child constructs their own mathematical knowledge and that they develop mathematical concepts as they engage in sense-making, mathematical activity. Their MR programme is based on sense making and mathematical activity and normally takes place alongside a teacher or other adult. In this way learners are not working on their own discovering knowledge per se but are assisted by a more knowledgeable other. This view coheres with those taken by each author in their individual studies.

The Learning Framework in Number (LFIN) developed by Wright and his colleagues (2006) provided us with a useful way of tracking and assessing mathematical progress over time. Wright et al. (2006) described the LFIN as providing a "blueprint for the assessment and indicates likely paths for children's learning" (p.7). This framework has been used to research and document progress in number learning of five to eight year old students in the first three years of schooling. As an intervention programme it involves intensive one-to-one teaching of low-attaining students but the programme has also been used with students of all levels of attainment (Bobis et al., 2005).

The four research studies detailed here were qualitative and drew on the case study research design. The data gathering method used for investigating the aspect of our research reported on here were structured interviews, particularly those used in the MR programme (Wright, Martland, & Stafford, 2006). It is beyond the scope of this paper to include the entire interview script but one item from the interview is given in Figure 6 below as an example:

If I tell you that eight times seven is 56	Show this card 8 x 7 = 56
Can you use these numbers and signs to make a division sentence?	8 7 56 = ÷
Can you make another division sentence?	

Figure 6. Sample interview task (Commutativity and inverse relationship in multiplication and division) (Wright, Martland, & Stafford, 2006, p. 182)

Each author used one, some or all of the LFIN aspects as an analysis tool for their study. In this section we detail three of the five LFIN aspects and the associated levels or stages, so as to illustrate how we determined where to position learners on the LFIN using data collected from the interviews. We specifically worked with a version of the LFIN that combines elements from Wright et al.'s 2006 and 2012 works. The key aspects of the LFIN are:

- Structuring numbers 1 to 20
- Number words and numerals (including forward and backward sequences)
- Conceptual place value knowledge (ability to reason in terms of tens and ones)
- Early arithmetic strategies (strategies for counting and solving simple addition and subtraction tasks)
- Early multiplication and division
(Wright, Ellemor-Collins, & Tabor, 2012; Wright, Martland, & Stafford, 2006)

Each of the key aspects of the LFIN are elaborated into a progression of up to six levels or stages with each model describing the characteristics of the levels or stages (Wright, Martland, Stafford, et al., 2006). These are detailed below for Conceptual Place Value, Early Arithmetic Strategies and Early Multiplication and Division.

Table 2. Conceptual place value

ASPECT C: Conceptual Place Value (CPV)		
Level Number	Level Descriptor	Characteristics
1	Initial concepts of 10 (ten as a count)	Not able to see ten as a unit composed of ten ones. The child solves tens and ones tasks using **a counting-on or counting-back strategy**. One 10 and 10 ones do not exist for the learner at the same time
2	Intermediate concepts of 10 (ten as a unit)	**Able to see ten as a unit composed of ten ones**. The child uses incrementing and decrementing by tens, rather than counting-on-by-one to solve uncovering board task. The child cannot solve addition and subtraction tasks involving tens and ones when presented as horizontal written number sentences
3	Facile concepts of 10 (tens and ones)	**Tens and ones are flexibly regrouped**. Ten is a unit that can be repeatedly constructed in place of 10 individual ones. Child is able to solve addition and subtraction tasks involving tens and ones when presented as horizontal written number sentences by adding and/or subtracting units of tens and ones

Table 3. Early arithmetic strategies

ASPECT D: Counting as a problem solving process (Early Arithmetic Strategies)		
Stage Number	Stage Descriptor	Characteristics (representing increasing levels of sophistication)
0	Emergent counting	**Cannot count visible items**. The child might not know the number words or might not coordinate the number words with the items
1	Perceptual counting	Can count only **visible** items starting from 1. Including seeing, hearing and feeling
2	Figurative counting	Can count **concealed items** but the learner will 'count all' rather than 'count on'.
3	Initial number sequence	Initial number sequence. The child can **count on** rather than counting from one, to solve + or missing addends. May use the **counting down** to solve removed items. (count-back-from)
4	Intermediate number sequence	**Count-down-to** to solve missing subtrahend (e.g. 17-3 as 16, 15 and 14 as an answer. The child is able to use a more **efficient** way to count down-from and count down-to strategies (count-back-to)
5	Facile number sequence	Uses of range of **non-count-by one strategies**. These strategies such as compensation, using a known result, adding to 10. Commutativity, subtraction as the inverse of addition, awareness of the 10 in a teen.

Table 4. Early multiplication and division strategies

ASPECT E: Early Multiplication and Division		
Level Number	Level Descriptor	Characteristics (representing increasing levels of sophistication)
0	Initial grouping and perceptual counting (Forming equal groups)	Able to model or share by dealing in equal groups but not able to see the group as composite units; **count each item by ones.**
1	Intermediate composite units (Perceptual multiples)	Able to model equal groups and counts using rhythmic, skip or double counting; counts by ones the number of equal groups and the number of items in each group at the same time **only if the items are visible**.

2	Abstract composite units (Figurative units)	Able to model and counts **without visible items** i.e. the learner can calculate composites when they are screened, where they are no longer rely on counting by ones. The child may not see the overall pattern of composites such and "3, 4 times".
3	Repeated addition and subtraction	Co-ordinates composite units in **repeated addition and subtraction**. Uses a composite unit a specific number of times as a unit e.g. 3 + 3 + 3 + 3; may not fully co-ordinate two composite units.
4	Multiplication and division as operations	**Two composite units are coordinated abstractly** e.g. "3 groups of 4 makes 12"; "3 by 4" as an array
5	Known multiplication and division facts strategies	**Recalls or derives easily, known multiplication and division facts;** flexibly uses multiplication and division as an inverse relationship, is able to explain and represent the composite structure in a range of contexts.

Our findings

In the next section of the paper we share findings from our four research studies undertaken over the last 3 years. We share the way in which the LFIN enabled our research and our analysis as well as how this framework enabled the developmental aspect of planning for future club activities and teacher development.

Analysing learner developmental levels for design of after school maths club activities

(see Stott, 2014)

In her doctoral study, Stott investigated how Grade 3 learners' mathematical proficiency progressed (or not) whilst participating in two after school maths clubs over the course of a year and offered insight into how mathematical proficiency may develop in Grade 3 South African learners. Stott used the LFIN as an analytic tool to track progress between March and November 2012 for 17 club learners in all five LFIN aspects. A key contribution of her study was the extension of the LFIN to obtain quantifiable data in the form of scores, so as to analyse progression of the club cohort of learners in addition to the progress of individual learners.

Bob Wright (2003) has specifically stated that the data derived from the one-to-one MR assessment interview "does not result in a score" (p.8), the interview data is always used to profile the individual child's stage of early number learning onto the LFIN using stages and levels. However, Stott argued that such scoring could be useful. Working as she does in many clubs (subsequent to her research clubs), it is useful for her to compare different clubs to each other, thus she generated quantifiable data which she called 'Mathematical Proficiency (MP) Interview Scores'. By working with percentages, she was able to usefully aggregate these scores in order to make comparisons across more than one club using tables and graphs. These types of comparisons across the whole club or sets of clubs are not easily noted from the aspect stages or levels detailed within the LFIN itself, as each set of stages or levels is profoundly different and one would not be comparing like with like (see Tables 2, 3 and 4 for examples).

In her research clubs she tried to balance the needs and progress of the whole group with those of the individual learners. After conducting the first series of interviews and generating the scores Stott was able to see where the club learners had achieved high scores and low scores and used this information to plan activities for the whole club that addressed areas of weakness.

The findings from Stott's study suggested that the learners assessed in both clubs made progress to varying degrees as evidenced by the Mathematical Proficiency interview results. The graph in Figure 7 below shows how Stott used the percentage scores generated from the interviews to draw comparisons between her research clubs. The graph shows the overall club percentage change figures for each LFIN aspect for both case study clubs. Of interest is that the scoring allows one to see the similarity in improvements across LFIN aspects across the two clubs.

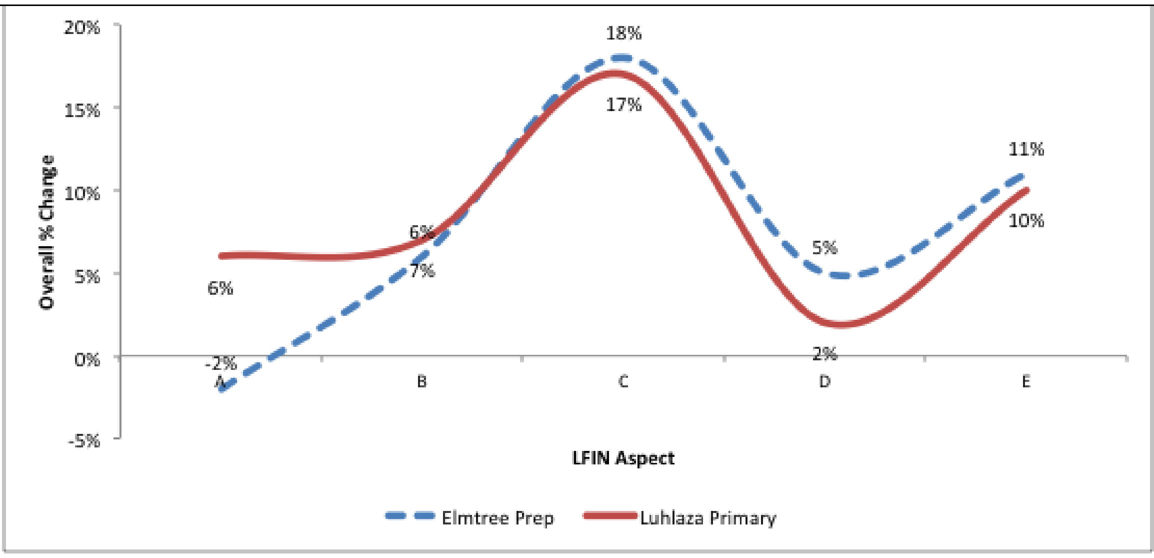

Figure 7. MP interviews: Club comparison - overall % change for each LFIN aspect

Analysing individual learner numeracy levels and relating this to the opportunity to learn

Here Graven shares how the analysis of two learner's interview responses and assessed levels of numeracy proficiency influenced their opportunity to learn and participate in subsequent club activities. Analysing the Wright et al. (2006) interviews conducted in February 2012 and again in November 2012 enabled Graven to note learner numeracy progressions over the year. Graven noted that Jade had progressed from her dependence on a 1-to-1 counting all strategy that dominated across questions in the first interview to counting and using more efficient counting on strategies in the second interview. Lebo had also progressed from occasionally needing to refer to concrete objects to knowing and using several number facts for enabling efficient solutions. Thus in terms of the LFIN, Jade progressed from a level 2 borderline 3 in early arithmetic strategies to a level 3 later in the year, whilst Lebo progressed from a level 3 to level 5 (see Table 3 above for detail of these levels). In conceptual place value, Jade remained at level 1 in both interviews but there was some evidence of her developing level 2 knowledge in one of the items in the November interview but this was not carried over to subsequent assessment items. Lebo progressed from level 2 to 3 during the year. This analysis of early arithmetic strategies and conceptual place value using the LFIN enabled Graven to notice how in several club activities, Jade was unable to participate fully in the way that Lebo did. An example follows.

The differential levels of numeracy progression meant that each learner brought different capabilities to the activities set by Graven. So for example in one session Graven asked learners to generate spider diagrams where they generate a set of sums (as the legs) that provide the answer of the number in the centre circle (the spider's body). For this activity Graven began with the number 10 in the circle and encouraged learners to generate sums that make 10. Jade participated actively in this and generated sums like 9+1 using her fingers. Lebo quickly generated several sums, without fingers, including subtraction sums. For the next activity Graven wrote 36 + 25 in the spider's body and, using place value cards, discussed how they could find the answer. Lebo was able to participate here and related the adding of the units and the tens to the place value cards. Jade on the other hand tended to look out the window as the place value discussion did not connect with her finger method of calculation.

Graven's reflection on this episode, based on her knowledge of Jade's level of numeracy proficiency at the start of the club (as Level 1 for conceptual place value and Level 2/3 for early arithmetical strategies) resulted in Jade being unable to contribute meaningfully to a conversation on place value which Lebo was able to contribute to. Furthermore following the place value discussion Lebo was able to participate fully in the activity that followed quickly generating multiple sums through manipulating numbers efficiently (e.g. 20 + 40 + 1). He was thus able to make the most of his opportunity to learn through the activity and generated new ideas and extended his thinking – all of which was enabled by his fluency (Lebo was Level 3 for early arithmetic strategies and at Level 2 for conceptual place value at the start of the club) with numbers. Graven's realisation here was that more individualised mediation was needed when providing an activity to a group of learners who are at different levels of numeracy proficiency. She realised that failure to do so could exclude learners at lower proficiency levels from participating in discussion and activities where higher levels of learning are introduced (and sustained by learners operating at higher levels of proficiency). This thus led to the realisation that certain club activities provided learners differential access to the opportunity to learn. This reflection is informing collaborative research currently underway with Heyd-Metzuyanim focused on exploring the relationship between forms of numeracy participation and the opportunity to learn.

An investigation of a Mathematics Recovery Programme for multiplicative reasoning
(see Mofu, 2013)

This part of the paper focuses on a Masters study undertaken by Mofu in 2013. The aim of her study was to inform mathematics teaching in her own school and to find ways to support primary school teachers at large in developing the strategies to teach and remediate multiplication reasoning. Mofu's experience in the classroom confirmed that learners experienced difficulties with multiplication. She observed that when working with multiplication, her grade 5 learners were still counting visible objects in ones. Some learners, when performing multiplication tasks, draw circles or small lines for counting and some just added the numbers. Thus, in addressing this problem her study examined what level of multiplicative reasoning was displayed by the learners in the case study group and how effective the use of the Mathematics Recovery programme was in the South African context when used to remediate a group of learners.

Using a qualitative case study approach, Mofu collected video recorded one-to-one oral interviews with the learners. A sample of six Grade 4 learners were purposively selected using a basic written assessment instrument to a class of Grade 4's which specifically looked at assessing their knowledge and understanding of multiplication. From the scored results

Mofu selected: 2 top scoring learners, 2 middle scoring learners and the 2 bottom scoring learners. These learners were invited to participate in an after school intervention programme aimed at supporting and remediating multiplicative reasoning. Mofu used the LFIN to profile the learners using pre and post intervention interview data and to determine their levels of multiplicative reasoning.

Learner progress in LFIN levels data was analysed using guidelines provided by Wright, Martland and Stafford (2006) as shown in Table 4 above. Table 5 below gives an overall picture of how the learners in her study progressed in terms of the LFIN levels from the pre (March 2013) to the post (April 2013) assessment.

Table 5. Learners overall progress in LFIN levels over time from pre to post assessment (Mofu, 2013 p. 49)

	Learner A		Learner B		Learner C		Learner D		Learner E	
	PRE	POST	PRE	POST	PRE	POST	PRE	POST	PRE	POST
LEVELS	2	3	2	3	1	3	3	4	4	5

Given the relatively short intervention in this study (4 sessions over 5 weeks), progress made from level one to another level was one of the most important results for Mofu. Her data showed that in the pre assessment, learners were counting in ones (positioning them at level 1) and relying on using constrained methods to solve multiplication tasks. After the intervention, the post assessment showed that constrained methods disappered and learners were able to count in equal groups and use more efficient and fluent methods to solve the multiplication tasks. The rate of progression in Mofu’s study was far greater than she expected; all learners progressed at least one level. Of note is that Learner C progressed from level 1 to level 3 in the short time, which represents a significant shift in her multiplicative reasoning.

Of interest is that Mofu drew on the efficiency spectrum for procedural fluency developed by Graven and Stott (2013). Their efficiency spectrum for procedural fluency ranged from restricted / constrained procedural fluency towards elaborated and fully flexible fluency. The strategies used by the learners in Mofu’s case study confirmed the notions of efficiency and fluency she had coded and analysed in the oral interview and showed an overlap of learner strategies. The learners displayed a range of responses from restricted / constrained procedural fluency towards elaborated and fully flexible fluency. This resonated with her sense that learner’s multiplicative proficiency or fluency needed to captured in its own right. Thus Mofu adapted the Graven and Stott (2013) spectrum for procedural fluency into multiplicative spectrums to help understand learner progress. Figure 8 below shows Mofu’s adapted spectrum of multiplicative proficiency.

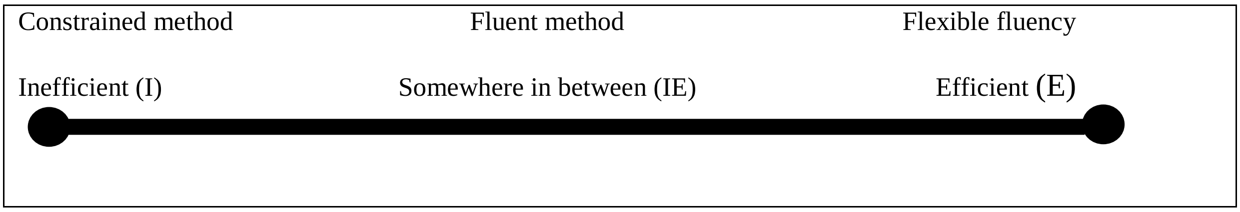

Figure 8. Spectrum of multiplicative proficiency for constrained, fluent and flexible fluency (Adapted from Graven & Stott, 2013)

In order to analyse multiplicative proficiency Mofu quantified the qualitative data to track possible progress using the spectrum discussed above. The progress of the learner was evident when they moved to the middle or upper end of the spectrum, which indicated increased fluency, flexibility and efficiency in multiplicative thinking. Figure 9 shows the positions of each learner according to the methods each used on the spectrum for the pre and post assessments starting with constrained (I-Inefficient) on the left, fluent (IE) methods in the middle and flexible fluency (E-Efficient) on the right. The values are the number of tasks where the learners showed the usage of different methods. So for example, Learner A progressed form using mostly constrained methods in the pre interview (in 5 questions) to more flexible methods in the post interview (in 5 questions)

	I Constrained	IE Fluent	E Flexible fluency
LEARNER A			
PRE	5	1	1
POST	2	0	5
LEARNER B			
PRE	4	3	0
POST	2	0	5
LEARNER C			
PRE	6	1	0
POST	2	2	3
LEARNER D			
PRE	3	1	3
POST	0	1	6
LEARNER E			
PRE	1	2	4
POST	0	0	7

Figure 9. Summary of spectrum methods for all learners across 7 tasks (Mofu, 2013 p. 57)

Mofu found that the use of the Maths Recovery (MR) Programme made it possible for the learners in her case study to progress in terms of multiplicative reasoning. The MR programme highlighted that, as teachers we need to understand the levels that the learners are operating at so as to assist them in their learning trajectory. During the intervention, Mofu gave the learners guided support, helping learners to think about multiplication and division, encouraging them to use their own strategies and make mistakes. Learners were encouraged to enter into discussion and engage in activities that involved active learning, problem solving and critical thinking were considered in the teaching strategies in the MR programme in keeping with the social constructivist notion that learning takes place in a social context before it is internalised.

A key aim of Mofu's research was to explore the extent to which the MR programme could be used to support learners in developing multiplicative reasoning and proficiency. As a teacher she learnt the importance of providing learning tasks that allow collaboration with peers, having access to concrete materials like arrays for multiplication and division. She found that the MR programme offered rich learning activities for teachers to use in interventions. Mofu also saw the usefulness of learning as an educator from the interview and see it as a useful developmental tool. Wright (2013) himself urged teachers and teacher educators to find ways to "incrementally trial and implement" (p.38) MR programme approaches. He stated that this professional learning is a "pathway to profoundly strengthening children's learning of basic arithmetic" (p.38) and that this can lead to young children achieving at significantly higher levels.

Mofu found that the key disadvantage of the LFIN was that it was labour intensive and time consuming to administer for more than a few learners. The assessment interviews took approximately one and a half hours for each learner. Additional time was spent coding and allocating learners to LFIN levels. Thus while Mofu would recommend that teachers conduct the interviews with a range of their learners in order to gain in-depth insight into learner levels and difficulties in multiplicative reasoning, it is not feasible to assess all learners in this way. However the implementation of the multiplication part of the MR programme to a group of learners holds potential for work in class. In her new role of Foundation Phase Mathematics Curriculum Planner, in the Eastern Cape, Mofu has subsequently conducted many fruitful workshops in this regard with Foundation Phase teachers in the Eastern Cape.

Exploring relationships between levels of numeracy reasoning, conceptual understanding and productive disposition (see Ndongeni, 2013)

This part of the paper reports on the findings of the fourth author's Masters research that focused on the relationship between '*conceptual understanding*' and '*productive dispositions*' (Kilpatrick et al., 2001) in the context of multiplication. Having noticed over a period of years that the Grade 7's in her school still relied on unitary counting and written tallies when dealing with multiplication and division problems this pointed to an important area of research. Ensor, Hoadley, Jacklin et al. (2009) and Schollar (2008) have argued that there is a lack of shift from concrete counting-based to more abstract calculation-based strategies and this seemed evident also in the case of multiplicative reasoning.

The study drew on the LFIN to establish learner levels of conceptual understanding in multiplication. Wright et al. (2006) argue that the topics of multiplication and division build on the students' knowledge of addition and subtraction, and also multiplication and division provide foundational knowledge for topics such as fractions, ratios, proportion and percentage, all of which are core and essential areas of mathematical learning typically addressed in the primary or elementary grades. Notions of conceptual understanding and productive dispositions were theoretically informed by Kilpatrick, Swafford, and Findell's (2001) five-stranded framework of mathematical proficiency. These strands are: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive dispositions. The fifth strand, productive disposition, is defined as "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (p. 131). This strand however is largely under researched (Graven, 2012).

In the study a purposively selected sample of six Grade 4 learners was used: two high, two average, and two low performers as indicated by performance on an initial basic assessment of multiplication. Individual interviews were conducted using the Wright et al (2006) instrument for exploring the nature of students' conceptual understanding of multiplication. For learner dispositions, an instrument adapted from Graven's (2012) productive disposition instrument was used and all these interviews were transcribed. The questions asked were structured in order to elicit the presence or absence of indicators of productive disposition in the context of multiplication.

Below is the summary by levels in the progression of multiplication over time with descriptors of learner responses that serve as indicators of each level.

Andile - Level 1: Forming Equal Groups or Initial Grouping
The child did no see groups as composite units and thus counted items mainly in ones instead of multiples. Thus he mainly used perceptual counting and sharing.

Viwe - Borderline between Level 1 and 2
The child used counting in 1's and in some instances in 2's but he still used perceptual counting because he was reliant on seeing items.

Nako - Level 2: Perceptual Counting in Multiples
The child used multiplicative counting strategies to count visible items arranged in equal groups but had difficulty in solving items where groups were screened.

Anda - Level 3: Figurative Composite Grouping
The child used multiplicative counting strategy to count items arranged in equal groups where individual items are not visible. So she was not dependent upon direct sensory experience. She did not use the composite unit a specified number of times.

Lulu - Borderline between 4 and 5
The child was able to use the composite unit a number of times and was not dependent upon direct sensory experience. She fell short of the next level because she did no see the inverse relationship of multiplication and division.

Sindy - Level 5: Multiplication and Known multiplication and division facts strategies
The child was able to immediately recall or quickly derive many of the basic facts for 'division as operations' She was also able to see and the inverse relationship of multiplication and division.

The analysis of learner levels of conceptual understanding using the LFIN and Kilpatrick et al.'s (2001) indicators of a productive disposition enabled the construction of individual learner stories that foregrounded the relationship between these. The frequency of Kilpatrick et al.'s (2001) key indicators of a productive disposition present in learner responses is provided in Figure 1 below:

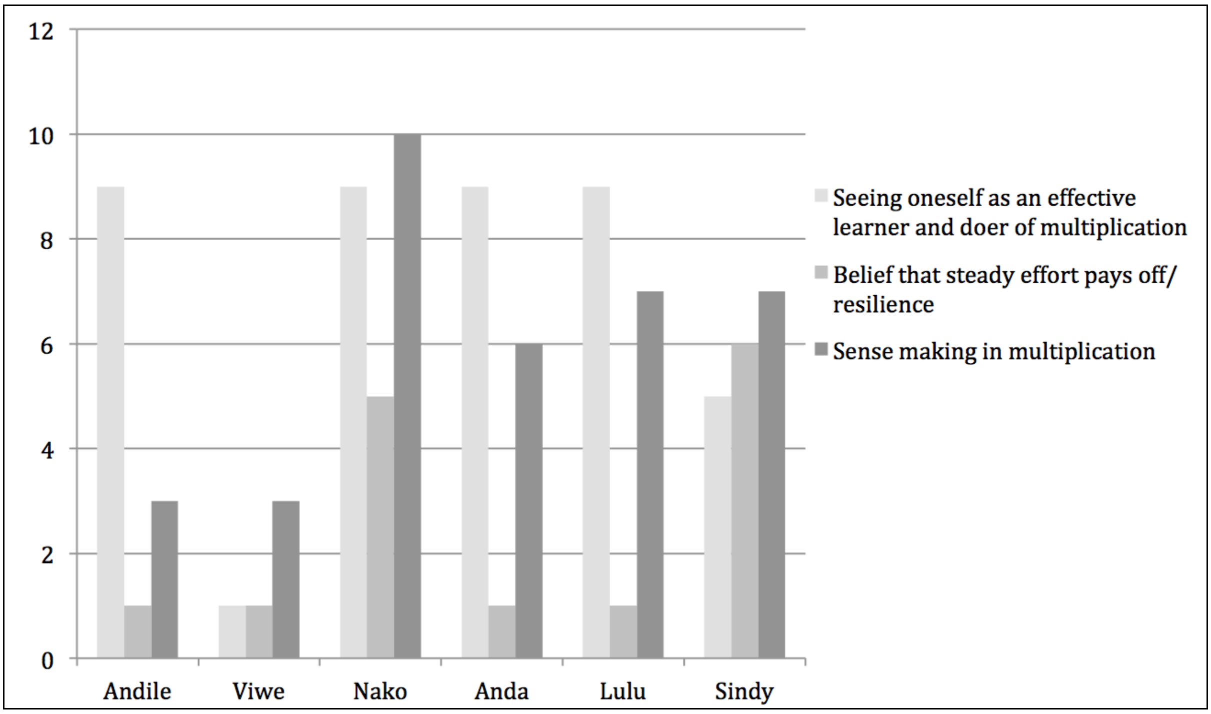

Figure 10. Key indicators of a productive disposition in case study learners (Ngongeni, 2013)

Of interest is the low presence of sense-making for learners (Andile and Viwe) matched with the lowest level of multiplicative reasoning in the LFIN (level 1) while seeing oneself as an effective learner and doer of mathematics and belief in steady effort did not seem to be clearly related to learner levels of conceptual understanding. So for example even while Andile was at Level 1 he said he saw himself as a strong mathematical learner. The study was limited in that it only had a small sample of learners but pointed to further research and the usefulness of the LFIN for assessing learner levels of conceptual understanding.

Concluding Remarks

This paper reported on each of our use of Wright et al.'s (2006) numeracy assessment and recovery framework as a tool for assessing learner levels of numeracy proficiency (and progress) across four research studies focused on learners in primary after school mathematics clubs in the Eastern Cape. The findings across these studies point to the usefulness of this tool for assessing learner levels of understanding and for planning subsequent interventions. In the paper we have made the case that while the ANAs are intended to provide teachers with useful information for planning future teaching they do not provide the teachers with opportunities to assess learner levels as the vast majority of learners are performing way below the grade level for which the ANAs are set and are thus unable to participate in several of the questions. Our paper has illuminated, through examples of data gathered across each of our research projects, the usefulness of identifying stages of numeracy development across their different research foci.

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