

MICLE Intermediate Phase Teacher Development Programme

SESSION ONE HANDBOOK

NAME:



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The original work from which these materials have been adapted:

Cortina, J.L. & Višňovská, J. & Zúñiga, C. (2014). Unit fractions in the context of proportionality: supporting students' reasoning about the inverse order relationship. Mathematics Education Research Journal, 26(1), 79-99.

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TEACHING SEQUENCE PART 1



Big learning goal:

Help learners understand the inverse order relation of unit fractions

e.g.
$$\frac{1}{2} > \frac{1}{5}$$

Resources:Storybook: Mama Khanyi and the PotsMeasuring sticks (+/- 24cm)Scissors, Writing Paper, PencilsStrips of coloured cardboard (+/- 1cm wide, +/- 20cm long)Rolls of white paper (e.g. the paper used in cash registers/tills)Worksheets 1A, 1B and 1C

1.1 Measuring using body parts

Big learning goal: Help learners understand the inverse order relation of unit fractions

Note:

At this time, we do not work with any fractions. Instead, we are working towards the 'big learning goal' by helping children to become familiar with situations where something smaller results in 'more' (hands measured) and something bigger results in 'fewer' (hands measured). This will become important to them later.

Specific goals:

- Learners explore measuring length using their body parts
- Learners notice that even when everyone measures with "the same unit", e.g. their hand, some hands are bigger and some smaller.
- Learners become aware of and summarise the **advantages** and **disadvantages** of using body parts to measure length. In particular, learners notice the different measurements that result when the same object is measured by different people (different size hands).

Resources: Worksheet 1A, pencils, notebooks

Grade(s): Third and beyond.

Introduction

The teacher starts with a whole class discussion in which learners talk about how things are measured. It is expected that they will talk about measurement instruments such as the ruler, the metre stick, and others.

The teacher can then ask the class about how people might have measured before those instruments were invented. It is expected that some learners will mention measuring with hands, feet, arm lengths, etc. If learners do not bring this up, the teacher can do so and mention that is how ancient people measured.

Individual (or small group) activity



Learners are asked to pretend to measure how ancient people measured, by using their hand length. Each group is asked to measure three or four objects in the classroom. If possible, they can also go outside to measure larger objects. Learners have to record the measures they obtain in their notebooks. Note:



Learners should already be familiar with measuring. You may need to remind them that:

(a) we start measuring by placing the hand exactly at the edge of the measured object,

(b) hands should be placed so that they do not overlap and so that there are no gaps between them,

(c) we start counting 'one' when the first hand is placed and count each one after that (some learners do not count the first one),

(d) need to measure along the straight line (e.g. a table edge), not crooked – in each instance need to discuss why this is needed (because the answer would be longer or shorter than the actual length).

WORKSHEET 1A CAN BE DONE AT THIS POINT

Whole class activity

The teacher presents the class with a problem such as this one:

The other day, Miss Zenani measured the blackboard using her hands. I paid attention and noticed that she was very careful when measuring. She counted eight hands. When she left, I measured it myself and counted six hands.

What do you think happened?

Is it possible for both of us to have measured the board correctly?

So why didn't we get the same measure?

Who do you think has the longer hand, Miss Zenani or me?

Note:



If these questions are proving challenging, or you notice that it is the same learners responding all the time, or if the learners are not yet used to talking in front of the entire class, the teacher can ask learners to first discuss their ideas in small groups and to come

up with responses that will then be shared. This provides time for more learners to engage with important ideas. This also provides a 'training' environment for talk.

<u>Writing opportunity</u>: Have learners write a short explanation (a sentence) about whose hand must be longer if Miss Zenani measured 8 hands, but you measured only 6 hands. This can be done individually or in pairs. **This should be done only AFTER learners have had the opportunity to discuss their ideas in a whole class or small group setting.**



The teacher can present the class with several problems like this. If children struggle to realise who has the longer hand, problems like the one below can also be used:

The teacher asks a learner to measure the board using her hand span.

Next, the teacher compares the length of her hand with the learner's, making sure that everyone can see which one is longer.

Then, the teacher asks the class whether if she were to measure the board, will she count more or fewer hands.

Finally, the teacher asks the class about why the person with the longer hand span (probably the teacher) counted fewer hands.

The teacher can follow up by asking the class if a person were to measure the blackboard and count two hands, would she have a short or a long hand. In addition, she can ask about someone that would count forty hands. Would she or he have a short or a long hand?

Whole class discussion

The teacher discusses the advantages and disadvantages of measuring using body parts. Regarding the **advantages**, it is expected that learners will bring up issues such as the convenience of having something with which to measure, all the time. Regarding the **disadvantages**, learners would mention that body parts of different sizes for different people. This makes is hard to use them to communicate the exact size of an object.

If the inconsistencies produced when using body parts to measure are still not clear for some learners, the teacher may present the group with situations such as the following:

Let me tell you about what happened to me the other day. I have a cousin that knows how to sew curtains, so I asked him to make some for me. I measured my window and counted twenty hand spans. So, I phoned and told him the measurement. He bought the fabric and made the curtains. But, what do you think happened?

<u>Writing opportunity</u>: Have learners discuss and then write about the curtain story and why there was a problem.



Notes:

1.2 Measuring with a standardised unit

Big learning goal: Help learners understand the inverse order relation of unit fractions.

At this time, we are working to create a need for learners to consider units of measure smaller than 1.

Specific goals:

- Learners recognise the need for measuring with a standardised unit
 - Learners can measure **accurately** with the stick (if this is not the case use the optional 'Accurate Measurement' activity below)
- Learners become aware of the problem of the remainder when measuring with a standardised unit (that is, when measuring with a standardised unit, almost always, a space remains that the unit will not cover exactly)

Resources: An unmarked, measuring stick, 24 cm long. The stick can be made of wood (about 9mm dowel) or plastic (such as a drinking straw), pencils, notebooks, story book: Mama Khanyi and the Pots, rolls of paper.

Grade(s): Third and beyond.

Introduction



The teacher explains that the class is going to learn about the history of an ancient people, and how they measured. The learners are told that, in the beginning, this group measured using their body parts, until the following story took place (pages

1-10 of the story: Mama Khanyi and the Pots). If you do not have the storybook, then you can read the story below:

There is a story about a famous potter that lived in those times named Mama Khanyi. She lived in a village called Matewu. She specialised in making traditional clay pots. She was so famous that she was regarded as an Emfundo, the name given to wise and skilled people.

Mama Khanyi had a daughter, Thembi, who helped her make the pots.



One day, Mama Khanyi had to fetch wood in the field, so Thembi stayed at the shop. Then, a group of elders from another village arrived. They were carrying an old traditional pot and wanted Mama Khanyi to make them a new one that had the exact same height. Thembi thought that she could help. She used her hand, just as Mama Khanyi had taught her, and carefully measured the old pot.



When Mama Khanyi arrived, she was sad that she had missed the visit of the elders. However, Thembi told her not to worry since she had carefully measured the old pot carefully with her hand.

Mama Khanyi made a pot that was three hands tall, which was the measure that Thembi told her. However, when the elders came to pick it up, what do you think happened? The two pots were not the same height. The elders were unhappy and Mama Khanyi was very upset.

The teacher can pause the story here and ask the class about what happened and why it happened. She can also ask them whether they thought the new pot turned out to be taller or shorter than the old one? If it was taller, how could that have happened? If it was shorter, how could that have happened?

The teacher can also ask the learners what Mama Khanyi can do to stop this from happening again.



The teacher can then continue with the story (pages 11 – 16), or read the following text:

That evening, Mama Khanyi sat outside her hut and started to think about what had happened. She thought about the many times that similar things had happened and wondered about what she could do.

While thinking about this, she fell asleep. She dreamed about talking to the moon and asking her what she could do. In her dream, the moon told Mama Khanyi that everything would be better if everyone would measure using the same standard. When she woke up, she found a white stick next to her, similar to the one I am going to show you now.







The teacher then reads further in the story book (pages 17 – 19), where the process of measuring a door using the stick is described.

These pages in the story book describe that when measuring, the learners should not leave gaps between the sticks or overlap the sticks, and they should measure in a straight line. The following diagrams show the CORRECT and INCORRECT way to use the stick:

Teaching Guide



Individual activity: The stick

The teacher gives every learner a measuring stick. The learners are asked to measure three objects. They have to write the name of the object and its measure in their notebooks.

Teacher activity: The Remainder

The teacher walks around the classroom while learners are measuring using the stick, paying attention to how learners deal with the remainder, that is, the spaces that are too short for the whole stick to fit in them.

Some learners might choose to ignore the remainder and produce "rounded off" measures. For instance, when measuring an object, they record "two sticks" even though the object is slightly longer than two sticks. Others might use some smaller units: "two sticks and four fingers". Others might use ambiguous units: "a bit more than two sticks". Finally, some learners might use fraction estimations: "two sticks and a half" or "two sticks and three fourths".

Optional Individual activity: Accurate measurement

If learners are not measuring accurately (e.g. they are leaving gaps, measuring in a skew line or overlapping the stick) ask the learners to explain what happened in the following scenario:

I have a space exactly four sticks wide in my kitchen that I can put a fridge into. I take two friends with me to the shop so that I have help carrying it. We measure the space for the fridge. I measure four sticks, one of my friends says that it is a little more than four sticks wide, and the other says it is a little bit less than four sticks wide. We all used the stick to measure. What do you think went wrong?

Teaching Guide



<u>*Answer</u>: The person who says it is more than four sticks overlapped the stick when measuring or placed the stick in a skew line when measuring. The person who said it is less than four sticks left gaps when measuring. (See the images on the previous page)

Whole class discussion

The teacher asks learners to show the ways in which they measured, making sure that the way in which they dealt with the remainder is clear in their responses.

The teacher aims to make the different kinds of strategies used by the learners clear to the whole class. The teacher also tries to make the limitations of each of the strategies noticeable. The teacher might ask the learners that used rounded measures, for example: Did the table measure exactly two sticks?

The main limitations that the teacher helps the class to notice are:

- Measures that are rounded off do not accurately account for the lengths of things. Two things of different sizes can end up being regarded as having the same length
- Ambiguous units do not accurately account for the lengths of things. Here too, two things of different sizes can be claimed to have the same size (e.g., "two sticks and a bit more", or "almost three sticks", or, "three and a half", when any remainder is called "a half").
- Using non-standard units, like fingers, brings back the same problem as using hands. Not everyone's fingers are the same size.
- Fraction estimations are only estimations How can someone be sure that it is really two sticks and three fourths?

Optional Individual Activity: Measuring and comparing heights

If there are learners who are still not clear that we need to have a way of describing the sizes of the remainders, allow children to use the stick to measure their height:



Learners mark their height on the chalkboard and then cut a strip of paper that has the same length. Next, they measure the strip with the stick and write the result in their notebook.

Learners should write their names and their measurement on the strip of paper and hand these in at the end of the lesson.

Teacher activity: The remainder

As learners work, the teacher walks around and tries to find measures that would lead someone to regard two learners of different heights as being equally tall. For instance, two learners of different height could both write down "four sticks and a half" as their height.



The teacher should collect the strips of paper and stick them up on the wall of the classroom, similar to this picture.

Whole class discussion

Several learners are asked to share their measures with the whole class. As they do so, the teacher tries to make it noticeable how the ways in which they are recording or naming the space where the stick no longer fits fully could lead someone to regard two people with different heights as being of the same height.

The teacher should select learners that have recorded the same measurements, for example, two learners who have both written "five and **a half**" but are not the same height, or two learners who have both written "five and **a bit**" but are not the same height.

The teacher concludes by commenting on how the suggestion of using a single standard (the stick) to measure did not solve the problem completely. It was still necessary to find a way of accounting, accurately and systematically, for the lengths of the spaces that the stick would not cover precisely.



Notes:



1.3 Constructing standardised subunits of measure

Big learning goal: Help learners understand the inverse order relation of unit fractions

Specific goals:

- Learners recognise the importance of having standardised subunits of measure
- Learners construct such subunits using strategies consistent with measurement division
- Learners realise that the subunits become smaller, as the number of times that these subunits need to be iterated to measure the whole unit increases

Resources: The measuring sticks; scissors; strips of cardboard that can be easily cut (the strips, of several colours, need to be about 20 cm long – a little shorter than the measuring stick); the story book

Grade(s): Third and beyond.

Introduction



Once learners have become aware of the limitations of their methods of describing the remaining spaces, the teacher reads on in the story about how Mama Khanyi and Thembi measured these spaces.

She mentions that they used pieces of plants to measure those spaces, which they called "obele". The English translation of the word "obele" would be "small". These smalls were made from a plant that grows straight and is easy to cut.

Individual Activity

Because the plant used by the Mama Khanyi is not available, they will use strips of cardboard.

The teacher asks the learners to measure again with the sticks. If the stick is insufficient to get an accurate measure, they have to cut a strip of cardboard that is as long as the space in which the stick does not fit exactly.

Learners can then write the measures in their notebooks, accompanied by a drawing representing the lengths of the cardboard strip they cut, as in the following example:

"The table measured three sticks and a small of this size:"

Whole class discussion

The teacher discusses with the class this way of measuring. She asks the learners whether this approach solves the issue of measuring accurately and making it possible to easily communicate the measures.

The teacher asks the learners for ideas about how the lengths of the strips could be named and written. The teacher also asks about how many of those strips would be needed, if every time you measured something you made and kept the 'small' and labelled it.

Overall, the teacher tries to help learners to realise that even though this is accurate, it is impractical as they would need an endless number of 'smalls'.

The teacher asks the learners for their ideas about how they could deal with this.



Note:

Ecarners are not expected to propose a sensible solution. This is another opportunity for them to consider how complex this task is.

Before starting the next activity, make sure that all the strips of cardboard are collected, so that learners do not confuse them with the ones that will be used next.

Whole class discussion

The teacher explains that the class is now going to learn about the method that Mama Khanyi came up with to produce smalls that she could easily label and identify by their size.

Each learner receives a red strip of cardboard, +/- 20 cm long and +/- 1cm wide.



Arrange the classroom so that more strips of cardboard are available for the learners in case they need to start over, but only one strip should be available at a time.

She tells them that Mama Khanyi had the idea of making something that the people called an "otibele". Translated to English, it means "small of two". Such a small would be shorter than the stick. Actually, it would be of such an exact length that when used to measure the stick, the stick would measure exactly two of those.



Use the suggested wording from pages 25 – 29 of the story book. Have a set of smalls ready to show to the learners. As shown below, one should be too long, one should be too short, and one should be the perfect length so that the stick would

measure exactly two of them.

First show the strip of cardboard that is too long. Show how two iterations of the strip clearly surpass the length of the stick.



Ask: Is this an otibele? ... Would a real otibele be longer or shorter than this? ...

Next, show the learners the strip of cardboard that is too short.



Ask: Is this an otibele? ... Would a real otibele be longer or shorter than this? ...

Next, show learners the perfect 'otibele' or 'small of two'. Show learners how two iterations are exactly the same length of the stick. Tell the learners that Mama Khanyi decided to colour it red so that she knew that this was an otibele, it was the perfect length to fit along the stick two times.



Learners will now make their own.

Individual activity

The learners make their own smalls of two. Those that finish quickly should help their classmates.

Do not allow learners to join strips together so that they have a piece that is the same length of the stick, that they then fold in half. Rather, encourage learners to estimate the length and cut the piece of cardboard and then measure the stick to see if it is correct. If it is too long, they can cut it shorter, if it is too short, they should start again.



Note 1:

An illustration is included at the end of this lesson plan to show the process the learners should follow when making the smalls. It uses the small of three as an example.

Note 2: See final note for reasons for discouraging 'bending' strategies.

Whole class discussion

Once all the learners have their small of two (otibele), the teacher asks the class if it would be enough.



Read further in the storybook (pp. 30) and ask learners: "Will it be now possible to measure everything using the stick and the small of two?

It is expected that learners will answer "no". If this is not the case, it is important to experiment if it would be enough or not. Learners should notice that there would still be many spaces in which neither the stick nor the small of two will fit exactly.

<u>Writing opportunity</u>: Have learners **describe** their work on creating the small of two and/or whether they will now be able to measure all lengths (and why or why not).



The teacher then asks the class which might be the next small that Mama Khanyi created. Some learners might think that it was one half as long as the small of two (1/4). Others might think it was the small of three.

The teacher tells them that it was the *etibele*, which means "small of three". She explains that an etibele would be of such a length that when used to measure the stick, the stick would measure exactly three of them.

Before learners actually start to make the etibele, the teacher should ask the class whether they think that it will turn out to be shorter or longer than the otibele. That is, whether the small of three will be shorter or longer than the small of two.

Learners should explain their reasoning.



<u>Writing opportunity</u>: Have learners write down their prediction and **reasoning** (e.g. why they think a small of two is shorter than a small of three). If you had no whole class discussion, ask learners to talk about their ideas in small groups prior to writing.

Individual and collective activities

A similar process is followed as more smalls are produced, until all learners fully understand that the next small must be shorter than the previous one.

Small of:	Suggested colour:
Two	Red
Three	Orange
Four	Yellow
Five	Dark Green
Six	Blue
Seven	Purple
Eight	Pink
Nine	Light Green
Ten	Grey

In some groups it might be necessary for learners to make six or more smalls. It is also possible that three or four are enough for everyone to realise that the more times a small fits into the stick, the shorter it has to be.



Some learners often engage in the activity of creating smalls with much enthusiasm and enjoy the needed estimation and precision. Some continue to make subsequent smalls while their peers are still finishing up. This allows the teacher to attend to learners who need more guidance while allowing meaningful engagement for 'fast' learners.

Note:

Note:

The use of the unusual terminology of 'smalls' or 'obele' is especially targeted at learners who might have learned fractions in the past. Learners who were previously unsuccessful with understanding fractions are often discouraged by the mention of the

word 'fractions' or use of the symbols. Use of 'smalls' and a different symbol system (introduced in 1.4) helps these learners to stay engaged.

Prior learning of fractions probably involved part-whole activities where food items (e.g. pizza) are equally shared. We do not want children to draw on such imagery, and instead, hope that they will start seeing unit fractions (smalls) as lengths that are in a specific relation to the length of the stick (whole).

This is also why we recommend discouraging use of bending strategies for creating 'small of two', which are typically attempted by adults, and especially by mathematics teachers). Rather than creating two 'halves', three 'thirds', four 'fourths', etc. we want learners to create only ONE strip of a specified length that can later be iterated to measure out multiples of its length.

For learning to reason later on, it is crucial that they create the smalls by the process of trial and error (repeatedly cutting and iterating the cut strips along the stick).



Optional

When this sequence was done in Mexico, prior to using the 'smalls' words (which include the number words, e.g. 'small of two') we used terms in the local language of the culture in which the story took place (Mayan, Teotihuacan) for 1 – 3 classroom sessions before transitioning to calling them 'smalls'. Parts of these words were made up as the languages did not always survive, but they sounded believable to the children.

When the sequence was trialled in South Africa, these words were adapted to sound believable to South African learners. In South Africa, these were the words used:

Small of:	Name in the original language:			
Two	Otibele			
Three	Etibele			
Four	Utibele			
Five	Atibele			
Six	Ambabele			
Seven	Enditibele			
Eight	Ahuitibele			
Nine	Itetibele			
Ten	Nedibele			

The children loved it and made lots of effort to use the strange words well.

In Australia, we chose not to do this because we did not have as much time to work with learners.

Conclusion



The teacher can conclude this lesson by reading the final pages of the storybook.

Teaching Guide

Illustration of the process learners are expected to follow when making the smalls, using the small of 3 as an example.

1. Three times (triple) the length of the small is longer than the length of the stick. The small is too long.



2. Three times (triple) the length of the small is shorter than the length of the stick. The small is too short.



3. Three times (triple) the length of the small is as long as the length of the stick, exactly. This is a small of three.



The stick and the smalls



1.4 Notating and comparing the length of the subunits (the smalls)

Big learning goal: Help learners understand the inverse order relation of unit fractions

Specific goals:

- Learners develop a need for a system of writing the sizes of the standardised units (smalls and the stick)
- Learners become familiar with a simple way of writing the standardised units
- Learners use this notation to compare, in a generalised way, the lengths of the smalls

Resources: The measuring sticks; the set of smalls that learners previously produced; worksheet 1B and 1C

Grade(s): Third and beyond.

Introduction

The teacher tells the class that when Mama Khanyi took orders for the pots she made, she liked to keep a note to remember which pots should be what size. Learners are then asked about how the sizes of the smalls could be easily written down.

Small group activity

Working in small groups of three or four, learners propose different ways of labeling the smalls. Amongst the ones that they might come up with are the following: So4, S4, (4) etc.

The teacher will now explain, and the class will think about, the system that Mama Khanyi used to easily write the sizes of the smalls. The learners will explore The Code for labeling the smalls.

The teacher explains that one of the problems Mama Khanyi had was that if she only used the number "2" to label the small of two, she could not tell the difference between the "2" meaning "two sticks," or "the small of two".

She realised that she could create a special code to label the smalls. This code involved placing the number inside a square when she was referring to a small. She labelled the small of two in the following way:



The teacher then writes several smalls on the board, using both the code and written language, as shown:



WORKSHEET 1B CAN BE DONE AT THIS POINT

Individual activity

The teacher writes the labels of several smalls using Mama Khanyi's code and asks different learners to read them out loud. Learners should copy them into their notebook and write next to the symbol what means (how we read it).

You can also dictate the names of several smalls (e.g., "small of six"), and ask the learners to write them down using the code:



Once the learners clearly understand the meaning of the code, presents a series of problems to the class, like the following:

The teacher writes on the board:



<u>Teacher</u>: Who can tell me what the first symbol means? <u>Lonwabo</u>: Small of three. <u>Teacher</u>: And the other one?

Zukiswa: Small of two.

<u>Teacher</u>: And which do you think will be longer? I want you to think about it. Who thinks that the small of three will be longer of the two? Why? Who thinks the small of two will be longer? Why?

For the first few examples, allow learners to look at the smalls they made to check which is longer.

Individual activity

Once the learners have made several comparisons as a whole class, you can write a list of pairs of symbols on the board and ask the learners to copy them in their notebooks.

Using *greater than* or *smaller than* symbols (> and <), learners should indicate which is the longer and which the shorter. The comparisons involve only the sizes of the smalls that the learners made and have on their desks.

Examples:



WORKSHEET 1C CAN BE DONE AT THIS POINT

Whole class discussion

The teacher leads the discussion of the answers that different learners gave to the comparisons. She favours (provides more time to discussing) explanations in which learners do not need to resort to the physical smalls. For instance: *Learner:* The small of six is shorter than the small of five because you have to make it smaller to get it fit six times, or else it will not fit, and the small of five only has to fit five times.

<u>Writing opportunity</u>: After discussing ideas, learners can be asked to summarise their explanation in writing.

Individual activity and whole class discussion

In the next round of tasks, at least one of the smalls involved in the comparison is not one of those that the learners physically produced. For instance:



When presenting these tasks, the teacher asks the learners to *imagine* making each of the smalls themselves, and to picture in their minds their length. She can ask some of the learners to gesture with their hands how long they think each of the smalls will be.

As learners become used to imagining the sizes of the smalls they did not physically produce, the teacher can present more challenging comparisons.



As the class discusses the solutions to these comparisons, the teacher constantly asks the learners to think about the size of the smalls based on how many times they have to be used to measure the whole stick. She also asks learners to think about which of the two compared smalls would have had to be used more times.

Notes on the development of notation in the sequence



When first introduced, for the learners, unitary fractions are physical things, namely, strips of relative size to the size of the unit (i.e., smalls of the measuring stick).





It is expected that some learners will initially produce labels to refer to the smalls by using initials. Hence, *the small of four* (which stands for the strip: a fourth, as well as for the length as long as that strip: one fourth)

This will be represented in ways similar to the following:



At this point, the teacher proposes writing a box as the special symbol for indicating that a number represents a small:



It is expected that some learners will next repeat the symbol in the box to represent the iteration of a small, as shown next:



[Meaning: three iterations of the small of four]

This is an *additive* way to express the length of 3 smalls of 4—one can imagine "+" symbols inserted in between the boxes and that each stands for *one small of four*. For most learners, this is the notation that makes the most sense because it represents the process they used to create the result as adding one more measurement piece each time. This is like repeated addition or like writing $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.

The teacher will then suggest a shorter notation to represent iterations of a small. This might appear in the learners' work before the teacher even suggests it. It could look like this:

In this case, 3 indicates the number of iterations and 4 indicates the small that was iterated. The teacher can encourage those who use this type of notation to place the number that stands for how many iterations (the multiplier) *above* the box instead, to prevent confusion between "3 sticks and one small of 4" and "3 smalls of 4".



This way of notating is more difficult for learners to understand, because it is *multiplicative* (more condensed; symbols do not have 1-to-1 correspondence with the process of how the length was created). The symbol outside the box is a *multiplier*—it indicates how many times the small is iterated. The symbol in the box indicates the *multiplicand*—the quantity (small) that is being iterated the stated number of times.

Learners who are only understanding the additive notation need a plenty of opportunities to notice and make sense of multiplicative symbols. "Unpacking" the symbol and linking it repeatedly to the process of measuring with the small is likely to support their thinking.

Finally, once learners are clear about the meaning of each of the numbers, the teacher proposes getting rid of three of the four lines on the box, so as to write the measures using the accepted fractions notation.



Once this way of representing fractions is introduced, the teacher may also introduce the usual fraction terms. Hence, she can explain to the learners that, in modern days, smalls of two are called "halves"; smalls of three, "thirds"; smalls of four, "fourths"; smalls of six, "sixths" and so on.

"Instead of saying three smalls of four, we now say three fourths."





TEACHING SEQUENCE PART 2



Big learning goal:

Help learners to understand that common fractions represent sizes that are either smaller than, greater than or equal to one unit.

e.g. $1 > \frac{1}{5}$ AND $1 < \frac{6}{5}$ AND $1 = \frac{5}{5}$

<u>Resources</u>: Storybook: Mama Khanyi and the Pots

Measuring sticks (+/- 24cm)

Set of smalls made by learners previously

Strips of white paper (+/- 1 and a half sticks long)

Rolls of white paper (e.g. the paper used in cash registers/tills)

Scissors, writing paper, pencils

Worksheets 2A and 2B
2.1 Measuring with the subunits (the smalls) and notating the measures

Big learning goal: Help learners to understand that common fractions represent sizes that are either smaller than, greater than or equal to one unit.

Specific goals:

- Learners recognise that subunits of a unit can be used to measure.
- Learners think about *how to express with notation* the lengths that are measured by iterating a subunit.
- Learners become familiar with a simple way of notating measures made by iterating a subunit.

Resources: The measuring sticks; the set of smalls that learners previously produced; strips of paper 38 cm long (1 and a half sticks long) made out of rolls of white paper tape; extra rolls of white paper tape.

Grade(s): Third and beyond.

Start/Launch

The teacher asks the class what they think the people in Mama Khanyi's village used the smalls for. It is expected that some learners will respond "to measure things". The teacher then asks what kinds of things they might have measured with the smalls.

The teacher then tells the class that one of things that the they measured with much care and precision were strips of cloth, which they used to decorate all sorts of important things. The learners are told that they will measure some strips of paper and pretend they were like the strips of cloth. The teacher hands out strips that are 38 cm long (1 and a half sticks), and asks the learners to measure them using their stick and smalls. She also asks the learners to write the measure in the way they think it might have been done.

Individual activity

Learners measure the strip and write the measure. They can do so working individually or in pairs or small groups.

Whole class discussion

The teacher asks the learners to show the different measures they got. It is expected that most of the learners would have combined the lengths of the stick and the smalls to cover the length of the paper strip.

Fraction as Measure

The following are examples of the measures that learners may produce:





Note:

It is possible that some learners produce incorrect measurements. For example, someone might say "a stick, a small of four, a small of three, and a small of five." At this point of the sequence, the main focus is on how to write these measurements, so it is not necessary to correct them at this time.

The teacher then tells the learners that Mama Khanyi preferred to use a single small to measure the lengths of strips, instead of combining them. The learners are asked to measure the strip again using a single small, and then write the measure in the way that Mama Khanyi might have done.

Individual activity

Learners measure the strip again, using a single small, and write the measure.

Whole class discussion

The teacher once more asks the learners to share the different measures they obtained. It is expected that most of the learners will write the measure repeating the symbol that represents the small:



[Meaning: six iterations of the small of four].

Other learners might have used a shorter way, using only two numbers: one that would represent the number of iterations and another the small that was iterated.



Note:

This shorter way of notating is harder for learners to understand, because it is multiplicative (the symbol outside the box is a multiplier – it indicates how many times the small is iterated). Because this is where we are heading mathematically, the teacher needs to provide learners with both the need for this notation (see the upcoming tasks) and enough time for making sense of it.

For learners who produce the left version of the short notation, confusion is possible between "3 sticks and small of 2" and "3 smalls of 2." The teacher can bring this up with these learners during individual work, or talk about the notation during a whole class discussion:

"This is a really good way but Mama Khanyi preferred to put the 'how many times' number on top, so as not to confuse it with a number of sticks."

If both ways (long and short) of writing the measures emerge in the classroom, the teacher can ask the learners about which Mama Khanyi would have preferred. If most of the learners think that it would be the first one, the teacher can present tasks where learners notice that the first type of notation can become quite long to write and inconvenient to read. For instance, she can ask them to measure the strip using a small of six (or a small of 8) and write the measure:



Fraction as Measure

She may also ask them to compare how "seventeen smalls of five" would be written.

If learners only used the first way of writing, the teacher can use the same tasks as above for them to notice that it can become impractical. She can then introduce the second way as the one that Mama Khanyi preferred.

In the case that none of the two ways emerge, the teacher may introduce both. The teacher can say that the first method was how Mama Khanyi originally wrote the measures, and the second was how she eventually preferred because the first way was impractical. The teacher might point out that the second way allows writing all kinds of measures using just a few symbols.

Whole class activity

Different learners come to the board and write the measures that the teacher reads out, using the shorter system.

For instance, the teacher dictates "Seven smalls of two" and the learners will write:

Alternatively, the teacher can write the measures on the board and ask specific learners to read their meaning out loud, from their desks. For instance, the teacher writes:



7

2

and the learners answer: "five smalls of six."

Individual activity

If the teacher is unsure if all the learners have understood the brief system, then they can dictate the names of several measures (e.g., "three smalls of four"), and ask the learners to write them down in their notebooks using the Mama Khanyi's code.

The teacher can then hold a whole class discussion in which the answers are revealed and analysed. This could include 'translations' between the long (additive) and short system for writing smalls.

The teacher may also write a list of measures, ask the learners to copy them into their notebooks and then write the names of the measures using words:



Three smalls of five

At this point, it is important that the teacher makes sure that all of the learners in the class are interpreting the numeral outside the box as the number of iterations that a small, defined by the number in the box, was iterated.







2.2 Creating the length of the reference unit (the stick) using the subunits (the smalls)

Big learning goal: Help learners to understand that common fractions represent sizes that are either smaller than, greater than or equal to one unit.

At this time, we are working to create a need for learners to consider whether the length of the unit can be produced by iterating the subunits.

Specific goals:

- Learners recognise that a length equal to the length of the reference unit can be produced by iterating the subunits
- Learners recognise that the number of times that a subunit needs to be iterated to measure the length of the reference unit is directly related to its size. Hence, when measured with the small of three, the stick will measure 'three smalls of three'; when measured with the small of four, it will measure 'four smalls of four', and so on.
- Learners anticipate how much would the reference unit measure when measured with any subunit, including those that that were not physically created (when measured with the small of sixteen, the stick will measure 'sixteen smalls of sixteen', and so on).

Resources: The set of smalls that learners previously produced. Rolls of white paper tape, scissors.

Grade(s): Third and beyond.

Start/Launch

The teacher starts the lesson by asking learners whether they have heard about villages that are remote and hard to access by road (outback or mountain settlements). It is likely that some of the learners will have a story about difficulties or time it can take to reach such places, *How long do you think it would have taken Mama Khanyi to travel to those places?* The teacher then proceeds to tell one of the Mama Khanyi's stories.

Once a year, Mama Khanyi prepared for a long journey, when she travelled with all her tools through the rural mountain villages, offering her wisdom and pottery skills. On one such occasion she reached the first mountain village, after two weeks of walking on the mountain paths.

When the excited villagers started to place orders for their new pots, Mama Khanyi noticed that her stick was no longer in her toolbox. It must have slipped from her toolbox on the way!

Fortunately, she still had her set of the smalls with her. But would this be enough for her to take the measurements for the villagers' orders? Could Mama Khanyi use her smalls to make a new stick, exactly the same as the original?

Whole class discussion

The teacher asks learners to estimate with their hands (without any measuring) how long the stick was. It is expected that the learners would have a clear idea about the length of the stick but would also be aware that estimation was not an acceptable method to create a new one. Learners should cut their length estimates as a strip of paper for later comparison with the actual length of the stick.

Individual activity

Learners now use their set of smalls and the paper tape to cut out a strip that they think would measure exactly one stick.

It is likely that some learners would complete the activity early, while others would need some time to work out how to do it. To provide more thinking time in the class, the teacher may ask the early finishers to write down an explanation for how they know that their strip is exactly one stick long. Alternatively, the teacher can ask them to measure out a one-stick-long strip using a different small than the one they originally chose.

Whole class discussion

Once all the learners have had some time to think about measuring out the length of the stick, the teacher will ask them to share and explain their methods for helping Mama Khanyi, and to decide whether the shared method would work.

It is likely that most learners worked with the small of two. The teacher should ask some learners to explain and re-explain why they think two iterations of small of 2 is as long as the stick. The reasoning in these explanations would likely try to capture the following process:

We made small of 2 to be such a length that the stick measures exactly two of them. That is why 2 iterations of small of 2 must be exactly as long as one stick.

In learner language, we would likely hear statements like:

"The small of 2 fits on the stick exactly twice, so I need to have 2 of them to make the stick."

During the learners' explanations, it can be helpful to use the stick to show how the small was initially created: *'such a length that the stick measures exactly 2 of them'* or how it *'fits on the stick*

exactly 2 times (twice).' The teacher can call on a learner to show this to the class by measuring an actual stick with a small of 2.



The learner can then bring the small of 2 (but not the stick) over to a board, measure two iterations, and mark the length as one stick: *'so I need to have 2 of them to make the stick'.*

Once the classroom seems to have reached a consensus – when learners no longer raise questions and counter-arguments – the teacher can ask the class whether this would be a good way to write down their suggestion:



The teacher would then continue: *Did anyone use something different than small of 2 to make a new stick? Could we use a different small and how many times would we iterate it? Could we do this with a small of 3? Could we use small of 4?*

If only a few learners used other smalls initially, time needs to be provided for small group work and explorations of the idea. As learners provide additional ways of producing the stick, and explain them, additional inscriptions will be added to the board.



Finally, the teacher asks about the smalls that were not physically created:

What if we wanted to use small of 15? How many iterations of that small are needed to get the length of the stick? How do you know this?

By this time, it is expected that learners will be aware of both the number pattern (as many smalls as is the 'name' of the small being used) and know why this pattern holds. As a result, they will be able to anticipate how many iterations are needed of *any* small to make the stick.

Fraction as Measure

Writing opportunity: After the conversation, learners can be asked to write down how they could create a new stick using different available smalls.

Alternative follow up task/story

If there is a need in the classroom to continue focusing on equivalences with the unit (the stick), provide additional opportunities for explanations:

Different people from Mama Khanyi's village visited a distant village over years and they all brought back measurements of the same statue, but these measurements all seem different.

These are the measurements they brought back. Which of these people were wrong?





Note:

The goal is that learners get fluent in both noticing and explaining that because the name of the small indicates the size that the stick measures when measured with the small, we need exactly that many iterations to get the length as long as the stick.



2.3 Creating lengths by measuring with the subunits (the smalls) and notating the measures

Big learning goal: Help learners to understand that common fractions represent sizes that are either smaller than, greater than or equal to one unit.

Specific goals:

- Learners recognise that subunits of measure can be used to *produce* specified lengths by iteration.
- Learners build flexibility in *expressing with multiplicative notation* the lengths that are measured by iterating a subunit.

Resources: The measuring sticks. The set of smalls that learners previously produced. Long strips of differently coloured paper, rolls of white paper tape, scissors, paper glue, large poster board sheets of paper (1 per group).

Grade(s): Third and beyond.

Start/Launch

The teacher asks the class whether they have seen how people dress for important traditional ceremonies and special occasions. It is expected that some learners will describe traditional clothing that they are familiar with. The teacher may share images from various cultures and bring attention to the rich patterns often created with repeated stripes of certain lengths, such as this Ndebele clothing.



Fraction as Measure

The teacher then explains, referring to the character from the story 'Mama Khanyi and the Pots', that Mama Khanyi's daughter, Thembi, became a designer of traditional clothing and explains that the people of Mama Khanyi's village particularly valued designs with patterns created from strips of cloth with lengths that had special meaning, so Thembi used strips of these lengths:



The teacher then discusses how to produce strips of the indicated lengths. This may start by asking learners: *How would you make a strip that measures this length* (pointing at one example)? *Which small would you use? How many times would you measure out with it?* or *How many times would you iterate it?*

The teacher will then share some of Thembi's patterns and invite learners to create new designs using the same lengths of strips (more examples are provided at the end of this lesson).



Individual activity

Learners individually (or in pairs) measure strips of paper by repeatedly iterating the small. They cut multiple copies of strips of the given lengths and imagine new designs. The teacher asks them to *write* the length of each strip (using brief notation) on it, so that it is easy to organise them.



Note:

This activity is intended to provide learners a chance to practise using the smalls as units of measure in their own right, with precision.

It is also an opportunity to recognise iteration as a key part of the measurement process and thus of comparison of relative sizes of different lengths.

It is possible that once learners measure out one strip of a given length, they will use this strip (not the small) to make the remaining copies of the same length. The aim of the activity if for learners to practice iterating the small, so try to discourage this.

Group work

Learners make groups of 4 and each group receives a large poster paper with an outline of a robe, where they will glue their designs Each strip needs to have a visible notation of how long it is, as this might be needed later during the whole class discussion. When the work is completed, posters are displayed in the back of the classroom.



Whole class discussion

The teacher asks learners to focus on how the lengths are written (robes and created strips should not be within learners' reach, but accessible if needed).

Karina had lots of cut-offs from a beautiful fabric that were each exactly one stick long. Which of the special lengths could she make using these cut-offs and which could she not make?

The teacher, holding the stick, asks:

Which of these strips do you think are longer than the stick and which are shorter? How can we tell without comparing them directly with the stick?



Learners need to answer the question and explain their reasoning: *"3 smalls of 2 would be longer than the stick because the small of two fits twice in the stick and you have three of them".* If learners are not sure, the teacher can encourage them to compare the stick with the strip on their robe or make the strip of the length in question.

<u>Writing opportunity</u>: Have learners write down their comparison (prediction) and **reasoning** (e.g., why they think *five smalls of six* is shorter than *the stick*).

Whole class discussion

The teacher asks learners to make comparisons of the strips made with smalls. At first, learners only compare the lengths that are shorter than the stick with lengths that are as long as or longer than the stick – for example, *Which of these two strips is longer? How can you tell?*



Next, learners are asked to make comparisons of strips they had not yet produced, for instance, *Which is longer?*



Here too, the comparisons are based on whether a strip is longer than, shorter than, or as long as the stick.

Finally, the teacher asks learners to compare measured lengths for which they did not make the smalls (e.g., 13/17 vs. 17/13).

<u>Writing opportunity</u>: Learners can be asked to first discuss in small groups and then to write down what they have learned so far about comparing different lengths measured with smalls.

worksheet 2b can be done at this point



Some exemplar patterns





Fraction as Measure

Teaching Guide

WORKSHEETS

1A - Eng nes:Date:	
the classroom with	her feet.
t when measuring t	he room?
measure the width o	of the classroom with their
group member will a the width of the cla	count more or fewer or the assroom?
Fewer	Equal
er count when mea	suring the room?
Teacher	Learner
gth of the playgrour	nd, who would count more
Leo	arner
	1A - Eng Date: The classroom with t when measuring t measure the width group member will a the width of the cla Fewer er count when mea Teacher gth of the playgroun Lea

Angi and Thembi went to the park. They counted the steps they took when they walked from the lake to the big tree. Angi counted 70 steps and Thembi counted 80 steps. Choose an answer.

Who takes the longer steps?

Angi

Thembi





Explain your answer:

 $$1 \mbox{A}$ - Eng$ Cut a length of paper that is the same as one of your group members' height.$



This person should use their hands to measure their height. Then, ask a second group member to also measure the length of paper with their hands.

How many hands did the first person count?

How many hands did the second person count?

Who has the longer hand? _____

If a Grade R learner used her hands to measure this height, will she count more or fewer hand spans than your group did?

More

Fewer

Explain your answer:

Name:	1A - Afr Datum:	
	Datom.	
Jou onderwyser gaan die Hoeveel voete het jou ond	breedte van die klaskamer derwyser getel?	met sy voete meet.
Kies 'n leerder van jou gro te meet.	ep om ook die breedte van	die klaskamer met hulle voete
Sal die leerder minder or n	neer voete tel?	
Meer	Minder	Dieselfde
Hoeveel voete het die lee	rder in jou groep getel?	
Was jou groep se voorspel	lling korrek?	
Wie het langste voete?	Onderwyser	Leerder
As die onderwyser en die l meet, wie sal die meeste v	eerder die lengte van die he voete tel?	ele speelgrond met hulle voete
C	onderwyser Leero	der
Verduidelik jou antwoord:		

1A - Afr

Angi en Thembi het park toe gegaan en hulle treë getel toe hulle van die meer tot die groot boom gestap het. Angi het 70 treë getel en Thembi het 80 treë getel.





Thembi

Verduidelik jou antwoord:

1A - Afr Sny 'n lengte papier wat dieselfde lengte as een van die leerders in jou groep is.



Twee leerders moet die lengte van hierdie stuk papier met hulle hande meet.

Hoeveel hande het die eerste leerder getel?

Hoeveel hande het die tweede leerder getel?

Wie het die langste hande?

As 'n graad R leerder hierdie papier met hulle hande meet, sal hulle meer of minder hande tel?

Meer

Minder

Verduidelik jou antwoord:

Write what each of Mama Khanyi's code symbols means:



1B - Eng

Show how these smalls would be written in Mama Khanyi's code. See the example: Small of four

4

Small of two

Small of seven

Small of three

Small of eight

Small of five

Small of nine

Small of six

Skryf die naam vir elke simbool wat Mama Khanyi gebruik het:



Naam:	1B - Afr	_Datum:
Teken Mama Khanyi se simbool vir elke	e 'klein':	
Klein van vier	4	
Klein van twee		
Klein van sewe		
Klein van drie		
Klein van agt		
Klein van vyf		
Klein van nege		
Klein van ses		

	1C - Eng
Name:	Date:

Which is longer?

Write the greater than (>) or the smaller than (<) symbols between the smalls.



Which is longer?	
Write the greater than (>) or the smaller than (<) symbols between the smalls.	

9	3	99	71
12	4	8	80
14	2	44	13
5	15	18	61
2	9	27	8
13	6	12	11
16	26	77	78
7	10	25	60



Watter een is langer? Vul > of < in.



Name:_____Date:_____

Write the name of the measures, according to Mama Khanyi's new code:



2A - Eng

Show how the following smalls would be represented in Mama Khanyi's code. Five smalls of six

5
6

Three smalls of two

Nine smalls of seven

One small of three

Eight smalls of eight

Two smalls of five

Six smalls of nine

Five smalls of six

ſr

Naam:_____Datum:_____

Skryf die name vir die volgende metings.



2A - Afr

Teken die simbole vir die volgende 'kleine':

Vyf kleine van ses

5
6

Drie kleine van twee

Nege kleine van sewe

Een klein van drie

Agt kleine van agt

Twee kleine van vyf

Ses kleine van nege

Vyf kleine van ses
5
5

Name:_

Date:_____

Which is longer?

Write the *greater than* (>) or the *smaller than* (<) symbols between the smalls.



5
5

Name:_

Date:_

Which is longer?

Write the greater than (>) or the smaller than (<) symbols between the smalls.



2B - Afr



- All

Watter een is langer?

VUI > Of < in.

4 3	 2 3	6 9	10 7
5 4	 4 6	4 10	3 2
3 4	 6 4	7 6	1 4
1 3	 4 2	8 6	75
2 8	 5 4	5 3	89

Watter een is langer?

 $v_{UI} > of < in.$

4	12	6	12
12	4	12	6
9	15	40	24
11	12	24	33
4	14	15	25
19	12	25	20
21	14	88	68
30	2	77	88
18	12	90	30
12	18	18	45