Research Proposal for Debbie Stott
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Rhodes University Education Department
Numeracy Chair
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Provisional Title:
The nature of learners' evolving mathematical identities and mathematical proficiency in relation to their participation in informal after-school Grade 3 Maths Clubs
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Dear Reader
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1 Introduction

I am a research student within the SA Numeracy Chair research community at Rhodes University. Within this Chair I am also the co-ordinator and learning programme designer for the after school Maths Club. From the beginning, I have been interested to explore these Maths Clubs from the learners' point of view.

One of the key problem areas for me is between the way that South African classrooms work and how the educational literature says things should be. There seems to be huge discrepancy here and I want to explore, from a learners' point of view within after school clubs how this might be different. We also have a large percentage of learners in this country who do not see maths as useful or sensible and who would give it up if they could. How can this situation improve?

Therefore the focus and purpose of this study is twofold. Firstly I wish to look at the nature of the evolving mathematical identity of learners who participate in these informal, voluntary maths clubs. Secondly, to investigate the nature of the evolving mathematical proficiency in learners who participate in these Maths Clubs. It is hoped, that this in turn, may yield insights for more formal classroom learning environments.

Other purposes are to contribute to the body of knowledge on after-school / out-of-school learning programmes as well as to attempt to address some of the more detailed issues discussed in the forthcoming sections.

This proposal will give you a sense of what frames this research study from a theoretical and learning perspective, my differing roles in the research as well as the proposed methodology for the study.

2 Key Problems

South Africa has recently seen a number of reports, papers and books, reporting on the current state of education. One book tells the story of the 'crisis' in the South African primary education sector and focuses particularly on literacy and numeracy (Fleisch, 2008). This view is echoed in a paper commissioned by the Presidency where Taylor, Fleisch and Shinder (2008) state that poor progress of students in high school and further education is due to the inadequate preparation of primary learners, particularly in literacy and numeracy.

Their findings suggest that access is not the main problem in South African schools at present, but that quality is. They believe that by improving the quality of primary schooling, particularly literacy and numeracy competence is a prerequisite to effecting quality gains on a long-term basis in secondary schooling, FET colleges and universities (Taylor, Fleisch, & Shinder, 2008).

Many reasons are given as to why quality is an issue. One of the causes of these problems are the multiple changes in the education systems that have taken place in South Africa over the last 17 years (Bloch, 2009; Fleisch, 2008; Taylor, Fleisch, & Shinder, 2008). These changes have had an influence on teaching methods and practices, which is where I wish to focus this discussion. Fleisch (2008) points out that prior to the implementation of Curriculum 2005 in March 1997, many classrooms and teaching practices here in South Africa used the rote-learning method of teaching, which focused typically on whole class teaching approaches. In mathematics classrooms this teaching was focused on procedural learning.

However, Curriculum 2005 painted a picture of classrooms where learners should participate enthusiastically in groups and take responsibility for their own learning (Brodie & Pournara, 2005). Does this view painted in Curriculum 2005 resonate with what happens in reality? Brodie and Pournara (2005) and Fleisch (2008) suggest not. When the curriculum was reviewed during 2000, the Chisholm Report (Chisholm et al., 2000) and Fleisch (2008) indicate that ideas about learner centeredness and collaborative learning were poorly interpreted and therefore were not very successful in practice. Fleisch states that classroom teaching methods have not changed dramatically, and that forms of 'rote method continue to dominate classroom interactions' (2008, p135). This means that teaching practice still focuses largely on teaching content. This quote from the Chisholm report illustrates this clearly: "*many learners in the classes observed still do not participate fully in the learning process since teachers are still providing a great deal of direct instruction and are still pre-occupied with content coverage*" (Chisholm et al., 2000, Chp 6). Fleisch quotes a number of South African researchers who make similar points; the strongest opinion is that of Harley and Wederkind:

> *"there is strong evidence that C2005 as a pedagogical project is working counter to its transformation aims. It is widening, not narrowing, the gap between historically advantaged and disadvantaged schools" (Fleisch 2008).*

In terms of numeracy learning, South African classrooms could still therefore be described in terms of what Boylan calls 'usual *school mathematics*' (Boylan, 2010, p.10). These classrooms can have some or all of the following features. The classroom works within a given set of practices that are not open to negotiation, the practices can have a regulatory or disciplinary intent, the experience for many of the learners is one of separation, individualisation and marginal participation, there are fixed hierarchies and authoritarian relationships between the participants and the environment is characterised by reproduction rather than production.

As Boylan (2010) points out, the dominant relationship in this type of classroom is hierarchical and authoritarian. The relationships between the learners in this type of classroom are well established. It has been noted that by the time a group of learners have been together in this type of class for a few years, the established learner identities and relations can become fixed or determined.

Askew (2008) believes that identities become sedimented and are maintained this way because of the established range of ongoing actions of all the participants, that the identities are a result of the transactions between the learner and the sociocultural context. This observation is supported to some extent by the study done here in South Africa by Graven and Buytenhuys (2011). The few extracts below from learner narratives give some examples of this:

> *"From since I can remember I have struggled with Maths. I would always try my best but never see results"*
> *"Since I was young I refused to do Maths homework, not because I didn't want to but because I simply did not understand the work that needed to be done."*

Worryingly, Graven also points out that 'negative non-participatory learner identities seem to appear in many learner stories quite early in their mathematical learning' (Graven, 2011). The danger of negative stories or negative labelling appearing early on in mathematical learning is that they become self-fulfilling prophecies (Sfard and Prusak, 2005) and deterministic and can shut down the space for future productive learning (Graven, 2011a).

These early forming, established identities can also lead to a competitive environment where individual learning is seen as the only way of being and where group work is not accepted because it means sharing answers that should be kept secret (Askew, 2008).

From these research studies, we see that is can be difficult to move away from traditional pedagogies and to implement new practices in the classroom. See for example Suurtamm and Vezina (2010). The literature suggests many reasons why this is the case. There is however another element that comes to the fore: implementation of these new practices often meets with resistance from the learners themselves.

Hunter (2008) points out that by changing the communication and participation practices expected in the classroom create challenges for learner. Askew's experience described in his 2008 article illustrates this quite clearly. When he and his colleagues tried to change the ways that lessons were organised by using 'true' collaborative or group work strategies, they met with resistance from the learners because they challenged the established norms and identities. Goos has also noted resistance from a number of learners when the teacher attempted to move them 'toward more independent and critical engagement with mathematical tasks'.

Similar situations are evident in South African schools. Graven and Buytenhuis (2011) explain how long it took to persuade Maths Literacy learners to take part in new forms of mathematical practice in this extract:

> *"It took about six months to get my initial group of learners to accept that I was on their side and that as a team we could achieve a new and positive maths experience. I positioned myself as a co-learner - as indeed I was. […] The only way to learn was going to be through engagement. At last they began to gain confidence and were willing to risk participation in discussions."*

It could be assumed that in addition to learners being inadequately prepared for further schooling in terms of numeracy, the identities within South African classrooms may be fairly predetermined. This and the issues mentioned above have led to the framing of my research questions as shown below.

3 Research questions

My primary research question is:

How do learners' mathematical identities and mathematical proficiency evolve in relation to their participation in informal after-school Maths Clubs designed as communities of inquiry?

We can then look at this primary question from two perspectives:

1. **How do learner's mathematical identities evolve (if at all) over the period of participation in the Maths Club?**
   - How does this evolving identity relate to the forms of participation, interactions and activities promoted in the clubs?

2. **How do learner's mathematical proficiency levels evolve (if at all) over the period of participation in the Maths Club?**
    - How do these evolving proficiency levels relate to the forms of participation, interactions and activities promoted in the clubs?

4 Is there a need for this research?

I believe that there is a need for this research from both a South African and international perspective. I give reasons for this in the following section.

In South Africa

In general, there is a lack of mathematical educational research in South Africa, especially at primary school level. A study undertaken by Venkat, Adler and colleagues on South African research produced between 2000 and 2006 revealed that mathematics teaching and learning represents the 'bread and butter' of work in the field of mathematics education and is located particularly in the context of curriculum reform. However, the empirical base of the research tends towards secondary level education, with a relative under-representation of empirical research at primary level (Venkat, Adler, Rollnick, Setati, & Vhurumuku, 2009). As my work is located in the primary level, this is a good opportunity to contribute to this empirical base.

South Africa needs confident learners who feel they can do maths so that they are able to go on and study mathematics in their further education and, more importantly, who are able to use maths in their everyday lives. Indeed, the South African curriculum documents call explicitly for this. In South Africa and worldwide it is seen that maths is the 'gatekeeper to many economic, educational and political opportunities for adults.

Furthermore, South African and worldwide calls for mathematical proficiency for every learner are universal themes (See Kilpatrick & Swafford, 2002; Kilpatrick, Swafford, & Findell, 2001; National Research Council, 2001) as this extract from 'Adding It Up' explains:

> *"For people to participate fully in society, they must know basic mathematics. Citizens who cannot reason mathematically are cut off from whole realms of human endeavour. Innumeracy deprives them not only of opportunity but also of competence in everyday tasks" (Kilpatrick, Swafford, & Findell, 2001, pg 1).*

It is important, therefore, to explore what factors enable and constrain the development of mathematical proficiency in South African learners in order to fulfil these needs and for this research study to attempt to contribute to aspects of mathematical practice n South Africa.

Finally, there is very little research on learner after-school clubs in South Africa, particularly at primary level. This is also echoed in the wider international community, where there is a small amount of work on after-school clubs (Papanastasiou & Bottiger, 2004). Again, these tend to focus on secondary school.

In a wider context

The articles written by Lubienski and Bowen (2000) and Cobb and Hodge (2011) highlight where mathematical research related to equity in mathematics education is lacking. The 2000 Lubienski and Bowen article quotes percentages from their study. 52% of research articles were about a specific mathematical topic, 37% focussed on elementary education, whilst only 2% on early childhood education and they noted that only 623 of the 3,011 articles related to one of the equity categories (gender, ethnicity, class and disability). Even in 2011, Cobb and Hodge point out that equity is still under-represented and has generally been marginalised within mainstream mathematics education research (p. 193).

There is an opportunity for this research to add to these under-represented areas in this wider context. The learners that will be participating in this research are in early childhood education, and may qualify in at least 2 or more of the equity categories. Furthermore, this research is not focused on a specific mathematical topic but on the nature of evolving identity formation and evolving mathematical proficiencies.

5 Purpose of the study / Rationale

The main focus and purpose of this study is twofold. Firstly I wish to look at the nature of the evolving mathematical identity and mathematical proficiency levels of learners who participate in informal, voluntary Maths Clubs.[1] Secondly, I wish to investigate how these evolving identities and mathematical proficiencies relate to the forms of participation, interactions and the activities that are promoted in the informal after-school Maths Clubs. In other words, I want to examine what enables, what constrains the evolving identities and proficiency levels, and how this evolution may or may not occur.

It is hoped, that this exploration may yield some insights for more formal classroom learning environments.[2]

For clarity, I will briefly unpack here what is meant by '*informal*'. Table 1 below shows how it is envisaged that the informal Maths Club environment to be. This has been contrasted to the more formal environment typically found in schools and discussed in the pages above.

Table 1 - Formal vs. informal learning environments

Formal maths classroom / environment	**Informal maths environment**
Participation is expected as part of formal schooling (in-school-time)	Voluntary participation during out-of-school time
Less learner choice over the activities that they work on and engage with	More learner choice over the activities that they work on and engage with
Curriculum as a guiding framework but not prescriptive	Curriculum as a guiding framework but not prescriptive
Largely acquisition based and often driven by teaching to assessments	Participation based, where participants are active and engaged
Teacher led and typical interactions between teacher and pupil only	Mentor led where mentor is seen as a full participant in the learning process
Assessment tends to be summative and results in ranked performance	Assessment is formative and integrated and is used to guide individual learning experiences for the participants
Prescriptive, teacher controlled classroom rules	Negotiated sociomathematical norms

Other purposes are to contribute to the body of knowledge on after-school / out-of-school learning programmes as well as to attempt to contribute to some of the issues discussed in the sections above.

6 Context of study

6.1 South Africa and Maths Education in South Africa

This larger context has been discussed in the section on Key Problems above.

6.2 Within SA Numeracy Chair

In response to the situation discussed earlier in terms of lack of quality in South African primary education, Taylor et al (2008) point out that the national and provincial Departments of Education have begun to focus on the problem of primary school learning. Driven by these same imperatives, national research funding organisations have begun to invest in research projects for this specific area. The SA Numeracy Chairs across South Africa including that at Rhodes are examples of this type of funding.

The objectives of the Chair are to improve the quality of teaching of in-service teachers at primary level and to improve learner performance in primary schools as a result of quality teaching and learning. It is anticipated that these objectives will be achieved by researching sustainable and practical solutions to the challenge of improving solutions in schools and by providing leadership in numeracy education.

6.3 Maths Clubs

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[1] The empirical field for this study will be Maths Clubs run within the context of the SA Numeracy Chair. These Maths Clubs have been conceptualised as fun, informal places to allow learners to have an experience that is different to the 'usual maths classroom' described later in this proposal and to see what affect this may have on the development of mathematical proficiency and mathematical identity formation. This context is further elaborated on page 6.

[2] From a situated learning perspective (see section 7 Theoretical Framework for the study) there is no assumption that there will be transfer or a causal relationship between the informal and formal learning environments. It may however be interesting to interview learners to see how they view the relationship between the two. This study could illuminate further possible avenues of research for post-doctoral study.

As mentioned before, the Maths Clubs are the empirical field for this study, so it is necessary to give a brief overview of these including my role in these clubs apart from being a researcher.

Background and Rationale for the Clubs

One of the responsibilities of the SA Numeracy Chair is to facilitate learner numeracy proficiency by running learner-directed and learner-oriented maths activities for the duration of the project. Examples of these activities include Maths Bonanzas, Maths Relays and Maths Clubs. The Maths Clubs, as mentioned earlier, have been conceptualised by Professor Graven as being informal places where learning can take place in out-of-school-time. A diagrammatic overview of these clubs is shown in Appendix A.

Learner activities are therefore a key part of the Chair sphere of activity and are a place where the Chair is responsible for working *directly* with learners rather than via the teachers.

Based on previous experience with Maths Clubs, Graven (2011b) argues that clubs are an opportunity for disrupting passive learning culture and deliberately working with learners to become confident mathematical participators. She believes that Maths Clubs might offer opportunities for the creation of active engagement, negotiation and participation for learners, where learners can live out different stories. She says that it would make sense to explore the opportunity clubs might offer in 'interrupting' the negative relationships spoken about in the problem section by re-authoring new positive experiences, stories and thus learner identities.

Graven is keen to point out that:

> *"This is not to deny the need for mathematics classrooms to provide the opportunity for more participatory and positive learner identities but rather that the extra curricula nature of such clubs might provide increased freedom to focus on the deliberate construction of positive participatory mathematical identities intentionally at the expense of covering the range of skills and knowledge required to 'get through' the curriculum* (Graven, 2011b).

My role in the clubs

As a member of the Chair research community, I have the unique opportunity to participate in a number of these Maths Clubs both as club leader / co-ordinator and researcher. Furthermore, working with the Chair, I have been assigned the role of Maths Clubs Co-ordinator and have been specifically tasked with the design of the Maths Clubs programme. This enables an ongoing, powerful reflexive practice to develop between the club learning programme design, implementation and research. While club design is not the focus of my research, it of course frames the way in which club practices and activities, forms of participation, interactions and sociomathematical norms will evolve. I thus elaborate on this in Appendix C.

Maths Club Design Process

As the Maths Club designer for the Chair, I wish to make mention of the design process for the Maths Club programme as this is an integral part of my reflexive praxis. However, for the purposes of my own scholarship and to maintain research focus, I have kept this description brief. A more detailed description, rationale for why I have chosen this design and how it may support this study can be found in Appendix B.

I have used and adapted a combined Zone Theory and professional development model used by Goos and her colleagues (Goos, 2006) summarised in Figure 4 in Appendix B to conceptualise and design the Maths Club learning programme. I have done this by inserting the contextual elements relevant to my context into the framework used by Goos and her colleagues.

Maths Club Pilot

The Maths Clubs will be piloted during the second half of 2011. This gives me another special opportunity. It will give me a sense of the appropriateness of my research questions and possible insight into the need for more. I will be able to see if my data collection methods are comprehensive and if not, where the gaps are. I can also use it as an opportunity to see how the data can be analysed and to see if my theoretical and methodological frameworks need to be adapted or extended. So in essence I have the chance to pilot this research and to make changes to elements of it before I start.

7 Theoretical Framework for the study

In preparation for this proposal I have examined a wide range of literature and from this have filtered out the readings that I think are relevant to this study at this point. As background information I have listed the areas that I have read and some of the key theorists and researchers identified in those areas. These are shown in Appendix C.

How did I filter the readings?

I determined the key readings forming the theoretical and conceptual framework for this study in three ways. Firstly, I examined the major framing of the Chair as these frame the broader context of this study. Secondly, I studied a collection of readings which I consider to inform the specific elements of this research study at this point in time. Finally, I reviewed in detail a set of readings that inform the design of the Maths Club programme which forms the empirical field for this study. The diagram in Figure 1 below shows how the areas of reading fit together.

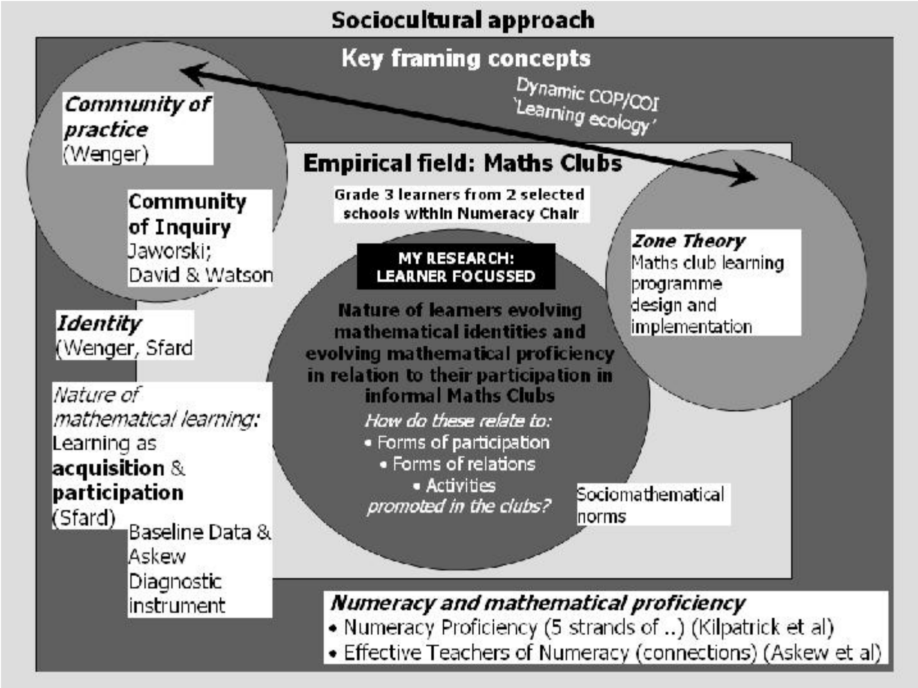

Figure 1 - Theoretical framework diagram

Having already made sense of a large part of the body of knowledge, I believe that the purpose of my research is then to evaluate it, apply critical judgements to various arguments and so on. Therefore, in this brief review I will look at each of above aspects in turn and describe what I have read. I will give an explanation of why they have been chosen for this study, why at this proposal stage they are seen as important to this study, and to show how they are expected to support the study.

The field of mathematics education research

Before delving into specifics, I would like to situate this study within a broader theoretical framework and talk briefly about the specific field of mathematics research. In the 1990's mathematics researchers focusing on learning started to move beyond individual cognitive development to considering sociocultural and situated aspects of mathematics learning. This is what Lerman (2000) refers to as the 'social turn in mathematics education research'. Boaler and her colleagues (Boaler, Ball, & Even, 2003) point out that mathematics researchers began to expand their frameworks and analytical perspectives beyond psychology, drawing on a wide range of disciplines including philosophy, anthropology and sociology' (p. 495).

Boaler et al (2003) also call attention to the fact that mathematics research comes under criticism for its lack of practical impact in educational settings. It is now more important than ever for the mathematical researcher to make a contribution to both theory and practice (p. 502).

Broader theoretical framework

One's view of reality (ontology) and how one acquires knowledge (epistemology) inform *what* one wants to research and *how* one does this. From an ontological perspective, this study views reality as socially constructed. This means that reality can be seen by multiple people who interpret events from differing perspectives, giving various perceptions of those same events. This coheres with an epistemological view that knowledge is gained through personal or collective experience giving a subjective meaning to knowledge. More specifically, the underlying foundation for this study is that knowledge is generated through collective meaning making as opposed to being generated on a personal basis in the individual mind.

Situated learning

This idea of collective meaning making is central to *situated theories of learning*, where learning is located in particular forms of experience and not simply in the mind. From this perspective, the learner and what is being learned are always situated in activities, processes and contexts. The focus is very much on how individuals interact in social practices (Prescott & Cavanagh, 2008) and could be regarded as '*learning-in-*

activity' (Hunter, 2008). Primarily, from this position, the idea of learning, whether successful or not, whether through interaction or not, is a product of collective doing (Ben-Yehuda, Lavy, Linchevski, & Sfard, 2005). The work of Vygotsky provides a key starting point for this perspective on learning in that sociocultural factors are essential in human development. A situated learning perspective can be seen as providing a bridge between cognitivist and sociocultural perspectives by giving 'primacy to the local, subjective and social constructed contexts' (Graven, 2002a). Learning from this view point is dynamic and is an 'integral and inseparable aspect of social practice' (Handley, Sturdy, Fincham, & Clark, 2006).

From this perspective, learning can be thought of as emerging from opportunities to participate in the practices of a community as well as developing an identity which provides a sense of belonging. Knowledge is therefore seen as mediated and also socially constructed.

Within this view, learning is therefore viewed as authentic participation in the discourse and practices of a community. Learners make sense of their experiences as they increase the range and level of participation in the norms and practices of the community (Lave & Wenger, 1991). Situated learning positions the 'community of practice' as the context in which an individual develops the practices and identities appropriate to that community.

Wenger's social theory of learning revolves around trying to find an answer to how these identities are created and changed through participation in such a community of practice (Wedege, 2009) and as well as how to understand learning as a **social** process (Wenger, 2000). Learning is located in the process of co-participation and not in the heads of individuals.

Wenger's (1998) social theory of learning builds on his initial work with Lave (Lave & Wenger, 1991) and is based on four premises:

- That people are social beings is a central aspect of learning
- Knowledge is about competence with respect to 'valued enterprises'
- Knowing is about active engagement in the world
- Meaning is ultimately what learning produces

Knowing and learning are further described by Wenger (2000). For him knowing is a matter of displaying competences defined in social communities. However we experience knowing in our own ways based on our experiences of life. Therefore socially defined competence is always in interplay with our experiences. It is in this interplay that learning takes place. Whenever competence and experience are in close tension and either starts pulling the other, learning takes place. Learning is therefore a dynamic thing, between people and the social learning systems in which they participate. It combines personal transformation with the evolution of social structures (2000, pp. 226-227).

Wenger's conceptual framework for his social theory of learning is made up of four components: *meaning* - learning as experience; *practice* - learning as doing; *community* - learning as belonging and *identity* - learning as becoming. This is shown in diagrammatic form in Figure 2. These elements are interconnected and are mutually defined by the other components. Wenger notes that one can switch any of these components with learning and make it the primary focus and the model would still make sense. Indeed, this is what many researchers do when using this framework as a tool for analysing and describing learning that takes place in a situated context. [3]

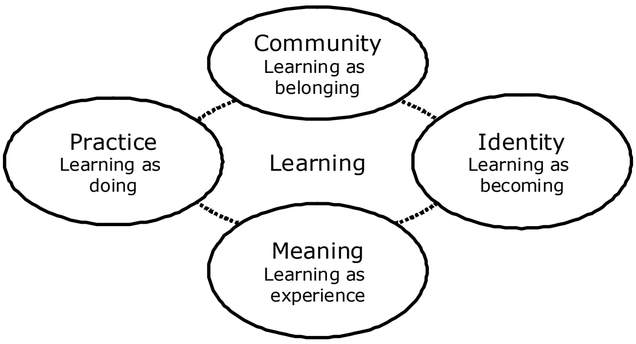

Figure 2 - Wenger's components of social theory of learning

Benefits and drawbacks of using Wenger's social theory of learning

Graven (2002b) points out that Wenger's work has been increasingly drawn on to describe learning in mathematics whilst Kanes and Lerman (2008) state that studies such as my own who wish to foreground the

[3] More detail on this is provided in section 8.7

social dimensions of mathematics education will find this work useful as an analysis tool. There is however much debate in the literature concerning this theory of learning and what it fails to adequately address. One of the issues is that the theory doesn't provide a way to understand how people fail to learn. Wenger indicates that people always learn something, even if it's learning how not to belong but he doesn't account for failure to learn. Some studies have addressed this failure to learn primarily from an identity standpoint (see for example Ben-Yehuda, Lavy, Linchevski, & Sfard (2005) and Boaler & Greeno, (2000)) whilst others have used social theories such as Bernstein to explain issues (see for example Cooper & Dunne (2000) and Lerman & Tsatsaroni (1998)). This is important for this study, as learners may fail to evolve in terms of mathematical proficiency and this will then need to be explained.

From this situated perspective then, we can see that the community of practice is the unit of analysis. The empirical field for this study has been conceptualised as a community of practice, therefore what follows is a discussion on the key elements of community of practice and identity.

7.1 Key Framing Concepts

Participating in a community

As we have seen contemporary situated perspectives consider that learning involves an increasing participation in a community of practice. Wenger's defines 'communities of practice are groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly' (Wenger, 2006). Although communities are everywhere, not all communities are communities of practice. Communities of practice are normally formed by people who engage in a process of **collective learning** in a shared domain of human endeavour. Furthermore he cautions that communities of practice should not be 'romanticised': they are 'born of learning, but they can also learn not to learn' (Wenger 2000, p. 230).

What assumptions are made about learning in communities of practice that are relevant for the purposes of this study? Wenger's theory of learning can be summarised using the following principles. Learning:

- is the ability to negotiate new meanings
- creates emergent structures
- is fundamentally experiential and social
- transforms our identities
- is a matter of belonging using different forms of participation

This final statement is very important as this notion of belonging is what Wenger (1998) refers to as '*modes of belonging*'. Our belonging to a community can take various forms at various levels. These different forms of participation are *engagement, imagination* and *alignment* (Wenger 2000, p.227). A more detailed description of these modes appear on page 12.

What are the necessary features of a community of practice in mathematics?

Watson and David (2008a) articulated the features of communities of practice in specifically *mathematics*. Drawing on this work and that of Wenger (Graven, 2002a; Wenger, 1998; 2006; Wenger, 2000), the key features of a *community of practice within mathematics* are:

- Participants work purposefully together towards the achievement of a common mathematical understanding
- There are shared, evolving ways of behaving, language, habits, values and tool-use
- The practice is essentially constituted by active participation
- Participants may evolve to function mathematically
- Through participation in the practice, participants create mathematical identities
- As a result of participating in the practice of the community, it is possible that all participants learn

In relating these features specifically to my research questions, point 5 highlights that participation can be the conduit or vehicle through which mathematical identity is created. Point 6 indicates that for all members of the community (including teachers or mentors), it is possible that mathematical proficiency may evolve.

In this study the Maths Club will be considered and designed to be a community of practice (and inquiry). This concept of a community of practice has been tested to some extent in South Africa with in-service teacher programmes and within Maths Clubs at secondary school level as illustrated by Graven (2002b; 2004; 2011b; 2002a). What is not yet clear is how this framing concept will work with younger, foundation phase children in this context of a Maths Club. As the data emerges from the empirical field, especially through the pilot, it will be possible to see how this theoretical framework might frame the research and how it might promote the development of mathematical proficiency.

Where do communities of inquiry fit into this picture?

Kanes and Lerman (2008) point out that a 'critical disposition' or 'critical reflective stance' is important to research in mathematics education, especially when working within the context of community of practice. Jaworski (2003; 2006a; 2006b) believes that the concept of a community of practice can be extended to being a community of inquiry by ensuring that the members of the community have a clear sense of belonging and that the belonging 'presupposes an inquiry stance'; a place from which to look critically, in a positive way, at whatever we are engaged in, asking critical questions that take us deeper into the substance of our activity (2006a, p.78-79). For Jaworski, inquiry is not the practice of the community but is an attitude and tool for looking critically at how goals are achieved within that community.

The Maths Clubs are framed and conceptualised as communities of practice with a strong bias towards inquiry. It is the intended aim that the participants in the Maths Club may develop increasingly inquiry dispositions. We can see that this inquiry stance may be different for different participants in the Maths Clubs. For example, for both mentors and learners it may become apparent as a search for knowledge and an attitude of questioning. For myself as the co-ordinator and researcher, it may manifest as a place from which I can look critically, in a positive way, at whatever I am engaged in, asking critical questions that take me deeper into the essence of my activity.

Identity

Wenger's (1998) notion of '*learning as becoming*' is integral to identity. For him, learning and identity are inseparable. Because learning transforms who we are and what we can do, it is an experience of identity. He states that it is not just an accumulation of skills and information but a process of becoming – becoming a certain type of person or of not becoming a certain type of person (p. 215).

Identity and specifically identity in mathematics is a vast field of research. Psychologists, anthropologists, sociologists and mathematics educational researchers each have a finger in this pie! Sfard and Prusak (2005) believe that the notion of identity is a "perfect candidate for the role of 'the missing link' in the researchers' story of the complex dialectic between learning and its sociocultural context" (p. 15) but they also point out that although many claim that identity has 'potential as an analytic tool for investigating learning', it is ill-defined and does not have an operational definition. Their operational definition is shown below[4]. They argue that it is therefore difficult to use identity as an analytical tool. In spite of this seemingly large hurdle, this research study will seek to use identity as an analytical tool and as such an aim of this PhD will be to contribute to this debate especially in regard to identity formation in early primary school mathematics learners.

A review of the literature on identity will reveal that there are many ways of describing identity. The final definition I will use to analyse my findings in this study will ultimately emerge from the empirical field as data is collected. In the meantime, however, based on the context within which this study is situated, a broad sociocultural perspective defines identity as being constructed by individuals as they actively participate in cultural activities (Nasir, 2002, p. 217-219).

The situated learning perspective brings a 'renewed or alternative focus on issues of identity' (Handley et al. 2006, p644). From this perspective, learning is not simply about developing one's knowledge and practice, it also involves a process of understanding who we are and in which communities of practice we belong and are accepted. Using identity from a situated perspective can be used as a way of understanding and explaining how learners act in different situations. Specifically, the pilot will provide opportunity for consolidating the theoretical perspectives on identity that I have fore-grounded in the following paragraphs.

In keeping with the framing concepts of this study, the community of practice theory previously discussed gives a view identity which is characterised as '*a constant becoming*' that defines who we are (Wenger; 1998, p.149). From this standpoint, identity:

- is a negotiated experience whereby we define who we are through our participation and by the way we and others reify ourselves
- is community membership
- is a learning trajectory whereby we define who we are by where we have been and where we wish to go
- is nexus of multi-membership whereby we define who are by the way we combine our various forms of memberships into one identity
- is defined just as much by the practices we engage in (participation) as the practices we do not engage in (non-participation).

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[4] Sfard and Prusak's (2005b) operationalised definition of identity is a set of reifying, significant, endorsable stories about a person. These stories, even if individually told, are products of a collective storytelling.

Perhaps the most significant feature of Wenger's theory in terms of identity is his description of three modes of belonging through which we participate in communities of practice. These are engagement, imagination and alignment. *Engagement* is about mutual participation (or choosing not to participate) in meaningful activities and interactions. *Imagination* refers to an open-minded disposition that requires a willingness to explore, take risks, and make connections in order to create new images of the world and ourselves. *Alignment* describes a process of coordinating perspectives and actions and finding a common ground from which to act. At this stage, I anticipate using these modes as the analytical tools for identity.

Wenger (1998; 2000) states that it is useful to distinguish between these modes for two reasons: one, from an analytical point of view, each mode contributes different aspects to the understanding of formation of personal identities. Two, from a practical point of view, each mode requires a different kind of '*work*'. Most of what we do involves a combination of the three so it is useful to try and develop these modes in combination. Each of the modes has a number of abilities and processes that typify the work of each mode. These are listed in the Methodology section.

Nature of mathematical learning

I have already explored learning to some extent by examining Wenger's theory of community of practice. From that we see learning as participation in a community. Now I look a little deeper into mathematical learning and specifically with learning as 'becoming a participant' in mathematical practice.

Beliefs about mathematical learning

In the last twenty years or so, mathematics educators and researchers have recognised that mathematics and mathematics learning is a social practice (Lerman, 2000). According to Siegel & Borasi (1994) many researchers have argued that becoming a successful mathematics learner is not simply about acquiring an established body of knowledge and skills. It is perhaps more importantly about taking on a set of beliefs, norms, world views and practices characteristic of the mathematics community. David and Lopes (2002) support this view and state that mathematical activity includes dimensions that are related to socialisation and metacognition. If we want learners to learn mathematics in a meaningful way, then they need to learn how to think mathematically. They need to be socialised into the practices, processes and ways of thinking that are characterised by the activity of mathematicians. Ball and Bass (2000) highlight that these types of beliefs 'define a problem space' for mathematics teaching and learning and that this 'shapes' practical decisions as well as the pedagogical issues that arise in practice. This supposes a shift from learning as 'acquiring mathematical knowledge' to learning as 'becoming a participant' in mathematical practice. These ideas are considered in this section.

Learning as participation vs. learning as acquisition

Sfard's much cited 1998 article, describes the differences between two metaphors. '*Learning as acquisition*' theories can be regarded broadly as mentalist in their orientation, with the emphasis on the individual building up cognitive structures. In contrast '*learning as participation*' theories attend to the socio-cultural contexts within which learners can take part (Sfard, 1998).

Her table of metaphorical mappings gives a schematic comparison of the two metaphors. From this we could draw the conclusion that the community of practice framework could rest upon the metaphor of 'learning as participation'. Learning takes place as a result of becoming a participant in a community and knowledge is an aspect of that practice. Sfard (2001) further draws attention to the fact that the participationist researcher will focus on the growth of mutual understanding and coordination between the learner and the rest of the community and the focus will turn to the activity itself and to its changing, interactional aspects. This is an important consideration for this study, as I will be exploring how learners' identities and mathematical proficiencies evolve in relation to their participation.

Acquisition metaphor		Participation metaphor
Individual enrichment	Goal of learning	Community building
Acquisition of something	Learning	Becoming a participant
Recipient (consumer), (re-)constructor	Student	Peripheral participant, apprentice
Provider, facilitator, mediator	Teacher	Expert participant, preserver of practice/discourse
Property, possession, commodity (individual, public)	Knowledge, concept	Aspect of practice/discourse/activity
Having, possessing	Knowing	Belonging, participating, communicating

Figure 3 - Sfard's metaphorical mappings (1998)

While some writers argue for the need for a paradigm shift away from (or even rejecting) acquisition perspectives in favour of participation, I would like to elaborate on my position for the purposes of this study. I agree with Sfard when she suggests that these metaphors are not alternatives but that each provides different insights into the nature of learning as she argues here:

> *"An adequate combination of the acquisition and participation metaphors would bring to the fore the advantages of each of them, while keeping their respective drawbacks at bay. Conversely, giving full exclusivity to one conceptual framework would be hazardous" (p. 11).*

This study blends notions of acquisition with notions of participation. One research question focuses on learner evolving identity formation by participation and belonging in a community thus bringing the participation metaphor into focus. On the other hand, the focus of the second research question on evolving numeracy proficiency will benefit from drawing on the acquisition metaphor. This individualises mathematical proficiency as assessments and activities would be measuring what mathematical proficiency has been acquired at a particular points in time. This approach is by no means unique to this study.

Researchers working in the fields of situated learning, community of practice and inquiry have highlighted that whilst communities of practice are excellent ways of researching the nature of teacher learning and participation, research that takes place with regard to the actual mathematical learning of primary school learners needs more tools for making sense of learning process. Research into mathematical learning can require a blending of both participation and acquisition metaphors. As we see, Sfard (1998) advocates a blended approach when she debates the learning as acquisition and participation metaphors. She asserts quite strongly that 'too great a devotion to one particular metaphor can lead to theoretical distortions and to un-desirable practices' (1998, p 4).

Studies by Goos, Galbraith, & Renshaw (2002), Jaworski (Jaworski, 2006b; Jaworski & Potari, 2009) and Askew (Askew, 2004) provide some mathematical examples where blending of these two metaphors have taken place.

Learning as mathematical participation

If learning is seen as participation, it is important to look more closely at what *mathematical* participation means. Firstly, we are asking learners to contribute in different ways to what they may be habitually used to. Boaler (2002) believes that the participation that is required of learners who learn in participation-oriented environments is very different to that expected of learners in more traditional, acquisition based classrooms. They are required to contribute *different aspects* of themselves as well as contributing **more** of themselves. Secondly, the argument is put forward by Boaler and Greeno (2000) that mathematics learning *is participation* in mathematical practices and they propose that learning of mathematics can be seen as a 'trajectory of participation' in the practices of mathematics. Thirdly, Ball and Bass (2000) suggest that when learners' interactions and participation are viewed from a *mathematical* perspective, particular features of mathematical learning become visible (p. 218). A fourth aspect is considered by looking at exactly what the practices are that we wish the learners to participate in (in the Maths Clubs). These are discussed under the sociocultural norms heading below.

Dowling (1996) and Wenger (1998) believe that if we focus on the patterns of participation that constitute learning, we can gain insights into the nature and extent of identification and belonging that the learners develop as they learn to be mathematics learners. David and Watson (2008) refine this focus by arguing that when we focus on learning as participation, we also need to look at the '*extent* to which the participation is mathematical in order to say anything about mathematical learning' (p. 49). They claim that it isn't enough to simply say that the learners participated in various practices. They advocate that we need to look for the kinds of mathematical activity that are *afforded* in classroom interaction sequences and how learners are *constrained* by teachers. These ideas will form the basis for initial focus questions for data analysis.

For this study it is important to be able to qualify the nature of participation and to be able to determine whether it is mathematical or not. For example, we could say that learners are participating in the clubs simply by showing up every week, but this does not give us a sense of either their mathematical participation in the clubs or of how their mathematical proficiency evolves. For that to be evident we need to look at how they participate in the practices put forward by the club, the extent to which that participation is mathematical, what they give of themselves and what they contribute to the community.

Sociomathematical norms

The next idea is that of socialising or initiating the learners into mathematical processes and ways of thinking. The term 'norms' was introduced to designate the mutual expectations that are established in the classroom through the interactions between the teacher and the students (Cobb, Wood, Yackel, & McNeal, 1992; Yackel & Cobb, 1996). Yackel and Cobb (1996) identified three norms in mathematics classrooms: mathematical, sociomathematical, and social.

For the purposes of my definition, I will focus on the *sociomathematical* norms which are described by Yackel and Cobb as the normative aspects of classroom activity that are related specifically to mathematics (1996). Hunter (2008) explains that these sociomathematical norms evolve from mathematical activity between *all* classroom participants as they are *negotiated* in the discursive dialogue.

These norms are important for the study, as they describe what practices we want the learners to participate in and what practices the club mentor will initiate the learners into. The mentor has a central role in this. These norms concern expectations about how mathematical activity should be carried out, concern the type of discourse and mathematical talk that is likely to foster productive participation in a mathematical community and relate to the mathematical qualities of participation and concern the expected forms of social interaction in a mathematical community. In addition these norms are important aspects of developing some of the strands of mathematical proficiency such as adaptive reasoning and productive disposition which are discussed below.

Numeracy and becoming mathematically proficient

The next area of focus is defining what numeracy is and what it means to become mathematically proficient as this is central to my second research question. I define numeracy as it appears in the Chair documentation. This working definition is obtained by combining, extending and slightly adapting two seminal studies of Kilpatrick et al (Kilpatrick, Swafford, & Findell, 2001) and Askew et al (Askew, M. Brown, Rhodes, Johnson, & Wiliam, 1997) which will be discussed below.

> *"The ability to process, communicate and interpret numerical information is a variety of contexts overlaid with strands of numeracy proficiency: understanding numeracy concepts, computing fluently (practically, mentally and procedurally), applying concepts to solve problems (in creative and inventive ways), reasoning logically (in creative and inventive ways) and engaging with mathematics – seeing it as sensible, useful and do-able (enjoyment and passion)" (Graven & Schafer, 2011, p.20).*

I now turn to the idea of *mathematical proficiency*. In light of public concern in the USA regarding how well children learn mathematics, the National Research Council produced a report called '*Adding it Up: Helping Children Learn Mathematics*'. The report maintains that mathematical procedural fluency has been the focus of instruction in the past and has been over emphasised. They contend that much more is needed to prepare learners for the world and for personal success. All students should and can be mathematically proficient. Proficiency is an important foundation for further instruction in maths as well as for further education in fields that require maths. But more importantly, they claim that in order for people to participate fully in society, they must know basic mathematics (Kilpatrick & Swafford, 2002; Kilpatrick, Swafford, & Findell, 2001; National Research Council, 2001).

Their research led them to adopt a 'composite, comprehensive view of successful mathematics learning'. They recognised that no term can completely capture all aspects of mathematics, so they chose the term '*mathematical proficiency'* to comprise what they think it means for anyone to learn mathematics successfully. They see mathematical proficiency as having *five interwoven* strands:

- *conceptual understanding*: comprehension of mathematical concepts, operations, and relations
- *procedural fluency*: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*: ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*: capacity for logical thought, reflection, explanation, and justification
- *productive disposition*: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

They emphasise that these five strands are separate yet interwoven and interdependent. Mathematical proficiency is only present when all strands are developed and proficiency develops over time. This has implications for "how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal" (National Research Council, 2001, p.5).

The importance of this notion of mathematical proficiency to this study is considerable. In the first instance, this study seeks to explore how evolving mathematical proficiency can be promoted in Maths Clubs and to understand how forms of participation, interactions and promoted activities enable or constrain this evolving mathematical proficiency. Secondly, it provides a framework or language for describing and analysing learners' mathematical proficiency trajectories over the one year period. Finally this view of mathematical proficiency addresses some of the wider contextual issues raised in the opening sections of this proposal.

This view of successful mathematical learning does not take social or cultural factors into account. However, in my view these are well accounted for by other key framing concepts described in this section.

Another key framing text is '*Effective Teachers of Numeracy'* (Askew, M. Brown, Rhodes, Johnson, & Wiliam, 1997). The extract from the Summary of Findings gives us an overview of the study:

> *"The study explores the knowledge, beliefs and practices of a sample of effective teachers of numeracy. It is one of a small number of projects where effectiveness is defined on the basis of learning gains: i.e. teachers were identified as highly effective if their classes of pupils had, during the year, achieved a high average gain in numeracy in comparison with other classes from the same year group (1997, p.3).*

Although this UK based study is now over 14 years old, it is still important to this study for two reasons. The first is that the definition of numeracy used in the Askew et al study has been adopted by the Chair as detailed above (See Askew et al 1997, p.10). Secondly, whilst the focus of this study was on mathematics teachers and teaching, widespread numeracy assessments of learners were carried out to determine the effectiveness of the teaching. The Chair has been given permission to use these diagnostic assessments and these may prove useful for a formative assessment tool of mathematical proficiency in the Maths Club as well as a data collection instrument. See the Methodology section for more information about this.

7.2 Maths Club Programme Design

A cohort of literature relating to the Maths Club design and implementation process is discussed in the Context section and in Appendix B. One other key issue is highlighted here.

Interactions

A look at the problem statement and sub research questions for this study will show that interactions between participants form a part of the research. I would now like to talk a little about these interactions. Interactions can be seen as the relations that take place between the participants in a classroom, and as discussed in the Key Problems section, these can become sedimented or fixed for a number of reasons.

Askew (2008) believes that despite the 'situatededness' of the discourse, mathematics education is still very much focussed on the processes and outcomes of learners' cognitive changes. This essentially means that the focus is still very much on the individual within the social context. He makes a case for another way forward in which a more dynamic, emergent model of classroom identities is created, one where teachers and learners co-construct the available 'positions' as necessary. By changing the narrative, there is potential for different configurations to be developed, configurations arising from 'attending to relations rather than individuals' (2008, p.65). This study will investigate if there is a relationship between these relations and the evolving mathematical identities and proficiencies of the learners.

Askew (2008) argues that relations within classrooms, between pupils & teacher, pupils and pupils and between teacher, pupils and maths are central to the ongoing negotiation of new ways of being. He maintains that 'relations are just as situated as learning'. In other words, the interactions between the participants are situated in particular contexts and are different in different contexts. As these relations play out, social identities emerge for learners and teachers (2008).

8 Research Method / Methodology

8.1 Research paradigm

The theoretical background for this enquiry points to an interpretive research paradigm. From this perspective, research must be observed from inside and through the direct experiences of the people involved. It recognises that:

- people actively construct their social world
- situations are fluid and changing rather than fixed and static; events and behaviour evolve over time and are richly affected by context - they are 'situated activities'
- events and individuals are unique and largely non-generalisable
- people interpret events, contexts and situations, and act on the bases of those events
- there are multiple interpretations of, and perspectives on, single events and situations
- reality is multi-layered and complex
- many events are not reducible to simplistic interpretation, hence 'thick descriptions' are essential rather than reductionism
- we need to examine situations through the eyes of participants rather than the researcher (Cohen, Manion, & Morrision, 2000, p.22)

This type of research aims to understand, explain and demystify social reality through the eyes of the differing participants, in other words to understand the subjective world of human experience. It focuses on understanding actions and meanings and not on the causes or the underlying political, ideological interests that shape participants behaviour. The conclusions reached will be relativist in that the truth will be relative to this particular frame of reference or context and will not be an absolute truth.

It is from this position and from the stated ontological and epistemological standpoints mentioned earlier, that the research methodology has been designed.

8.2 Style of research

The research will be a longitudinal, qualitative case study using two Maths Clubs. These clubs will be run at two of the Chair's 15 selected schools. I have chosen a case study style as it allows one to portray, analyse and interpret the complexity and uniqueness of real people and their situation in the Maths Clubs. It can be used to provide an in-depth study of a particular learning environment. Its aim is the production of a nuanced description of the context, and an account of the interactions that take place between learners and other relevant persons in the educational context.

The case study allows an understanding of a particular subjective experience and will show what can be learned from this particular case. Generally, results from case study research cannot be generalised but they can be used to powerfully illuminate other situations.

The learners included in these clubs will be members of the Grade 3 classes of the selected schools and I will be the co-ordinator of both clubs. The research will be largely qualitative supplemented with quantitative research in the form of ongoing assessment of learner mathematical proficiency and will be a longitudinal analysis with data collection occurring at different points over a period of 1 year from November 2011 to November 2012.

8.3 Sampling

According to Cohen et al (2000), my sample will be a '*non-probability sample*' as I will be targeting a particular group, in the full knowledge that it does not represent the wider population; it simply represents itself. Furthermore it will be of the '*convenience*' sampling type which is also known as accidental or opportunity sampling which involves choosing the nearest individuals to serve as respondents (2000, p. 102). Within the Chair, I have 15 selected schools to choose from to run clubs in. I have chosen to case study two schools. This will be done based on those being closest to the University in Grahamstown. One school will be a township school and the other will be an ex-model C school. Club participants will be drawn from the Grade 3 learners in these schools.

8.4 Ethics

As I will be working with young learners in the Maths Clubs, it will be especially important to pay attention to ethical issues. There are a number of ways that I will go about doing this.

Firstly, let's look at the issue of *informed consent*. In the first instance, I will seek parental consent. This will be achieved by sending letters to the parents explaining the research and the anticipated learner involvement. The letters will be available in three different languages. The parents will be required to sign these letters of consent. The parents will be provided with contact details and they will be encouraged to phone or meet to discuss any concerns they may have.

Secondly, written teacher, principal and district permission will be required for each school and class that wishes to allow learners to participate in the Maths Clubs.

Finally, I will explain to the learners how they will be involved in the research and in the Maths Clubs. This explanation will be offered in English, Afrikaans and isiXhosa and delivered by the use of translators directly to the learners. An open discussion also will be held and will be used to explain to the learners how the clubs work and that they can withdraw from the club and / or the research without consequences.

The second issue regards *anonymity*. As the learners will be recorded via field and journal notes, via video and occasional voice recordings, it is important that no names are used in subsequent publication of data including the PhD thesis. To this end, learner and school names will be changed.

The third issue is about *voluntary participation*. Club participation is voluntary. Learners will be asked if they wish to participate and teachers' advice in this regards will be sought. Learners will be able to leave the club at anytime if they, their teacher or parent wishes them to do so. Alternatively they are free to remain in the club but withdraw from providing data for the research. If this occurs, I will ensure that learners who are not involved in the research work in different locations to those that are so that they are not included in any video or voice recordings.

8.5 Data collection methods

Data collection will take place over one year plus an additional pilot period of 6 months. I have a variety of data collection methods which are described in detail below. I will use an integrated, on-going data collection and analysis approach to help maintain control over the process and allow other possibilities and contrary and alternative explanations to present themselves.

Once again, the pilot will provide the opportunity collect, sort and attempt to make some sense of the data and to see if the method of analysis I have chosen works. A detailed discussion follows with regard to the rationale for choosing specific data collection methods and the data collection time frame.

Rationale for data collection instruments

It has been important to select my data collection instruments carefully for a number of reasons. Working in interpretive research paradigm, I need to ensure that I am not attempting to collect data that doesn't cohere with the paradigm. In addition, due to the complex nature of my role within the clubs, I need to ensure that I can maintain research focus as well as validity and reliability in the data collected. Cohen et al. (2000, p.79) indicate that data collected from a case study is in-depth and detailed and comes from a wide variety of data sources. These considerations are therefore reflected in the selection of the data collection methods described below.

- Non-worded learner questionnaires
- Informal learner interviews
- Learner focus group interviews
- Mathematical task-based interviews
- Observations: structured peer and participant-as-observer observations
- Video and voice recordings
- Combined Field Notes and Reflexive Journal
- Copies of learner work examples
- Baseline mathematical proficiency assessments (written and verbal interviews)
- Askew et al diagnostic instrument – formative assessment of mathematical proficiency
- Documents generated by the Chair in connection with the Maths Clubs

The reasons why I have chosen these particular instruments are given below. They are grouped under three headings namely: instruments for investigating identity, participation, interactions and activities; instruments for assessing learner evolving mathematical proficiency and overall instruments.

1. Instruments for investigating identity, participation, interactions and activities

Learner questionnaires will be designed that are primarily non word-based with the aim to find out how the learners currently feel about mathematics, school and themselves. Cohen, Manion and Morrison (2000) explain that when the respondents are children, word-based questionnaires can be 'off-putting'. They point out that in these circumstances it is acceptable to design a questionnaire that might include visual information and ask participants to respond to this visual information. For example pictures, cartoons might be used. There are some issues with this approach but I will trial it during the pilot. I intend to ensure reliability and validity by verifying the data collected via these questionnaires with data gleaned from the learner interviews, focus groups and Maths Club observations.

Learner interviews. Formal interviews with the learners may be seen as threatening to the learners so I anticipate that the interviews will be what Cohen et al call '*interview guide approach*' where the topics and issues to be covered are specified in advance in outline form but the interviewer decides the sequence in the course of the interview to keep the interview conversational (2000, p. 271). I want to use the interviews as a descriptive tool to gain a sense of the learners: who they are, how they feel about mathematics, the Maths Clubs and so on. The design of these interviews will be informed by the pilot as this will allow me to determine the topics and questions I wish to ask and how the learners may react to this type of situation.

Learner focus groups are a form of group interview, where the group discuss a topic supplied by the researcher. The focus is on the interaction within the group rather than with the interviewer. They allow views of the participants to emerge and the interaction provides the data. I will therefore use them to allow the learners to discuss a certain topic and they will be useful as a way of deriving insights from the learners and for providing triangulation with learner interviews and questionnaires. In addition they may be a way of getting more genuine responses from the learners rather than simply responses given in the interview situation where learners may give the answers they think you want. They can also help learners avoid feeling uncomfortable or threatened by a one-to-one situation. I have planned various formal focus groups for during the collection period. However, Goodchild (2002) suggests that learners can be selective in their memories and of recalling events that were significant for them. To this end, I anticipate that I will do informal focus-groups at the end of every Maths Club session.

Task-based interviews. In my role as club leader / mentor and as part of the initiation into the sociocultural norms in the club, I will naturally be regularly engaging the learners in conversations about their activities whilst they are working as well as asking them questions to explain their thinking. McKnight et al (2000, p.75) propose these specifically mathematical *task-based interviews* as a way to explore learners' approaches to and thinking about problem solving and other mathematical tasks.

Ongoing session and group work observations. These will mostly be done by myself in my role as researcher and will be mostly unstructured and will be noted in my combined field notes and reflexive journal. However, because I do have a complex role in the clubs, it is important to understand researcher roles in observations. Cohen et al point out that in observations, researcher roles can lie on a continuum. At one end lies the *complete participant,* moving to the *participant-as-observer,* thence to the *observer-as-participant,* and finally to the *complete observer* (2000, p 305). In order to carry out these observations, I will need to move from 'complete participant' to 'participant-as-observer' and back again. Cohen et al explain that unstructured observations are far less clear on what they looking for and therefore allow for observations of what is taking place before deciding on its significance for the research (2005, p 305).

Structured peer observations will be carried out by Chair and Maths Club colleagues. As structured observations are meant to know in advance what it is they are looking for, at this stage I anticipate they will primarily be used provide data on practices and interactions in the clubs. For example, I may ask observers to look at learner-learner interactions or learner-mentor interactions. These types of observation are systematic and can be used to generate numerical data which can then facilitate the making of comparisons between settings and situations, and frequencies, patterns and trends can be noted or calculated (Cohen et al., 2000, p 306). Peer observers will be passive, and will not be involved in the club. They will simply note down the incidence of the factors being studied. Observations tend to be entered on an observational schedule which will need to be designed and trialled before it is used to gather data. This design and trail will take place once again within the Maths Club pilot.

Voice and video recordings will be frequent and ongoing and are designed to capture the day-to-day activities and details of the clubs. Video recordings first will tend to be set up to record a Maths Club session or aspect of the session. The video record will act as a 'complete passive observer' in the club to free me up to take up my role of club mentor. Cohen et al claim that the role of the complete observer is typified in the one way mirror, by using a video camera, voice recorder or photograph to make the observations (2000).

The positioning of video recorders can thus become an important methodological issue as the data can be seen as invalid and unreliable. Cohen et al (2000) note that if the video recorder is always fixed in the same place, it might be seen as a selective observer, if it is movable, it can be seen as a *highly* selective observer. It is important therefore to introduce some kind of systematisation into video observations so that reliability is increased. The pilot will give me the opportunity to work on how to systemise this aspect of data collection.

2. Instruments for assessing learner evolving mathematical proficiency

In the discussion about the nature of mathematical learning on page 12, I discussed how this study blends the participation and acquisition metaphors. The 2 instruments discussed below focus on data collection for determining learners' evolving mathematical proficiency and are primarily connected to research question number 2.

Baseline numeracy proficiency assessments (written and verbal interviews). The Chair has used baseline US AID[5] assessments to assess a sample of grade three learners. The assessments are comprised of two parts: written assessments which are 'naked' sums without words and oral interviews, in which the subtests are linked to international curricula and are highly predictive of future success in mathematics (Brombacher, 2011).

The written assessments have been taken by all Grade 3 learners in participating schools, whilst 6 learners from each Grade 3 class have been interviewed. This baseline data will continue to be collected through the Chair for the duration of the project. Whilst it has not been collected specifically for the Maths Clubs, I can access this data by being part of the Chair. It can therefore provide Maths Club mentors and me as a researcher with a means of tracking learners' mathematical proficiency trajectories.

The Askew et al instrument is a diagnostic assessment and as stated earlier, the Chair has been given permission to use these. These give me a tool for ongoing formative assessment of the learners. These can be administered verbally to a whole class or individually as required. These tests assess:

- Understanding of the number system, including place value, decimals and fractions

[5] Developed by Research Triangle International in the United States. These have been adapted by Aarnout Brombacher for African contexts.

- Methods of computation, including both known number facts and efficient and accurate methods of calculating
- Solving numerical problems, including complex contextualised word problems and abstract mathematical problems concerning the relationships between operations (Askew et al, 1997, pp 15-16).

The baseline assessments used by the Chair are not tailored to match the 5 strands of proficiency model discussed earlier. However, the Askew instrument looks specifically at all but 1 of the 5 strands of proficiency (that of productive disposition), making them an ideal tool for this study.

Learner work examples will also contribute to the view of numeracy proficiency trajectories and allow me to see how learners approach problem solving activities as well as more procedural mathematics problems. Examples will include rough workings at the time of the activity or task engagement as well as any work done in workbooks. These examples will either be photocopied or photographed. In addition, these examples will provide additional data to validate video and audio recording.

3. Overall instruments

There are two instruments that pertain to the whole study. One will be my *combined field notes and reflexive journal*. I have already started to use this and intend to write in it regularly. It will help me make sense of the different perspectives I have on the research as described in section 6.3 on page 6 as well as being a place where I can air my excitement, frustrations and other feelings that show themselves over the course of the research project.

The second will be any *documentation* that is generated with regards to the Maths Club design process and its implementation. For example, I may have to write a status report for the Chair Advisory Board. This will become data for the study.

Data collection frequency and schedule

I have drawn up 2 further tables in connection with data collection.

- Table 2 shows how often I anticipate collecting each different type of data as well as an approximate total number of collections for each type.
- Table 3 shows the anticipated data collection schedule for November 2011 through to December 2012.

It is anticipated that the Maths Clubs will run one afternoon a week at each school during term time. As two Maths Clubs have been selected as the sample, data collection will take place in both of these to allow comparisons between the two schools chosen in the sample.

Due to the quantity of data that is to be collected, rather than leaving data analysis to the end, I anticipate at this stage, that data collection will be **on-going** and analysis will occur chronologically as the data is collected as suggested by (Merriam, 1998). This approach will allow me to maintain focus, identify coherent patterns and themes linked to the theoretical framework and will allow flexible revision of those emerging themes, if necessary.

Table 2 – Anticipated frequency and totals of data collections

Data collection method	Anticipated frequency of collection	Total collections per club (total)
Video recordings	Weekly	35 (70)
Voice recordings (per group)	Weekly & possibly per group of learners	35 minimum (70 minimum)
Structured peer observations	4 times in the year	4 (8)
Formal focus group interviews	Twice in the year	Dependent on numbers of learners
Task-based interviews	Ongoing	35 minimum (70 minimum)
Informal focus group interviews	At close of every club session	35 (70)
Learner interviews	Twice in the year	Dependent on numbers of learners
Learner questionnaires	Twice in the year	Dependent on numbers of learners
Askew instrument	Formally applied twice in the year but could be used as required	Dependent on numbers of learners
Combined field notes & reflexive journal	Ongoing	
Documents	Regular reports for the Chair, frequency unknown at present	Unknown

Table 3 – Anticipated data collection schedule for 2011/2012

Timescale	Data Collection methods		Review Chair Baseline Data	Askew Instrument	Learner Interview	Learner Questionnaire	Learner Work Examples	Ongoing Video recording	Ongoing Voice / audio recordings	Ongoing task-based interviews	Structured Peer Observation	Ongoing session & group Observations	Formal Focus Groups	Ongoing Informal Focus Groups
November 2011		Ongoing transcription and data analysis. See section on Data Analysis below for rationale	✓											
January 2012				✓1st	✓1st	✓1st		✓	✓	✓				✓
February 2012								✓	✓	✓				✓
March 2012								✓	✓	✓	✓1st			✓
April 2012								✓	✓	✓				✓
May 2012								✓	✓	✓	✓2nd			✓
June 2012								✓	✓	✓			✓1st	✓
July 2012								✓	✓	✓				✓
August 2012								✓	✓	✓	✓3rd			✓
September 2012								✓	✓	✓				✓
October 2012								✓	✓	✓			✓2nd	✓
November 2012				✓2nd	✓2nd	✓2nd		✓	✓	✓				✓
December 2012			No data collection											

8.6 Validity and reliability

Anfara and colleagues point out that a common criticism directed at qualitative research is that it fails to adhere to standards of reliability and validity (2002). I want to talk in particular about how I intend to use triangulation of methods as a strategy for avoiding this.

Triangulation and trustworthiness will be maintained through the differing and varied data collection methods and through the use of constant comparative methods which allow interplay between the data and theory and sustained engagement with all participants in the Maths Clubs. Table 4 below shows how the data sources correlate to the research questions and indicate that there are at least three different data sources for each question. This also ensures that I have aligned my research questions with my data collection methods.

Table 4 - Triangulation of data sources

Research questions	Field Notes & Journal	Design documents	Chair Baseline Data	Askew Instrument	Learner Interview	Task-based interviews	Learner Questionnaire	Learner Work Examples	Video recording	Voice recordings	Structured Peer Observation	Observations	Focus Groups: formal & informal
1. How do learner's mathematical identities evolve (if at all) over the period of participation in the Maths Club?	X				X		X	X	X	X	X	X	X
1b) How does this evolving identity relate to the forms of participation, interactions and activities promoted in the clubs?	X	X			X	X	X	X	X	X	X	X	X
2. How do learner's mathematical proficiency levels evolve (if at all) over the period of participation in the Maths Club?	X		X	X		X							
2b) How do these evolving proficiency levels relate to the forms of participation, interactions and activities promoted in the clubs?	X	X			X	X	X	X	X	X	X	X	X

I now look at validity. I wish to focus on *internal* and *external* validity. Although these tend to be more quantitative research terms, their underlying concepts are relevant to the qualitative field. Internal validity (or credibility) can be achieved through accurately describing the phenomena being researched, via careful transcription, prolonged engagement in the empirical field; persistent observation (in order to establish the relevance of the characteristics for the focus); and triangulation of methods, sources, investigators and theories (Anfara, K. M. Brown, & Mangione, 2002; Cohen, Manion, & Morrision, 2000). This study will have prolonged engagement in the field, multiple methods of observation and I have attempted to triangulate where possible as shown in Table 4 above. A member check (or respondent validation) is another strategy normally used to ensure internal validity. However, the learners in this study are too young to take on this role. The structured peer observations are therefore a vital element in ensuring internal validity and giving another view point of the data.

External validation (or transferability) is the degree to which the results of a study can be generalised to a wider population, cases or situations. This is a problematic aspect of qualitative research, especially in case studies. This is further exacerbated by the fact that I am working in the interpretive paradigm and am attempting to understand a subjective experience. In order to ensure that external validity in my study is addressed, I will provide what is called a '*thick description*'. Schofield explains that this is a 'clear, detailed and in-depth description so that others can decide the extent to which findings from one piece of research is generalisable to another situation' (in Cohen et al., 2000, p. 110).

Reliability refers to the extent to which research findings can be replicated and is a notion rooted in quantitative research. Cohen et al lay out options for a viable approach to ensuring reliability in qualitative research by cohering to real life, context and situation-specificity, authenticity, comprehensiveness, detail, honesty, depth of response and meaningfulness to the respondents (2000, p. 120). These suggestions will serve as guides for me in this study as I collect and analyse the data.

8.7 Data Analysis

Merriam (1998) recommends simultaneous data collection and analysis for generating categories and building theories. As previously stated, it is my intention to do this. Therefore, data collection will be on-going and analysis will occur chronologically as the data is collected. This approach will allow me to maintain focus, identify coherent patterns and themes linked to the theoretical framework and will allow flexible revision of those emerging themes, if necessary. In the early stages of the research, the data analysis will consist of identification of patterns and theme generation. As the research progresses and unfolds, data analysis will become more in-depth and will connect in more fundamental ways with the theoretical framework.

This approach allows me to incorporate advice put forward by Lerman (2003), whereby one constantly zooms in and out of specific parts of the complex teaching and learning processes in order to gain different perspectives on the research.

As I understand it at present, as data is collected, these steps need to take place on an on-going basis:

- Data transcription
- Identification of patterns
- Theme generation linked to theoretical framework
- Final analysis of that data in terms of the theoretical framework

Data transcription

This will be on-going and will involve transcribing anything that involves oral speech into a written form. This will include voice and video recordings, interviews as well as events that occur during observations. Cohen et al (2000) point out that transcriptions inevitably lose data from the original encounter and are therefore an interpretation of the data. In an attempt too avoid too much loss of data I will try to capture what was being said, the speaker's tone of voice and inflections of the speaker(s), mood, and speed of speech as well any other events that were taking place at the same time. This is where field notes, entries from my field notes and reflexive journal and feedback from observations will help to gain a fuller picture. Another advantage of on-going collection and analysis is that transcriptions are done whilst they are still fresh in the mind. I will transcribe directly onto the computer as this will allow the attachment of labels, sorting, searching and collating the data in many different ways.

Identification of patterns and theme generation

Anfara et al (2002) ask: what exactly does it mean when a researcher writes 'themes emerged'? They point out that there is 'no right way' of analysing data but suggest a method of code mapping with three iterations of analysis. In the first iteration, initial codes appear from the data and allow meaning and insight to be brought to the words and actions of the participants. In other words, examine what is there and label it (Patton in Anfara, K. M. Brown, & Mangione, 2002). Pattern variables can be established during the second iteration. The third and final iteration brings the analysis to a level of theory development and connection with the theoretical framework.

At this point in my work, I anticipate that I will use this kind of approach. I will test this out during the pilot and determine if it works for this study and for the various data collection methods.

Final analysis of the data

If I take the approach just described, once I get to the second and third iterations, I will need to look to my theoretical frameworks for analytical tools. In keeping with the theoretical underpinnings of this study, I will briefly describe those that seem to present themselves at this time.

Wenger's (1998) social learning theory

I will use this theory discussed earlier to analyse and describe the data collected from the case study[6]. Within situated learning, the unit of analysis is quite often the community of practice itself. In this case, the community of practice is the Maths Club. Hence, using the notion of a zoom lens proposed by Lerman (2003) mentioned above, I will use this learning theory as a tool to examine various aspects of the community of practice and seek to understand the potential learning that may be taking place. The major focus will primarily be on identity.

Looking at my research questions, I essentially have 2 major areas to examine whilst simultaneously seeing how these relate to participation, interactions and activities in the clubs.

1. Evolving mathematical identity

A focus on learning as becoming (identity) can be used as the lens to describe the nature of evolving identity formation. In addition it can be used to look at the enablers and constraints of this evolving identity. I anticipate using Wenger's modes of belonging: alignment, imagination and engagement to describe the nature of the evolving identity as well as to understand how this identity relates to the forms of participation, interactions and activities promoted.

For each mode of belonging, Wenger (1998, pp. 184-187) details various **abilities and processes** that form part of the work of identity formation. Those that are relevant to this study have been listed in Table 5 below.

Table 5 - The work of each mode of belonging

Engagement: the ways that we engage with others and how those interactions reflect who we are Doing things together, talking, producing artifacts	**Imagination**: Constructing an image of ourselves, of our communities and of the world in order to orient ourselves, to reflect on our situation, and to explore possibilities	**Alignment**: becoming connected through the coordination of their energies, actions and practices. Making sure that our local activities are sufficiently aligned with other processes that they can be effective beyond our own engagement
Abilities		
• Ability to take part in meaningful activities and interactions • To produce shareable artefacts	• To disengage, to move back and look at engagement through eyes of the outsider • Ability to explore, take risks and create unlikely connections	• Ability to coordinate perspectives and actions to direct energies to a common purpose
Processes (or work of the mode)		
• Mutual engagement in shared activities • Accumulation of a history of shared experiences • Development of interpersonal relationships • Shaping identities in relation to one another	• Recognising experience in others (e.g. being in someone else's shoes) • Developing methods of seeing ourselves in different ways • Sharing stories, explanations an descriptions • Gaining access to other practices through visiting, talking, observing and meeting • Creating models, producing representational artefacts • Generating scenarios, exploring other ways of doing things, other identities	• Investing energy in a directed way • Negotiating perspectives, finding common ground • Convincing, inspiring and uniting • Defining broad visions and aspirations • Devising procedures and control structures that can be used in other communities

[6] *Meaning* is a way of talking about our ability to experience the word as meaningful; *practice* is a way of talking about a shared history of learning; *community* is a way of talking about the social configurations in which our enterprise is defined and *identity* is a way of talking about how learning changes how we are (Wenger, 1998)

Taking these abilities and processes, I have been able to put together a useable model for analysis. These take into account the kinds of mathematical participation, interactions and activities that are promoted in the clubs.

2. Evolving mathematical proficiency

To understand and describe this question, I will use the 5 strands of the mathematical proficiency framework discussed earlier, namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. As I have previously argued, this aspect will focus on the acquisition of competence based on the 'acquisition metaphor' described earlier. However, it is important to remember that this acquisition will be done through participation in this community of practice and will take into account the enablers and constraints afforded by the activities promoted.

To support this view, I turn to Chapter 9 in the '*Adding It Up*' report (Kilpatrick, Swafford, & Findell, 2001). The authors view the teaching and learning of mathematics as the product of interactions among the teacher, the students, and the mathematics. They believe that the knowledge, beliefs, decisions, and actions of teachers affect what is taught and ultimately learned. However the students' expectations, knowledge, interests, and responses also play a crucial role in shaping what is taught and learned. Moreover, they believe that teaching and learning takes place in a context and that 'what goes on in classrooms to promote the development of mathematical proficiency is best understood through an examination of how these elements - teachers, students, content - interact in contexts to produce teaching and learning' (2001, pp. 313-315).

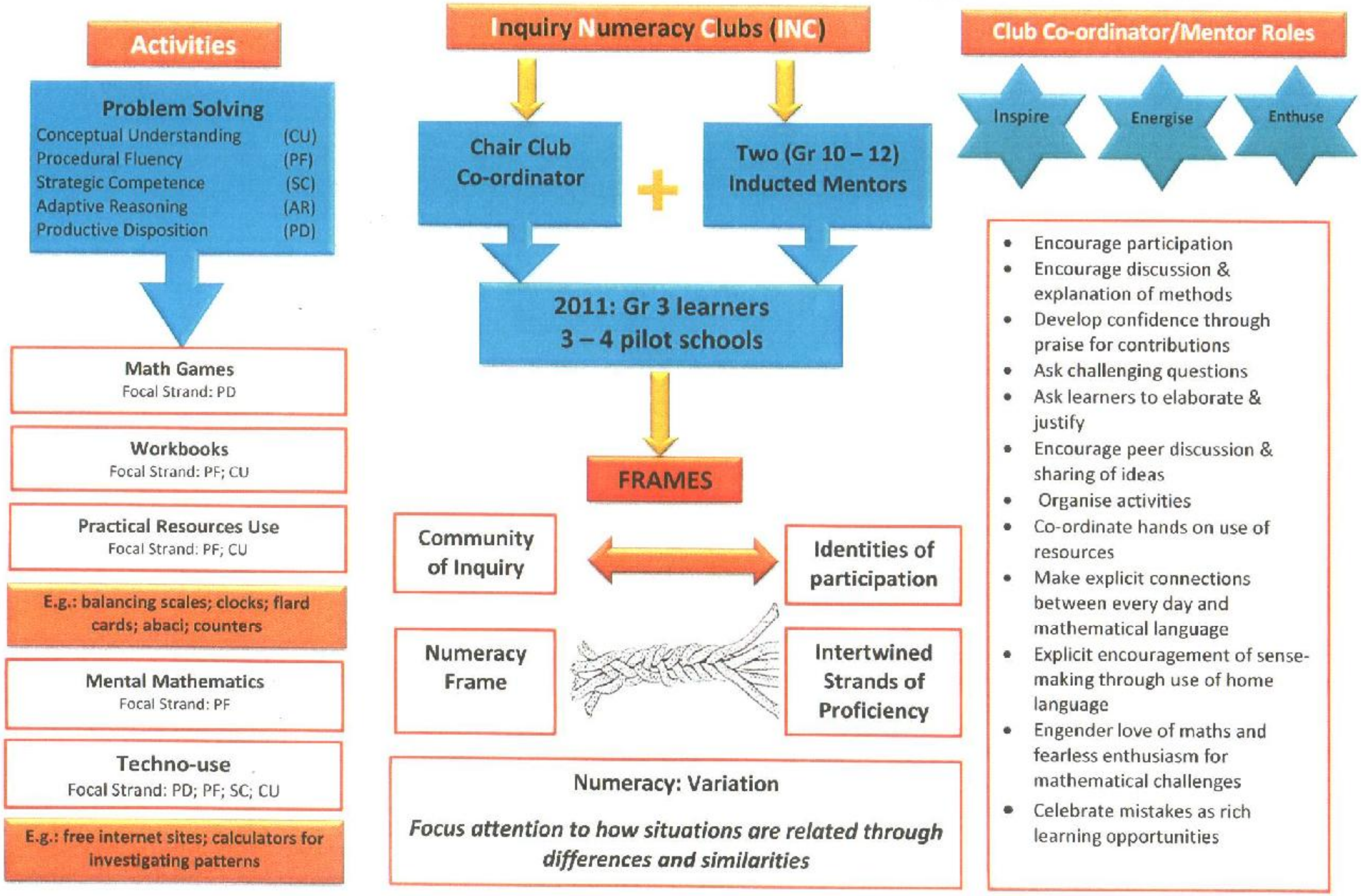
Finally I would like to detail some analysis indictors for each of the 5 strands of mathematical proficiency:

Table 6 - Mathematical proficiency indicators - drawn from (Kilpatrick, Swafford, & Findell, 2001)

Proficiency Strand (description)	**Analysis Indicators** ***Learners show evidence of:***
Conceptual understanding: integrated and functional grasp of mathematical ideas	• Verbalising understandings and connections amongst concepts • Being able to represent mathematical situations in different ways • Linking related concepts
Procedural fluency: knowledge of procedures, when to use them, skill and flexibility in using them accurately and efficiently	• Efficiency and accuracy in performing basic computations in number • Efficiency and accuracy in mental procedures and with pencil/paper • Being able to estimate • Flexibility with using a variety of tools • Ability to select appropriate tool for a given situation / problem
Strategic competence: ability to formulate mathematical problems, represent them and solve them	• Knowing a variety of representation and solution strategies for routine and non routine problems
Adaptive reasoning: ability to think logically about the relationships amongst concepts and situations	• Being able to give informal explanations and justifications of their work • Intuitive, inductive reasoning based on coding, patterns, analogy and metaphor
Productive disposition: ability to see sense in mathematics, seeing it as useful and worthwhile, being an effective learner and doer of mathematics	• Confidence in their ability to do and knowledge of mathematics • Keenness to work at mathematics

9 Appendix A

Maths Clubs Overview



10 Appendix B

Zone Theory

The ZFM, ZPA and ZPD can be seen as structures through which an adult or more knowledgeable other constrains or promotes the learner's thinking and acting and as such the ZFM/ZPA combination *interactively generates* the environment in which that learner develops (Blanton, Westbrook, & Carter, 2005). Blanton et al (2005) draw attention to a view that "the ZFM and ZPA are dynamic, interdependent constructs that are continually being reorganized in the learning process". Galligan also uses the word interdependent to describe the three zones and summarises the theory skilfully. "Valsiner's three zones constitute an interdependent system between the constraints put on the environment of the learner and the actions being promoted for the learner" (2008, p.2).

Blanton et al (2005) do however raise the question as to **how** these zones actually **interact** with the ZPD.

One of the many ways of showing this interaction is to use a model where these zones are represented as a Venn diagram which shows how the zones overlap and relate to each other to create 'learning spaces' (Galbraith & Goos, 2003; Goos, 2004; 2006; Goos, Galbraith, & Renshaw, 2002b). Other representations include set notation which was used by Valsiner himself and circles within circles. For the purposes of this discussion I will work with the zones as a Venn diagram.

Galligan (2008) argues that the use of Zone Theory has been relatively narrow, more often than not used in relation to teacher development or teacher practice. She puts forward an argument that the theory can offer much more, and could be applied in the wider context of development of mathematical understanding in children and adults. This is a positive argument for my selection of this model in this context as well giving an additional benefit in possibly being able to contribute to the body of knowledge on Zone Theory.

Overview of Zone Theory

Whilst the ZPD may be a well known concept to most in educational contexts, the ZFM and ZPA may not be, so I have provided a brief overview of the three zones and their relationships.

Zone of Promoted Action (ZPA)

This is the set of activities, objects, or areas in the environment by which an adult or more knowledgeable other attempts to persuade a learner to act in a certain way. The ZPA describes what the adult is promoting. However the learner is under no obligation to accept what is being promoted as in the case where learners may not wish to actively participate (Blanton, Westbrook, & Carter, 2005).

The ZPA should also be in a learner's ZPD. For example, having poor mathematics skills in a class which assumes basic mathematics skills may result in the learner's inability or reluctance to participate or learn. On the other hand those learners who believe they already have the necessary skills may not participate either (Blanton, Westbrook, & Carter, 2005; Galbraith & Goos, 2003; Galligan, 2008; Goos, Dole, & Makar, 2007a; Goos et al., 2010).

Zone of Free Movement (ZFM)

While acknowledging learners' freedom of action and thought, the ZFM represents cognitive structure and environmental constraints that limit the freedom of these actions and thoughts. This environment is socially constructed by others (teachers, administrators, and the curriculum writers) and the cultural meaning they bring to the environment. The ZFMs themselves can either be set up by these 'others', the students themselves or through joint action.

In essence, the ZFM is a function of what is allowed for the learner by the adult. On the one hand, the way an adult organises the ZFM anticipates the nature of the child's thinking about the concept being taught at the moment and in the future. In this sense, the ZFM ultimately channels the direction of development for the child, providing a framework for cognitive activity. (Blanton, Westbrook, & Carter, 2005; Galbraith & Goos, 2003; Galligan, 2008; Goos, Dole, & Makar, 2007a; Goos et al., 2010)

Zone of Proximal Development (ZPD)

Much has been written about the ZPD across educational disciplines and researchers have described it in different ways. (See for example: Bliss, Askew, & Macrae, 1996; Boaler, 2000b; Lerman, 2003; Solomon, 2007; Yelland & Masters, 2007). Lerman gives a simple overview of the ZPD:

> *Vygotsky's well-known notion offers a description of the whole process of learning, whether it be from a teacher/authority, a peer, or a cultural product such as a text book, as well as a tool for*

studying learning. A wide range of studies have examined aspects of the zone of proximal development (ZPD) (Lerman, 2003)

The differences and nuances of the ZPD are numerous, complex and open to debate. I will look at the aspects of the concept that are relevant to the Maths Club design process.

The key characteristics of the ZPD for me are highlighted in the subsequent paragraphs. Goos (2006) states that Valsiner re-interprets Vygotsky's original ZPD concept and regards it as "a set of possibilities for development that are in the process of becoming actualised as individuals negotiate their relationship with the learning environment and the people in it." Lerman (2003) believes that from a social community of practice perspective, the ZPD may be conceptualised as a "symbolic space involving individuals, their practices and the circumstances of their activity". He believes that learners can be "pulled into their ZPDs by a combination of the activity, the actors, and appropriate communication" (p. 103).

Many authors (see Blanton, Westbrook, & Carter, 2005; Galbraith & Goos, 2003; Galligan, 2008; Goos, Dole, & Makar, 2007a; Goos et al., 2010) emphasise that while the ZPD directs attention to what can potentially be learned, its usefulness is dependent on other enabling conditions. Firstly, teaching activity must be appropriately directed, and secondly the environment must be supportive of the intended learning. This emphasis is very important, especially when one is involved in designing a learning programme.

Another key aspect of the ZPD highlighted is that it recognises the status of learners *existing* understanding, which enables matching of tasks and teaching approaches. It is critical that my design takes this aspect into account, so that we can pitch the learning programme at the correct level for the learners.

Extending Zone Theory into designing professional development programmes

The Zone Theory model is further developed by Goos, Dole and Makar (2007), when they use it as an overlay to a framework used for designing professional development programmes created by Loucks-Horsley, Love, Stiles, Mundry, and Hewson in 2003. This framework incorporates a series of stages, which are illustrated in Figure 4 below.

Loucks-Horsley et al created the framework "to capture the decision making processes that are ideally involved in planning and implementing programs" (Goos, Dole, & Makar, 2007b). They explain that the rectangular boxes represent a generic planning sequence and that inputs into the planning process are shown when it is most important to consider them. Goos et al have re-interpreted these planning inputs in terms of Valsiner's zone theory to incorporate the social setting and actions of participants. Their model is shown in Figure 4 below.

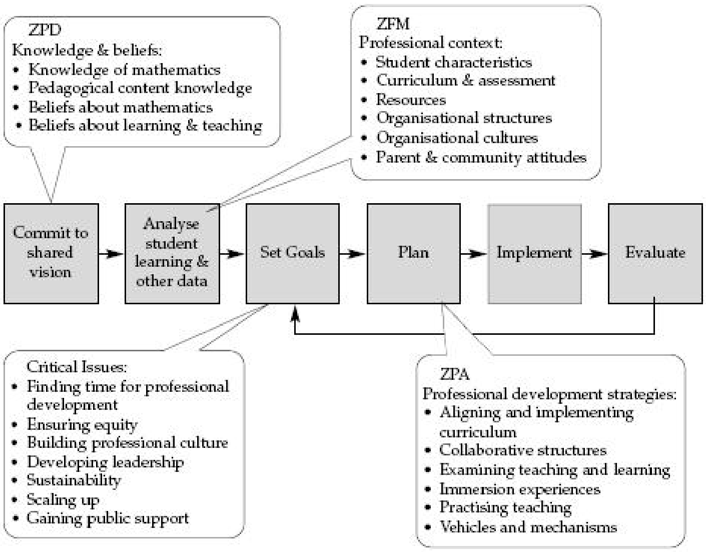


Figure 4 - Goos et al (2007b)

Why have I chosen to use this model?

There are a number of reasons. Firstly, the framework allows me to take into account various aspects of the SA Numeracy Chair and Maths Club programme. These are listed briefly below:

- The needs and the vision of the Numeracy Chair are an integral part of the design and include the core research frames for the project.
- Critical issues that contribute to the programme are noted and made visible.

- It is a process that is easily explainable to others and will contribute to ensuring that the Maths Clubs are sustainable after the research project finishes.
- Contextual and environmental enablers and constraints, availability of resources (both human and physical), organisational structures, club norms and cultures are taken into account at the beginning and during the design process. These make up the aspects that fall under the Zone of Free Movement (ZFM).
- The actions I want to take and the activities that I want to promote in the clubs in order to facilitate the achievement of new skills and knowledge are accounted for and documented in the Zone of Promoted Action (ZPA).
- Using the Zone of Proximal Development allows me to take cognisance of current learner knowledge, skills and strategies as well as think about where we want the learners to get to.

Secondly, the framework provides me with a process for setting goals, planning and evaluating the ongoing learning programme in the Maths Clubs. This gives me ownership of the process as well as a way of ensuring that it fits into the objectives of the Numeracy Chair. Also because the process is iterative and cyclical I can evaluate what is working and what isn't and use this to plan and implement subsequent actions and activities in the clubs. Furthermore, for me, the decision to use this model and ZPD framework aligns with the concept of a 'learning ecology' discussed above which assumes that the Maths Club learning environment is a dynamic and growing entity.

Does the framework help with this research study?

Barbara Jaworski (2007) talks about developmental research as being complex and suggests that there are two ways of engaging with a design process. One way is to study the design process itself and the other is to use the design process as a research process.

If we look at the Goos framework from the perspective of a research process, it offers me a few options.

Goos (2006) highlights that different *configurations* of the ZPD, ZFM and ZPA will arise in every study and it is these differing configurations that can be used to analyse and discuss the data collected during the study. Whilst there are many potential ways to analyse data, this opens a possible way for me to analyse the results of my study as well as bringing congruence to my work. Again, this is something that I can try out during the Maths Club pilot to determine the extent to which it suits my research. Galbraith and Goos point out other ways that Zone Theory can be used:

> *"On the one hand interacting zones may be used as lenses through which to identify and interpret mathematical, technological, and pedagogical attributes of lesson designs and presentations. As such they provide a means to conceptualise and communicate consistencies, inconsistencies, similarities, differences, opportunities, and logical extensions with respect to teaching and learning activities. Among other purposes they provide frameworks for enhancing the analytic quality of lesson observation and planning, and for principled evaluation of classroom segments in teacher education programs"* (Galbraith & Goos, 2003).

They go on to say that Zone Theory is just one way of gaining a perspective on observed and planned events, and on approaches in teaching - learning situations. But they stress that it is also an approach that goes beyond description, by providing us with analytical tools that support the development of theoretically coherent and systematic frameworks to interpret, evaluate, and design actions (Galbraith & Goos, 2003).

Further, the enhanced ZPD interactions framework proposed by Goos (2004) potentially allows me to unpack and focus on the ***interactions and interactions*** between the classroom participants as these are key elements of my research questions.

I believe that I may be able to add to the body of work on Zone Theory by using it to design, implement and research numeracy learning.

11 Appendix C

Literature Review Summary

Table 7 - Literature Review Summary

Area of reading	Reading references (all detailed in References at the end of the proposal)
Social theories: Archer, Bordieu, Giddens, Bernstein, Wertsch and Valsiner, Activity Theory, sociocultural frameworks	(Archer, 2010a; 2010b; Avis, 2007; Bakhurst, 2009; Bernstein, 1999; Elder-Vass, 2010; Giddens & Pierson, 1998; King, 2010; Lynam, Browne, Reimer Kirkham, & Anderson, 2007; Penuel & Wertsch, 1995; Ritzer, 2005)
Zone Theory	(Blanton, Westbrook, & Carter, 2005; J. P. Brown, 2003; Galbraith & Goos, 2003; Galligan, 2008; Goos, 2006)
Learning theories	(Donovan, Bransford, & Pellegrino, 1999; Salomon & Perkins, 1998; Vasta, S. A. Miller, & Ellis, 2004)
Nature of mathematics, numeracy and socio mathematical norms	(Askew, M. Brown, Rhodes, Johnson, & Wiliam, 1997; Boaler, 1997; Cooper & Dunne, 2000; Donovan & Bransford, 2005; Hunter, 2008; Lampert, 1990; Lerman, 2003; Ma, 2010; National Research Council, 2001; Wright, Martland, & Stafford, 2006; Wright, Martland, Stafford, et al., 2006; Yackel & Cobb, 1996)
Identity and mathematical identity	(Askew, 2008; Ben-Yehuda, Lavy, Linchevski, & Sfard, 2005; Black, 2004a; Boaler, 2002; Boaler & Greeno, 2000; Graven, 2004; Nasir, 2002; Penuel & Wertsch, 1995; Sfard & Prusak, 2005a; 2005b; Wenger, 1998; Wenger, 2000)
Community of Inquiry and Practice, Situated Learning	(David & Watson, 2008a; Graven, 2002a; 2004; Jaworski, 2006a; 2006b; Kanes & Lerman, 2008; Wenger, 1998; 2006; Wenger, 2000)
Classroom Discourse and talk	(Black, 2004b; Black & Varley, 2008; Hunter, 2008; Mercer, Wegerif, & Dawes, 1999; Walshaw & Anthony, 2008)
Teacher Learning, Mathematical Knowledge for Teaching, Learning programme design	(Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001; Goos, 2005; 2008; 2009; Goos et al., 2010; Hill, Ball, & Schilling, 2008; Shulman, 1987)
Research Methodologies & mathematical research	(Anfara, K. M. Brown, & Mangione, 2002; Ball & Forzani, 2007; Boaler, Ball, & Even, 2003; Cobb & Yackel, 1996; Cohen, Manion, & Morrision, 2000; David & Lopes, 2002; Lerman, 2000; C. McKnight, Magid, Murphy, & M. McKnight, 2000; Merriam, 1998; Siegel & Borasi, 1994)
South African context primary education	(Fleisch 2008; Taylor et al. 2008; Bloch 2009; Chisholm et al. 2000)}
South African mathematical context	(Chisholm et al., 2000; Graven, 2002a; 2002b; Graven & Buytenhuys, 2011; Graven & Schafer, 2011; Venkat, 2010; Venkat & Graven, 2008)

My role in the clubs (Powerful praxis)

It is possible that in theory, these multiple roles could cause tension in this research and I intend to use my reflective journal to document as much about how the roles impact on each and on the process. However, Graven writes about her own experiences in a similar situation and states that the "theoretical tension was turned into a research advantage" (Graven, 2004). She explains further:

> *"I was expecting some tension to emerge in relation to my role as an 'INSET co-ordinator' and my role as 'researcher.' Instead I discovered a powerful* ***praxis*** *in the duality of being both INSET worker and researcher. My own learning in terms of becoming a more experienced 'INSET provider' was maximised by the ongoing reflection, which was stimulated by the research" (Graven, 2002, pg 2, emphasis added).*

Graven uses the word '*praxis*'. For me this means a way of doing things or a way of translating theoretical ideas into action. This is insightful as it gives one a whole new way of looking at one's roles in a project and how those roles interact with each other.

For Graven, the dual role gave her a number of advantages, which I have listed below.

- It enabled a form of action-reflection practice
- It gave form to her research and the process
- The ongoing reflection was stimulated by her research
- Her own learning was maximised by the ongoing reflection

These insights are encouraging as to how multiple roles can bring powerful praxis to a research project. By being aware of these insights before I start this research, I will hopefully be able to make the most of this opportunity.

Askew et al instrument

I include below, an excerpt from the Askew et al (1997) report to give an overview of the assessments and what aspects of numeracy they cover:

> *"a set of three 'tiered' tests for different age ranges […] were developed and trialled. These were based on a diagnostic test assessing mental facility with numbers, and the ability to apply this, […]. The test had been shown to have high indices of validity and reliability.*
>
> *Aspects of numeracy which were covered in the tests were:*
>
> - *Understanding of the number system, including place value, decimals and fractions*
> - *Methods of computation, including both known number facts and efficient and accurate methods of calculating*
> - *Solving numerical problems, including complex contextualised word problems and abstract mathematical problems concerning the relationships between operations.*
>
> *An aural mode of testing was chosen where the teacher read out questions and pupils wrote down answers in specially designed answer books. This was done mainly to control the time pupils were allowed for each question; a wholly written test would not have enabled efficient methods to be so readily distinguished from more primitive time-consuming strategies based mainly on counting. Reading out questions was also more appropriate for younger children and weaker readers, enabled repetition where necessary, and maintained concentration for all groups" (Askew, M. Brown, Rhodes, Johnson, & Wiliam, 1997, pp 15-16).*

Further examples of these tests can be found in Appendix 1.3 in their report.

Detailed Dates for Data Collection

Timescale	**Collection Methods** (includes ongoing transcription & analysis)
November 2011	• Review Chair Grade 3 baseline assessment data
January 2012 Term starts: 11 January Start Maths Club week commencing 16 Jan	• 1st Learner questionnaires (week commencing 16 Jan) • 1st formal application of Askew instrument: oral baseline assessments (week commencing 23 Jan) • 1st Learner interviews (week commencing 23 Jan) • Ongoing weekly video and voice recordings of sessions (weeks commencing 16, 23, 30 Jan)
February 2012	• Ongoing weekly video and voice recordings of sessions (weeks commencing 6, 13, 20, 27 Feb)
March 2012 Excluding school holidays: 23 March to 10 April	• 1st Structured peer observations • Ongoing weekly video and voice recordings of sessions (weeks commencing 5, 12, 19 Mar)
April 2012	• Ongoing weekly video and voice recordings of sessions (weeks commencing 16, 23, 30 Apr)
May 2012	• 2nd Structured peer observations • Ongoing weekly video and voice recordings of sessions (weeks commencing 7, 14, 21, 28 May)
June 2012 Excluding school holidays: 22 June to 16 July	• Ongoing weekly video and voice recordings of sessions (weeks commencing 4, 11 June) • 1st Focus group interviews (week commencing 18 June)
July 2012	• Ongoing weekly video and voice recordings of sessions (weeks commencing 23, 30 July)
August 2012	• 3rd Structured peer observations • Ongoing weekly video and voice recordings of sessions (weeks commencing 6, 13, 20, 27 Aug)
September 2012 Excluding school holidays: 28 Sept to 8 Oct	• Ongoing weekly video and voice recordings of sessions (weeks commencing 3, 10, 17, 24 Sept)
October 2012	• Ongoing weekly video and voice recordings of sessions (weeks commencing 15, 22 Oct)

Timescale	**Collection Methods** (includes ongoing transcription & analysis)
	• 2nd Focus group interviews (week commencing 29 Oct)
November 2012	• Final structured peer observations • Ongoing weekly video and voice recordings of sessions (weeks commencing 5, 12, 19, 26 Nov) • 2nd formal application of Askew instrument: oral baseline assessments • 2nd Learner interviews • 2nd Learner questionnaires
December 2012 Schools finish 7 Dec	• No data collection

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