Chapter 2: The social turn in mathematics education research

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INTRODUCTION

In this introductory chapter, my task is to give an account of the growth of interest in social elements involved in teaching and learning mathematics over recent years, to account for that growth, and to give an overview of the main areas of research that make up the current intellectual climate in the 'academy' of mathematics education, from the perspective of the social. The first task will involve looking at the relationship between mathematics education and its surrounding disciplines. The second task, accounting for the growth of social theories, will be partly an archaeology and partly a personal view of how and why the concerns of researchers and many teachers have moved from largely cognitive explanatory theories to a greater interest in social theories. The third task, giving an overview of current ideas, will occupy the major part of this paper. In that overview I do not pretend that I have managed to incorporate all the work that is going on currently that positions itself in the 'social'. That would require much more space and time than is available. Instead I will try to identify what I see as the main directions, their common perspectives and their differences, and propose a synthesis.

KNOWLEDGE PRODUCTION IN MATHEMATICS EDUCATION

The field of knowledge production in the community of mathematics education research, as with other curriculum domains, gazes for the most part on the mathematics classroom as its empirical field, although also on other sites of learning and social practices defined as mathematical by observers (Hoyles, Noss & Pozzi, 1999). Researchers in mathematics education draw on a range of disciplines for explanations, analyses and curriculum designs. The process of adopting theoretical frameworks into a field has been defined by Bernstein (1996) as *recontextualization* (Bernstein, 1996), as different theories become adapted and applied, allowing space for the play of ideologies in the process. Prescribing teaching strategies and the ordering of curriculum content on the basis of Piaget's psychological studies is a prime example of recontextualization. Psychologists, sociologists, mathematicians, and others might therefore look at work in mathematics education and at educational studies in general, as derivative. At the same time, however, we should also look on the process as *knowledge production*, in that new formulations and frameworks emerge in dialectical interaction with the empirical field (Brown & Dowling, 1998) and are therefore *produced* in the educational context. The development of radical constructivism as a field in mathematics education research on the basis of Piaget's work is an example of what is more appropriately seen as knowledge production. The adaptation of the ideas of radical constructivism, or any other theoretical framework, into pedagogy, however, is a process of recontextualization where the play of ideologies is often quite overt.

I propose that there are three levels of knowledge. At the first level the surrounding (sometimes called foundation) disciplines of psychology, sociology, philosophy, anthropology, (in our case) mathematics, and perhaps others. At the second level, mathematics education and other curriculum areas of educational research. At the third level curriculum and classroom practice. The process of recontextualization takes place in the movement and adaptation of ideas from one level to the next. One could use this framework to examine changes in practice that are prompted by research findings. In the late 1970's in the UK, a major study of concept hierarchies in school mathematics influenced the content of both textbooks and government curriculum documents. This would be a case of recontextualization from the second to the third level. It is not useful, however, to examine changes in the field of mathematics education as a consequence of changes in, say, mainstream psychology or mathematics. For this reason one should call work at the second level knowledge production, not recontextualization (Bernstein, 1996). Educational research has more of a horizontal relationship to the domains I have described as being at the first level, rather than a hierarchical relationship to them. This chapter will be concerned mainly with knowledge production in the field of mathematics education, not with recontextualization into pedagogy. I will be suggesting that there has been a turn to social theories in the field of mathematics education, and examining the reasons why.

The range of disciplines on which we draw, which should be seen as *resources* for knowledge production, is wide and one might ask why this is so. I do not mean to imply that mathematics education is different to other fields of knowledge production in educational research: all fields have their similarities and overlapping ideas and each field has its unique features.

> Educational research is located in a knowledge-producing *community*... Of course, communities will display a great deal of variation in their cohesiveness, the strength of their 'disciplinary matrix', and the flexibility of the procedures by which they validate knowledge claims. Education as a field

> of research and theorizing is not firmly rooted in any single disciplinary matrix and therefore probably lies at the weak end of the spectrum, although I think this need not in itself be seen as a weakness. (Scott & Usher, 1996, p. 34)

Few areas of educational research are 'home grown' (curriculum studies may be one of the few) and it is typical for all communities in educational thought to draw upon other disciplines. The mathematics education research community seems particularly cohesive and active, as evidenced for instance by the fact that the mathematics education group is now the largest division in the American Educational Research Association. The procedures for validating knowledge claims that have emerged in recent decades, including peer review of journal articles, conference papers, research grant applications, and doctoral thesis examinations, are becoming more flexible and the criteria more varied. The numbers of journals and conferences are increasing, and one can expect that the development of on-line journals, and perhaps video-conferencing too, will accelerate the increasing flexibility. A framework for a systematic analysis of the productions of the mathematics education community has been sketched as the first stage in a program to map the elaboration of pedagogic modes over time (Lerman & Tsatsaroni, 1998).

The mathematics education research community appears to be particularly open to drawing upon other disciplines, for at least four reasons. First, mathematics as a body of knowledge and as a set of social practices has been and remains of particular interest to other disciplines such as psychology, sociology and anthropology as it presents particularly interesting challenges to their work. It is not surprising that one of the major challenges for Piaget was to account for the development of logical reasoning, nor that Piaget's account of knowledge schemata used group theory as its fundamental structure. Similarly, it is not surprising that Scribner, Cole, Lave, Saxe, Pinxten and others found the study of mathematical practices of great interest in their anthropological and cross-cultural studies. Second, mathematics has stood as exemplar of truth and rationality since ancient times, giving it a unique status in most world cultures and in intellectual communities. That status may account for mathematics being seen as a marker of general intellectual capacity rather than simply aptitude at mathematics. Its symbolic power certainly lays mathematics open to criticisms of its gendered and Eurocentric character, creating through its discursive practices the reasoning logical norm (Walkerdine, 1988). Third, mathematics has played a large part in diverse cultural practices (Joseph, 1991) including religious life, music, pattern, design and decoration. It appears all around when one chooses to apply a mathematical gaze (Lerman, 1998a). Finally, there is the apparent power of mathematics such that its use can enable the building of skyscrapers, bridges, space exploration, economic theories, ‘smart’ bombs and ... I should stop as the list descends into ignominy.

Until the last 15 years mathematics education tended to draw on mathematics itself, or psychology, as disciplines for the production of knowledge in the field (Kilpatrick, 1992). Analyses of mathematical concepts provided a framework for curriculum design and enabled the study of the development of children's understanding as the building of higher order concepts from their analysis into more basic building blocks. Behaviorism supplied the psychological rationale both for the building blocks metaphor for the acquisition of mathematical knowledge and for the pedagogical strategies of drill and practice, and positive and negative reinforcement.

Piagetian psychology called for historical analyses of mathematical (and other) concepts, based on the assumption that the individual's development replays that of the species (ontogeny replicates phylogeny). It was argued that identifying historical and epistemological obstacles would reveal pedagogical obstacles (Piaget & Garcia, 1989; for a critique see Radford, 1997; Lerman, 1999; Rogers, in press). This again emphasized the importance of mathematical concepts for education. In terms of psychology, the influences of Piaget and the neo-Piagetian radical constructivists are too well known to require documentation here, and I would refer in particular to the detailed studies of children's thinking (e.g. Steffe, von Glasersfeld, Richards, & Cobb 1983; Sowder, J., Armstrong, Lamon, Simon, Sowder, L., & Thompson, 1998). Both the disciplines of mathematics and psychology have high status in universities, and locating mathematics education within either group is seen as vital in some countries in terms of its status and therefore funding and respectability. Psychology has well established research methodologies and procedures upon which mathematics education has fruitfully drawn. Evidence can be seen, for instance, in the proceedings of the International Group for the Psychology of Mathematics Education (PME) over the past 22 years and in the Journal for Research in Mathematics Education (JRME).

Interest in the implications of the philosophy of mathematics, for mathematics education research was given impetus by Lakatos' *Proofs and Refutations* (1976) partly, I suspect, because of the style of the book, which is a classroom conversation between teacher and students. More important, though, is the humanistic image of mathematics it presents, as a quasi-empiricist enterprise of the community of mathematicians over time rather than a monotonically increasing body of certain knowledge. The book by Davis & Hersh (1981) which was inspired by Lakatos has become a classic in the community but others (Kitcher, 1983; Tymoczko, 1986; Restivo, van Bendegem, & Fischer, 1993) have become equally influential. A number of researchers (Dawson, 1969; Rogers, 1978; Confrey, 1981; Nickson, 1981; Lerman, 1983; Ernest, 1985, 1991) have studied aspects of teaching and learning mathematics from the humanistic, quasi-empirical point of view. That mathematical certainty has been questioned in the absolutism/fallibilism dichotomy is not due directly to Lakatos since he never subscribed to that view. With Popper, Lakatos considered knowledge to be advancing towards greater verisimilitude, but identifying the process of knowledge growth as taking place through refutation, not indubitable deduction, raised the theoretical possibility that all knowledge might be challenged by a future counter-example. In mathematics education the absolutist/fallibilist dichotomy has been used as a rationale for teaching through problem solving and as a challenge to the traditional mathematical pedagogy of transmission of facts. Fallibilism's potential challenge to mathematical certainty has led to mathematical activity being identified by its heuristics, but to a much greater extent in the mathematics education community than amongst mathematicians (Hanna, 1996; Burton, 1999a). This is another illustration of the recontextualizing process from the field of production of mathematics education knowledge, driven perhaps by democratic tendencies for pedagogy amongst some schoolteachers.

Whilst there is a substantial body of literature in social studies of scientific knowledge, there has been much less written about mathematical knowledge, although Bloor (e.g. 1976) is an early exception and Rotman's (1988) and Restivo's (1992) work more recent. Science education research draws heavily on social studies of scientific knowledge: in mathematics education that resource is still in an early stage.

THEORIES OF THE 'SOCIAL'

Studies in epistemology, ontology, knowledge, and knowledge acquisition tend to focus on how the *individual* acquires knowledge and on the status of that knowledge in relation to reality. Theoretical frameworks for interpreting the *social* origins of knowledge and consciousness began to appear in the mathematics education literature towards the end of the 1980s. Shifts in perspectives or the development of new paradigms in academic communities are the result of a concatenation of factors within and around the community. In the title I have called these developments the *social turn* in mathematics education research. This is not to imply that other theories, mathematical, Piagetian, radical constructivist or philosophical have ignored social factors (Steffe & Thompson, in press; Lerman, in press a). Indeed I have suggested above that the philosophical orientation was coincident with a humanistic, democratic concern by teachers and researchers at that time. Elsewhere (1998c, p. 335) I have discussed Piaget's and von Glasersfeld's emphasis on social interactions as providing a major source of disequilibrium. The *social turn* is intended to signal something different, namely the emergence into the mathematics education research community of theories that see meaning, thinking, and reasoning as products of social activity. This goes beyond the idea that social interactions provide a spark that generates or stimulates an individual's internal meaning-making activity. A major challenge for theories from the social turn is to account for individual cognition and difference, and to incorporate the substantial body of research on mathematical cognition, as products of social activity.

In making the *social turn* the focus of this chapter I have created my object of study. It becomes tempting, then, to pin down the emergence of that object in time, although in a ‘playful’ sense. The year 1988 saw the appearance of several texts that have become significant in the social turn in mathematics education research. Jean Lave's book *Cognition in Practice* (1988) challenged cognitivism and transfer theory in mathematics learning. In that book she described studies of the ‘mathematical’ practices of grocery shoppers and dieters which raised fundamental questions about mathematical practices in out of school practices being seen as merely the application of school techniques. The strategies and decision-making procedures that people used in those situations had to be seen as situated within, and as products of, those social situations. Further, the process of learning the strategies and decision-making procedures in the community of dieters, for example, should be seen as part of who one is ‘becoming’ in that practice. Terezinha Nunes (Carraher, 1988) gave a plenary address at PME in Hungary, reporting on the work of her group, in which she identified differences between street mathematics and school mathematics. For example, she demonstrated that the former is oral, the latter written, and that street mathematics "is a tool for solving problems in meaningful situations" (p. 18). That students who traditionally fail in school mathematics were seen to be successful in street situations made the challenge to knowledge as decontextualized schemata more powerful. Valerie Walkerdine's *Mastery of Reason* (1988) located meanings in practices, not as independent of them, and demonstrated that the notion of a 'child' is a product of a discursive practice, that is produced in language and particular social practices. Her Foucauldian analysis of classroom mathematics placed issues of power and the social construction of identity and meanings on the agenda. Alan Bishop's *Mathematical Enculturation* (1988a) gave a cross-cultural view of mathematical practices and attempted to give some universal parameters for their analysis. In the same year Bishop was editor of a special issue of *Educational Studies in Mathematics* on cultural aspects of mathematics education. These writers, and others, had published some of their work before 1988 but the coincidence of these major publications leads me to emphasize that year. It is clear that the community had to be receptive to these ideas for them to gain purchase. In that same year, one day of the Sixth International Congress on Mathematical Education in Hungary, called 'Day 5', was devoted to Mathematics, Education and Society, the result of the efforts of Alan Bishop and colleagues to bring social and cultural issues to the attention of the international mathematics education community. In 1986 a research group had been set up in the UK by Marilyn Nickson and myself called the group for "Research into Social Perspectives of Mathematics Education" (Nickson & Lerman, 1992). These are just two indicators of the receptivity of the mainstream community. It has to be said, though, that the receptivity of the mathematics education community to social theories was due more to political concerns that inequalities in society were reinforced and reproduced by differential success in school mathematics, than social theories of learning. Ethnomathematics, which was introduced as a new direction by Ubiritan D'Ambrosio at the Fifth International Congress on Mathematical Education in Adelaide in 1984 (D'Ambrosio, 1984), was a key element in the papers presented on Day 5 four years later, and can also be said to have played a large part in creating an environment that was receptive to the social turn.

The other key element in current socio-cultural theories in mathematics education is the work of Vygotsky and his colleagues, but it is a little harder to trace the beginnings of Vygotskian influences in mathematics education. Forman (in press) reminds us that Vygotsky's work only became available to the world community with desalinization in the Soviet Union at the end of the 1950s and only slowly and gradually were translations made available. The impact of his revolutionary ideas took time to emerge, Bruner and Wertsch being particularly important figures in that process (see Bruner, 1986; Wertsch, 1981). The significant differences between Vygotsky's theories, and those of Piaget which were, and still are, dominant, took even longer to reach recognition. People working in the field of education for children and adults with special needs (e.g. Donaldson, 1978; Feuerstein, 1980), in studies of self-regulation, and in language development took to Vygotsky's theories at an early stage. Cole, Engeström and others, including Lave, influenced by activity theory (Cole, 1996; Cole, Engeström, Vasquez, 1997), drew partly on studies of mathematical practices. However, the significance of Vygotsky's work only came to be appreciated by the mainstream mathematics education community much more recently.

The evidence I have found of Vygotsky's work becoming known within mathematics education suggests, again, that the late nineteen-eighties may be seen as something of a marker. From a search without the aid of electronic means, it appears that the first mention of Vygotsky in references:

- in PME proceedings was Crawford (1988);
- in *Educational Studies in Mathematics* in a review of Wertsch (1981) by Crawford (1985), but the first mention in an article was Bishop (1988b);

- in the journal *For the Learning of Mathematics* was Cobb (1989);
- in the *Journal for Research in Mathematics Education* was English (1993);
- in the *Journal of Mathematical Behavior* was Schmittau (1993).

The social turn in mathematics education has developed from, I suggest, three main disciplines or resources: anthropology (from e.g. Lave); sociology (from e.g. Walkerdine); and cultural psychology (from e.g. Nunes; Crawford). Each contains a number of streams, of course, and each has a number of influences. I have proposed (Lerman, 1998c) that there are some common themes and I will try to indicate later how these can be brought together into a fruitful and coherent research direction by a consideration of the unit of analysis for research in mathematics education. For now it will suffice to say that fundamental to the social turn is the need to consider the person-acting-in-social-practice, not person or their knowing on their own. I will frame this discussion by looking at aspects of situated theory, with critiques opening spaces for elaborations from sociology and from cultural or discursive psychology.

SITUATED KNOWING

Situated theories have generated great interest and received much critical attention in recent years (e.g. Kirshner & Whitson, 1997; Watson, 1998; Andersen, Reder & Simon, 1997; Greeno, 1997). Lave and Wenger (also Lave, 1988; Lave & Wenger, 1991; Lave, 1997; Wenger, 1998) have given radically different meanings to knowledge, learning, transfer and identity. Lave's studies of the acquisition of mathematical competence within tailoring apprenticeships in West Africa led her to argue that knowledge is located in particular forms of situated experience, not simply in mental contents. Knowledge has to be understood relationally, between people and settings: it is about competence in life settings. One of the consequences of this argument is that the notion of transfer of knowledge, present as decontextualized mental objects in the minds of individuals, from one situation to another, becomes perhaps untenable but at the very least requires reformulation. That argument seems to create special problems for mathematics education. Perceptions of mathematics as a discipline are predicated on increasing abstraction and generality across applications, and mathematical modeling is precisely the application of apparently decontextualized knowledge to almost any situation. Widely held perceptions of child development and of the acquisition of mathematical knowledge also are predicated on a move from the concrete to the abstract, whereby decontextualized mental schemata are constructed and can be used formally, at the appropriate stage of intellectual development. But these are not serious challenges to situated theory. The various sub-fields of the professional practice of mathematicians can be seen as particular social practices. To apply a mathematical gaze onto a situation and to identify and extract factors and features to mathematics is the practice of mathematical modeling. It has its masters and images of mastery, its apprenticeship procedures, its language, and its goals, just like any other social practice. Learning to 'transfer' mathematics across practices is the practice. The belief that the mathematics found in practices by the gaze of the mathematical modeller is an ontologically real feature of those practices is perhaps an extra block to seeing modeling as a social practice (see Restivo, 1992, for examples of sociological, practice-based accounts of the development of abstract mathematical structures).

The practices of the school mathematics classroom are certainly very different to the practices of mathematicians, or those who use ‘mathematics’ in the workplace, at least because school mathematics is not the *chosen* practice of students in classrooms. We can say, however, that learning to read mathematical tasks in classroom problems, which gives the appearance of decontextualized thinking, is again a particular feature of the practice of school mathematics for the ‘successful’ students (Dowling, 1998). It is effected by an apprenticeship into the practices of classroom mathematics that carry cultural capital (Bourdieu, 1979). The agents of the apprenticeship are the teacher and the texts, but also the acceptance or acquiescence of those students who become apprenticed.

In the next three sections I will examine aspects of situated theory: the need for a consideration of how subjectivities are produced in practices, as argued by Walkerdine and others; the particular nature of the practices of the mathematics classroom and the implications it has for notions of apprenticeship; and the problem of a suitable mechanism in Lave’s theory of learning (1996, p. 156). In the concluding section I will discuss the unit of analysis for the study of individuals in social practices, in an attempt to bring the critiques together into a synthesis of the social turn.

Subjectivity - Regulation in Practices.

> A community of practice is an intrinsic condition for the existence of knowledge, not least because it provides the interpretive support necessary for making sense of its heritage. Thus, participation in the cultural practice in

> which any knowledge exists is an epistemological principle of learning. The social structure of this practice, its power relations, and its conditions for legitimacy define possibilities for learning (i.e., for legitimate peripheral participation). (Lave & Wenger, 1991, p. 98)

Walkerdine (1997) suggests that what is missing in Lave's analysis of the subject in practices is subjectivity, the regulation of individuals within practices. In the move away from the notion of an individual transferring decontextualized knowledge from one practice to another, to the notion of knowledge and identity being situated in specific practices, Lave's work might seem to suggest that all individuals are subjected to those practices in the same way. There appears to be a goal for the learning which is characteristic of the practice, and apprenticeship into it is monolithic in its application. However, Walkerdine shows how the notion of 'child' is produced in the practices of educational psychology (1988; see also Burman, 1994), differentially positioning those who conform – white boisterous males, and those who don't – non-white people, girls, quiet boys and so on. Significations matter, they are not neutral meanings: situating meanings in practices must also take into account how those significations matter differently to different people. Practices should be seen, therefore, as discursive formations within which what counts as valid knowledge is produced and within which what constitutes successful participation is also produced. Non-conformity is consequently not just a feature of the way that an individual might react as a consequence of her or his goals in a practice or previous network of experiences. The practice itself produces the insiders and outsiders. Analysis of apprenticeship in particular workplace settings might appear not to reveal differing subjectivities produced in the practice. Women and people of ethnicities other than the majority might not choose to become tailors, and those becoming excluded may be forced to leave or may choose to see themselves as not suited to that job/identity. In fact in recent decades the entrance of women and people of color into high-powered workplace situations which were all white male domains has highlighted the subtle and not so subtle ways in which those situations have excluded others by virtue of the manner in which those workplaces and their practices are constituted.

The classroom, being a site of a complex of practices, requires a careful consideration of subjectivities. One kind of analysis has been offered by Evans (1993, in press) in which he argues that Foucault's work on the architecture of knowledge captures the way in which individuals are constructed by and within those practices. Evans suggests that discursive practices are not clearly bounded, they are continually changing, and one moves from one discursive practice to another through chains of signification. In a series of interviews, he asked mathematical questions set in different social contexts and identified the discursive practice that was called up by the question in its context, for a particular person. He criticizes the simplistic notion that giving real world contexts for mathematical concepts provides 'meaning' for students, a 'meaning' that supposedly exists in some absolute sense and is illustrated by or modeled in that real world context. He identifies school mathematics practice as one of a range of practices that might be called up for an individual. When that happens, if the interviewee was successful at school she or he might focus on the mathematical calculation required and answer correctly; more frequently the identity called up would be one of low confidence and lack of success. In another analysis of the production of subjectivities through the discursive practices of the mathematics classroom, Morgan (1998) analyzed the written productions of school students in their mathematics lessons according to the ways in which the teachers framed the task through their use of official discourse (what is expected by examiners), practical discourse (whether it can be understood by non-mathematicians) or professional discourse (what mathematicians might expect).

Much sociology of education presents macro-theories about social movements and the reproduction of disadvantage in schools. Walkerdine's and Evans' accounts draw on sociological theories of post-structuralism which describe the emergence of discursive practices, the production and maintenance of elites in and through those practices, and the techniques and technologies whereby power and knowledge are produced. Their work enables the use of Foucault's theory to look into specific practices at the micro-level of the mathematics classroom. Dowling's (1998) sociology of mathematics education owes its origins to Bernstein (e.g. 1996), who offers a language for the description of the pedagogic mechanism through which education reproduces social inequality as positionings in the classroom. Dowling carried out a study of a series of four parallel school mathematics texts that are written according to the authors' assumptions of the potentialities of different abilities. He demonstrates how the texts are in fact productive of those differing potentialities, and how the assumptions of ability coincide with the different modes of thinking produced in the stratification of society according to social class, identified through different forms of language. Cooper and Dunne (1998) also use Bernstein's theory to demonstrate how questions set in everyday contexts in national mathematics tests in the UK disadvantage working class children. In another use of sociological theories in mathematics education Brown (1997) draws on the work of Habermas to develop a theory in which individual learners reconcile their constructions with the framing of the socially determined code of the mathematics teacher.

In general, sociology provides resources for identifying the macro-social issues that bear on schooling but not always for making links between them and the micro-social issues that concern us in relation to the classroom. I have argued elsewhere (Lerman 1998b) that studying individual children or groups of children can be seen as moments in the zoom of a lens in which the other, temporarily out of focus, images must also be part of the analysis. Specifically, Walkerdine brings subjectivity into the study of subject-in-social-practice and I go along with her (and Agre, 1997) in seeing it as a necessary element. Individual trajectories in the development of identities in social practices arise as a consequence of our identities in the overlapping practices in which each of us functions but also emerge from the different positions in which practices constitute the participants. We can capture the regulation of discursive practices by talking of the practice-in-person as the unit of identity, as well as the person-in-practice. I will return to the question of the unit of analysis below.

The Practices of the Mathematics Classroom.

> A community of practice is a set of relations among persons, activity, and the world, over time and in relation with other tangential and overlapping communities of practices (Lave & Wenger, 1991, p. 98).

The classroom is clearly a site of many over-lapping practices. Whereas the mathematics teacher's goal may be to initiate learners into (what she or he interprets as) mathematical ways of thinking and acting, learners' goals are likely to be quite different. We must therefore ask how we can extend the notion of apprenticeship to incorporate the mismatch of goals. If we are to extend the valuable insights of the notion of communities of practice into the field of knowledge production in mathematics education, the nature of those goals and of the classroom practices must be analyzed, and I will turn to this task here. First, the way in which what constitutes ‘school mathematics’ is produced requires some examination, in terms of the play of values and ideology. Second, the range of goals of the participants, both those present (teacher and students) and those physically absent (state/community/media/school) must be elaborated. At the very least we must ask who or where are the masters in these multiple practices?

School mathematics. Bernstein's work over a number of decades has focused on how power and control are manifested in pedagogic relations. In particular he has looked at how the boundaries between discourses, such as those of the secondary school curriculum, are defined, what he calls the classification rules, and how control is effected within each discourse, the framing rules. As a principle, pedagogic discourse is the process of moving a practice from its original site, where it is effective in one sense, to the pedagogic site where it is used for other reasons; this is the principle of recontextualization. In relation to work practices he offers the example of carpentry which was transformed into woodwork (in UK schools), and now forms an element of design and technology. School woodwork is not carpentry as it is inevitably separated from all the social elements, needs, goals, and so on, which are part of the work practice of carpentry and cannot be part of the school practice of woodwork. Similarly, school physics is not physics, and school mathematics is not mathematics. Bernstein argues that recontextualization or transformation opens a space in which ideology always plays. In the transformation to pedagogy, values are always inherent, in selection, ordering and pacing.

In relation to mathematics, those values may include preparation for specific workplaces, but this is likely to be at the later stages of school for a small minority of students, at least in the UK. Other European countries have very different attitudes to vocational education. The school mathematics curriculum may include specific mathematics for everyday life: shopping, paying taxes, investing savings, bank accounts, and pensions, but again these issues will become meaningful to students at the later stages of schooling. The content of a mathematics curriculum which is to provide the skills necessary for either or both of these contexts would be very limited. In any case, the problems of transfer and contextualisation of knowledge suggest that the teaching of these skills in the classroom for use elsewhere would be highly problematic. For the most part, curriculum is driven by a view of education which may be: an authoritarian view (Ball, 1993), the inculcation of an agreed selection of culturally valued knowledge and a set of moral values and ways of behaving; a neo-liberal view (Apple, 1998), producing citizens prepared for useful, wealth-producing lives in a democratic society; a more old-liberal agenda (Hirst, 1974) of enabling children to become educated people able to fulfil their lives to the best of their abilities; or a more radical agenda (Freire, 1985) of preparing people to critique and change the society in which they engage. It may also be driven merely by inertia. Schools as institutions are there, they occupy children all day whilst some parents and guardians work, and the mathematics curriculum, in terms of topic content, remains very similar to that of 50 years ago. Whatever the ideological/value-laden intentions for teaching on the part of the school/community/state, the teacher has her or his goals too, which may or may not align with the institutional intentions. Initiatives such as the NCTM standards in the US or the National Numeracy Strategy in the UK (to take two examples with very different orientations) provide yet other sets of values that regulate the teacher's behavior.

The mathematical practices within a class or school, the way in which they are classified and framed, the state/community/school values which are represented and reproduced, and the teacher's own goals and motives, form the complex background to be taken into account by the research community (see Boaler,1997, for an exemplary study of different school practices). According to Lave, mathematics itself should be seen not as an abstract mathematical task but as something deeply bound up in socially organized activities and systems of meaning within a community. Nor, for that matter, should it be seen as a single practice. Burton (1999a) has found that mathematicians identify themselves by their sub-field, as statistician, applied mathematician, mathematical modeller, or topologist. In relation to school mathematics one must be aware of the particular nature of the identities produced. Boaler (1997) has shown how different approaches to school mathematics produce different identities as school mathematicians. She suggests also that the identities produced in one of the two schools in her study, Phoenix Park school, which used a mathematics curriculum built around problem solving, overlap with students' mathematical practices outside of school, but there is less evidence for this as Boaler relies on students' accounts, given in school, of such overlap. Boaler uses both Bernstein's analysis in terms of classification and framing and Lave's communities of practice as resources to explain her findings. Recently Boaler (1999) has talked of the particular practices of the two schools as offering constraints and affordances (Greeno & MMAP, 1998) as a way of interpreting the students' behaviors which resulted in them working to succeed, in the distinct terms of each school.

In summary, as researchers we need to examine the background that frames the mathematical practices in the classroom, irrespective of their allegiances (reform, authoritarian or other), and draw on the resources offered by Lave, Walkerdine, Greeno and others to study the ways that school mathematical identities are produced. In the next part I examine accounts which incorporate individual trajectories through those social practices (Confrey, 1995).

<u>Participants' goals.</u> Lemke and others point to the paths of particular people's learning by referring to individual trajectories (Lemke, 1997). People come to participate in social practices from an individual set of socio-cultural experiences. Individuality, in this sense, "is the uniqueness of each person's collection of multiple subjectivities, through the many overlapping and separate identities of gender, ethnicity, class, size, age, etc., to say nothing of the 'unknowable' elements of the unconscious" (Lerman, 1998b, p. 77). Lemke (1997) refers to the ecosocial system in which people function, and Engeström and Cole (1997) refer to the under-researched resistance of some actors in activities. More important to students than learning what the teacher has to offer are aspects of their peer interactions such as gender roles, ethnic stereotypes, body shape and size, abilities valued by peers, relationship to school life, and others (McLaughlin, 1994). The ways in which individuals want to see themselves developing, perhaps as the classroom fool, perhaps as attractive to someone else in the classroom, perhaps as gaining praise and attention from the teacher or indirectly

from their parents, leads to particular goals in the classroom and therefore particular ways of behaving and to different things being learned, certainly different from what the teacher may wish for the learners (Boaler, 2000). Winbourne (1999; see also Winbourne & Watson, 1998) has given an account of individual children's mathematical (and other) activities that set the children in the context of the multiple social and cultural practices in which they are positioned and that influence who they are at different times in the mathematics (and every other) classroom. Santos and Matos (1998) analyze the knowledge development of students in terms that take account of their social relations. Brodie (1995) and Lerman (in press b) offer similar analyses from different perspectives.

All these accounts give social origins to the individual trajectories which clearly manifest in the classroom (Wenger, 1998). The origins of individual meanings being located in socio-cultural tools roots individuality or voice in its proper framework. It is not the individualism of private worldviews, which has dominated the debate around subjectivity and voice in recent decades but power/knowledge as constituted in discourses. Discourses which dominate in the classroom, and everywhere else for that matter, distribute powerlessness and powerfulness through positioning subjects (Evans, 1993). Walkerdine's (1989, p. 143) report of a classroom incident in which the emergence of a sexist discourse bestows power on five year-old boys, over their experienced teacher, dramatically illustrates the significance of a focus on discourse, not on individuals. In some research on children's interpretations of bigger and smaller, Redmond (1992) found some similar evidence of meanings being located in practices.

> These two were happy to compare two objects put in front of them and tell me why they had chosen the one they had. However when I allocated the multilinks to them (the girl had 8 the boy had 5) to make a tower . . . and I asked them who had the taller one, the girl answered correctly but the boy insisted that he did. Up to this point the boy had been putting the objects together and comparing them. He would not do so on this occasion and when I asked him how we could find out whose tower was the taller he became very angry. I asked him why he thought that his tower was taller and he just replied "Because IT IS. " He would go no further than this and seemed to be almost on the verge of tears. (p. 24)

Many teachers struggle to find ways to enable individual expression in the classroom, including expressing mathematical ideas, confronting the paradox of teachers giving emancipation to students from their authoritative position. But this can fruitfully be seen as a dialectic, whereby all participants in an activity manifest powerfulness and powerlessness at different times, including the teacher. When those articulations are given expression, and not denied as in some interpretations of critical pedagogy (Lerman, 1998d), shifts in relations between participants, and crucially between participants and learning, can occur (Ellsworth, 1989; Walcott, 1994).

Learning is predicated on one person learning from another, more knowledgeable, or desired, person, in Lave’s terms the master. As Lave has pointed out, there are many overlapping practices in any one practice. This is particularly the case in the classroom since not many students’ goals are aligned with the teacher’s and very few wish to become teachers of mathematics.

Models of mastery. Lave and Wenger's (1991) notion of mastery was not focused on school classrooms (see also Wenger, 1998) but clearly offers valuable insights which require development if we wish to use them in the formation of appropriate theoretical frameworks. Learning seen as increasing participation in practices, the gradual attainment of mastery, is a rich description of identity development which has been shown to be appropriate to at least some aspects of the classroom (Lave, 1996).

Classroom practices include those overlapping identities produced in relation to the mathematics, such as abilities, as in Walkerdine's and Dowling's analyses, and purposes for mathematics. For instance, purposes may include minimum certification for continuing study; a key to careers and further education and training courses; or markers of recognition of general intellectual potential. Classroom mathematics practices also produce the more specific identities as, for example, good at number but not algebra, as competitive or collaborative in performance, etc. The complex of classroom practices also covers those outside of the intention of the teacher, as discussed above, particularly in relation to peers, and most importantly the differential regulation of different students within those practices. The teacher may perform the role of 'master' for some students in relation to some aspects of what we might call the mathematical identities produced, most often specifically the mastery leading to further study of mathematics, although we are referring here to mastery in terms of school mathematics. But the teacher will not stand as the master for most of the students for most of the classroom social practices that are important for them. How, then, might we extend Lave and Wenger's notion?

I suggest that it may be fruitful to refer to *multiple models of mastery* offered in the complex of classroom practices. Expertise/mastery may be represented in a person or not, hence *models*, and those masters may be present in the classroom or not. In terms of what can be called role models, other students might perform many of the roles that students may desire to emulate. The teacher's personal style is often reported as having been a significant factor in people's identification with, or rejection of, aspects of schooling including mathematics. In relation to people absent from the classroom, parents' stories of, for instance, their ability or lack in relation to mathematics can function as model for a student and a sibling or valued other similarly. So too images of who students want to become can act as models, perhaps including media personalities. This identifies the need for more complex studies of individual trajectories in the classroom, perhaps through narrative accounts (Winbourne, 1999; Santos & Matos, 1998; Burton, 1999b), examining who are the models and what are the practices that are important to individual students.

<u>A mechanism for learning.</u> Lave argues that learning may be represented as increasing participation in communities of practice (Lave, 1996). She writes that she finds the following three features of a theory of learning to be "a liberating analytical tool" (1996, p. 156) for discussing learning as social practice:

1. *Telos*: that is, a direction of movement or change of learning (*not* the same as goal directed activity),
2. *Subject-world relation*: a general specification of relations between subjects and the social world (not necessarily to be construed as learners and things to-be-learned),

3. *Learning mechanisms*: ways by which learning comes about (p. 156)

She argues that the *telos* of her two case studies, the tailors' apprentices and legal learning in Egypt in the 19th century, is to become masters of tailoring or law and to become respected participants of the everyday life of their communities. The discussions above, concerning recontextualization, the multiple practices at play in the mathematics classroom, regulation, and the need for an analysis that offers multiple models of mastery, suggests that we might need to refer to *teloi*, and the plural *subjects-worlds relations* as well as *regulative processes*. Here I wish to address Lave's third feature, that of learning mechanisms. Whatever mechanism is used, whether it is used as an explanatory framework or as an ontological statement, it must take account of the differences between workplace apprenticeships and the classroom, as well as being able to account for both. In the classroom, the teacher intends to teach: this is her or his function, however it is interpreted and realized. The difference to the situation of the master tailor is quite dramatic.

In many places in her writing Lave (e.g. 1997) proposes that one should focus on learning and make a separation of it from teaching. Lave is referring here to school teaching as the culture of acquisition, offering compartmentalized knowledge, and learning at a distance, drawing, that is, on the notion of transfer (p. 27/28). I suspect many teachers and certainly most, if not all, in the mathematics education research community would subscribe to a move away from that view of teaching. In looking at a (socio-cultural) mechanism for learning, however, the teacher has to be placed firmly into the picture. Here I will turn to Vygotsky's work, as his mechanism for learning captures at least some of the features called for by Lave and others.

Vygotsky provided a mechanism for learning with four key elements: the priority of the intersubjective; internalization; mediation; and the zone of proximal development.

- “Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, *between* people (*interpsychological*), and then *inside* (*intrapsychological*) . . . All the higher functions originate as actual relations between human individuals.” (Vygotsky, 1978, p. 57)
- “The process of internalization is not the transferal of an external to a pre-existing, internal "plane of consciousness"; it is the process in which this plane is formed.” (Leont'ev, 1981, p. 57)
- “Human action typically employs "mediational means" such as tools and language, and that these mediational means shape the actions in essential ways . . . the relationship between action and mediational means is so fundamental that it is more appropriate, when referring to the agent involved, to speak of "individual(s)-acting with mediational means" than to speak simply of "individual(s)"". (Wertsch, 1991, p. 12)
- “We propose that an essential feature of learning is that it creates the zone of proximal development; that is, learning awakens a variety of developmental processes that are able to interact only when the child is interacting with people in his environment and in collaboration with his peers” (Vygotsky, 1978, p. 90)

Central to all these features of Vygotsky’s mechanism for learning is the role of the teacher, although in various guises. It may be a more informed peer; a parent who has no explicit intention to teach; a master creating, together with the apprentice, a zone of proximal development; a text, a production of the culture from which one can learn; or indeed a teacher whose explicit intention is to enable the student to do something, be someone or know something that he or she could not do, could not be or did not know. All human development is led by learning *from others*, from the culture that precedes us.

Vygotsky's theories have been a huge stimulus to research in all kinds of domains of education (e.g. Forman, Minick & Stone, 1993; Cole, 1996; Cole & Wertsch, 1996; Wertsch, 1997, to name just a few recent works) and this includes mathematics education (for a review which relates to reform-related research see Forman, in press; see also Lerman, 1998b, 1998c) and some, hopefully productive controversy (Lerman, 1996; Steffe & Thompson, in press; Lerman, in press a). Recent work on discourse studies (Forman, in press; Forman & Larreamendy-Joerns, in press; Krummheuer, 1995), dynamic assessment (Brown and Ferrara, 1985; Day, forthcoming), and learning in the zpd (Lerman, in press b; Meira & Lerman, submitted) are just some illustrations of the continuing interest in developing Vygotskian theories.

To what extent, though, does Vygotsky's perspective provide the mechanism to which Lave refers? Where Piaget offers equilibration as the mechanism for learning, Vygotsky proposes the zone of proximal development. For Lave learning is transformation through increasing participation in social practices, and a mechanism for learning would need to take account of the goals of the individual in joining, or being coerced into joining the social practice, and the specificities of the practice in terms of situated meanings and situated ways of being. The mechanism would need to take account of the factors that contribute to the individual trajectory through the practice, including what an individual brings to a practice in terms of their prior network of experiences, and the regulating effects of the practice. Vygotsky was not directly concerned with social practices. At the time of the Russian revolution the singular discourse of dialectical materialism, and the drive for progress from a feudal society to communism did not allow for the availability of other theoretical resources. His early death in 1934, at the age of 38, precluded any engagement with more relativistic social theories. However, Vygotsky's psychology is a cultural psychology (Cole, 1996; Daniels, 1993) and it opens up spaces for different analyses than those which appeared during Vygotsky's life.

Vygotsky's work is generally taken to be about the <u>individual</u> <u>learning</u> in a <u>social context</u>, but I have suggested in this section that his theories make it clear that the zpd offers more than that. First, in that consciousness is a product of communication, which always takes place in a historically, culturally and geographically specific location, <u>individuality</u> has to be seen as emerging in social practice(s). Vygotsky's personal history as a member of a discriminated-against minority, the Jews, whose culture is carried in specific languages (Hebrew and Yiddish) and practices, which is obviously about identity, was a key factor in forming his thinking about development (Kozulin, 1990; Van der Veer & Valsiner, 1991). Second, I have argued that all <u>learning</u> is from others, and as a consequence meanings signify, they describe the world as it is seen through the eyes of those socio-cultural practices. In his discussion of inner speech Vygotsky makes it clear that it is the process of the development of internal controls, metacognition, that is, the internalization of the adult. Again, these are mechanisms that are located in social contexts. Finally, the zpd is a product of the learning activity (Davydov, 1988), not a fixed 'field' that the child brings with her or him to a learning situation. The zpd is therefore a product of the previous network of experiences of the individuals, including the teacher, the goals of teacher and learners, and the specificity of the learning itself. Individual trajectories are therefore key elements in the emergence (or not) of zpds (Meira & Lerman, submitted).

In fact Lave suggests that the need for learning mechanisms "disappear(s) into practice. Mainly, people are becoming kinds of persons" (1996, p. 157). The process of accounting for 'becoming kinds of persons' still calls for a mechanism, however, and I am proposing here that internalization through semiotic mediation in the zpd is a suitable candidate.

CONCLUSION: UNIT OF ANALYSIS

Perhaps the greatest challenge for research in mathematics education (and education/social sciences in general) from perspectives that can be described as being within the social turn is to develop accounts that bring together agency, individual trajectories (Apple, 1991), and the cultural, historical and social origins of the ways people think, behave, reason and understand the world. Any such analysis must not ignore either: it should not reduce individual functioning to social and cultural determinism nor place the source of meaning making in the individual. In order to develop such accounts researchers can choose to begin from the development of the individual and explain the influences of culture, or from the cultural and explain individuality and agency (Gone, Miller & Rappaport, 1999). I have argued here for the latter. In my review I have used Lave and Wenger's situated theories as a foundation and attempted to open spaces, through critique, for the development of their theories for our needs in mathematics education research. I have argued for consideration of the regulating effects of discursive practices. I have discussed the multiple practices at play in the mathematics classroom, most of which are not the intention of the teacher. As a result, the notions of mastery and legitimate peripheral participation need careful analysis in order to extend them to the classroom, and I have suggested that narrative methods of research are proving to be most fruitful in research. I have suggested that Vygotsky's notion of the zone of proximal development, when set within a discursive/cultural psychology that was not fully available to him, in terms of intellectual resources, during his lifetime, can perhaps provide the mechanism of learning to study the process of people 'becoming kinds of persons'.

> "The study of the mind is a way of understanding the phenomena that arise when different sociocultural discourses are integrated within an identifiable human individual situated in relation to those discourses" (Harré & Gillett, 1994, p.22).

Individuality and agency, then, emerge as the product of each person's prior network of social and cultural experiences, and their goals and needs, in relation to the social practices in which they function. I proposed the metaphor of a zoom lens for research, whereby what one chooses as the object of study becomes:

> "A moment in socio-cultural studies, as a particular focusing of a lens, as a gaze which is as much aware of what is not being looked at, as of what is… Draw back in the zoom, and the researcher looks at education in a particular society, at whole schools, or whole classrooms; zoom back in and one focuses on some children, or some interactions. The point is that research must find a way to take account of the other elements which come into focus throughout the zoom, wherever one chooses to stop." (Lerman, 1998b, p. 67)

But the object of study itself needs to take account of all the dimensions of human life, not a fragment such as cognition, or emotion. Vygotsky searched for a unit of analysis that could unify culture, cognition, affect, goals, and needs (Zinchenko, 1985). According to Minick (1987), Vygotsky moved from "the 'instrumental act' and the 'higher mental functions' ... to the emergence of 'psychological systems'" (p. 24) and then to his third and final formulation, that "the analysis of the development of word meaning must be carried out in connection with the analysis of word in communication" (p. 26). Further on, Minick said "In 1933 and 1934 Vygotsky began to reemphasize the central function of word meaning as a means of communication, as a critical component of social practice" (p. 26). Minick pointed out (p. 18) that there is a continuity among these three stages and that they should be seen as developments, each stage incorporating the other and extending it. In the second stage, Vygotsky and Luria had carried out their seminal study (Luria, 1976) on the effects of language development on the higher mental functions, a classic piece of research (Brown & Dowling, 1998) and characteristic of Vygotsky's approach in that stage. What was missing was "the child's practical activity"

(Minick, 1987, p. 26), and in the third stage he argued for the importance of incorporating goals and needs into the unit of analysis.

> Of course, by "relationship" Vygotsky meant here not a passive relationship of perceiving or processing incoming stimuli, but a relationship defined by the child's needs and goals, a relationship defined by the forms of social practice that "relate" the child to an objective environment and define what the environment means for the child. (p. 32)

The defining and prior element is the social practice, that the child's goals and needs are a crucial factor in the learning process, and that what the environment signifies is also defined by the social practice, not by the child. Minick stated that by this formulation:

> Vygotsky was making some significant strides toward the realization of the goal that he had established in 1924 and 1925, the goal of a theoretical perspective that would allow a unified analysis of behavior and consciousness while recognizing the unique socio-historical nature of the human mind. (p. 33)

The first part of a unit of analysis is provided by Lave's work and incorporates Vygotsky's goal, that of person-in-activity. Vygotsky's book title *Mind in Society* is of the same essence. I have argued in this paper for a theory of teaching and learning mathematics that incorporates Lave and Wenger's communities of practice notion with the regulative features of discursive practices and the consequences of the multiple practices that manifest in the classroom. I want, therefore, to extend the unit of person-in-activity to incorporate these bodies of work. When a person steps into a practice, she or he has already changed. The person has an orientation towards the practice, or has goals that have led the person to the practice, even if she or he leaves the practice after a short time. One can express that change by noting that the practice has become in the person. In order to incorporate these developments I want to suggest that the unit of analysis be extended to person-in-practice-in-person or, to give credit to Vygotsky, mind-in-society-in-mind (Slonimsky, 1999).

Finally, I want to propose a task for the reader, first suggested by Slonimsky (1999), to search for a suitable metaphor for mind-in-society-in-mind. The search is a productive activity, in that proposing metaphors and working with them to locate meanings in the two domains linked by the metaphor develops the potentialities of the meaning and use of, in this case, the notion mind-in-society-in-mind. By way of a first attempt, I offer the image of a shoot on the side of a growing plant. What is required for a suitable metaphor is, at least, that the metaphorical referent has a history (development of the plant to that point, genetic material), that it allows for an individual trajectory (one cannot predict its growth), and that it allows for experiences of overlapping practices (other plants taking nutrients, perhaps a wall or fences which alter the growth).

Research that works with person-in-practice, or mind-in-society, as a unit of analysis, such as activity theory (Cole, Engeström, & Vasquez, 1997) and some of the work on development in the zone of proximal development, would need to hold a focus on agency and the regulating effects of the practice(s). The notion of mind-in-society-in-mind is yet further indication of the extent of the contextualisation of human activity.

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