PROGRESSIVE PROGRESS AND PROFILING: PROGRESSION SPECTRA FOR TEACHERS

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A wide range of research points to the need for coherence and progression in the teaching of mathematics. We know that many of the assessments our learners write, do not give teachers any diagnostic information about where our learners are at. The assessments simply tell us that our learners are not at the expected grade level. There is thus a need for tasks and assessments that allow teachers to understand exactly what level their learners are at, so that they can identify resources and tasks that will help to progress learners to where they should be. In this presentation, I present learner progression spectra that are accessible for teachers which enable them to both profile learners over time and to provide useful ideas of ways to move learners along the spectrum from using constrained methods to more fluent ones.

Context and background

The South African Numeracy Chair (SANC) project at Rhodes University works towards improving mathematical proficiency in primary school learners through teacher development programmes and direct learner interventions such as afterschool maths clubs. The SANC project bases its notion of mathematical proficiency on Kilpatrick et al.'s (2001) definition of mathematical proficiency, which comprises five intertwined and interrelated strands of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition.

Graven and Stott (2012) argued that while this provides a powerful conceptualisation of mathematical proficiency, it is difficult to evaluate learner progress in mathematical proficiency in this way as the interrelationships in the fully conceptualised and idealised form of mathematical proficiency requires all five strands to be present and for each of them to be fully present in relation to their elaborated definitions. Graven and Stott (2012) argued that in order to assess learner numeracy progression over time, the fully idealised versions of each strand need to be unpacked so as to identify varying levels of learner attainment.

In this presentation, my focus is on procedural fluency and conceptual understanding and how progression in these can be identified using progression spectrums. *Conceptual understanding* is the ability to use multiple representations, estimating, making connections and links and understanding properties of number systems (i.e. number sense). *Procedural fluency* is the ability to solve a problem without referring to tables and other aids, using efficient ways to add, subtract, multiply and divide mentally and on paper, understanding when it is appropriate to use procedures or not (as not all calculating situations are alike).

A number sense approach can be useful to think about the relationship between these two strands. A child with number sense has the ability to work flexibly with numbers, observe patterns and relationships and make connections to what they already know, to make generalisations about patterns and processes. Number sense also includes a positive attitude and confidence (Anghileri, 2006). In the Foundation Phase the development of number sense includes the meaning of different kinds of numbers, the relationship between different kinds of numbers, and the effect of operating with numbers. In the Intermediate Phase this development of number sense and operational fluency should continue, with the number range, kinds of numbers, and calculation techniques all being extended.

Learner trajectories and progression

In order to assess learner progress in mathematical proficiency, we need to be able to assess the extent to which a learner may or may not have mastered a particular aspect of mathematics at different points in time and ultimately to understand if a particular strand or interrelationship of strands is evident. Graven and Stott (2012) argue that to "consider that a student might be either procedurally fluent or not procedurally fluent for example, is less useful than to gauge the extent to which they are mastering the fluency" (p.148).

A wide range of research points to the need for coherence and progression in the teaching of mathematics (for example Askew, Venkat, & Mathews, 2012; Schollar, 2008). We know that many of the assessments our learners write, do not give us as teachers any diagnostic information about where our learners are at (Graven & Venkat, 2014). The assessments simply tell us that our learners are not at the expected grade level. Stott, Mofu and Ndongeni (2017) argue that the idea of early intervention "can be problematic" (p.62) for teachers if they do not know what the learner is struggling with. There is thus a need for tasks and assessments that allow teachers to understand exactly what level their learners are at, so that they can identify resources and tasks that will help to progress learners to where they should be. However, teachers are unlikely to identify useful resources or generate resources with carefully inlaid progression if they do not have an understanding of how learners progress through school mathematics from Grade R to Grade 7 (Stott et al., 2017).

Learning trajectories are one way of thinking about this progressive progress in that they describe the broad learning path that learners typically follow, which is not always entirely linear. Essentially, "what is learned in one phase, is understood and performed at a higher level in a later phase" (van den Heuvel-Panhuizen, 2008 p. 14). Major benefits of thinking about mathematics in terms of learning trajectories are: 1) The boundaries of the grade classes can be broken down and a language is provided for discussion for teachers and 2) the cohesion and connections between the mathematical work of the different grades is clearer (van den Heuvel-Panhuizen, 2008). There are a number of progression models in the literature. I will briefly talk about three such trajectories which will inform the subsequent discussion.

Across a number of research studies, the SANC project have found the work of Wright, Stafford, Stanger and Martland (2006), originating in Australia, on defining levels of mathematical progress in their early Learning Framework in Number (LFIN) to be particularly useful. Our research and development teams have used this framework for analysis of learner levels of mathematical proficiency in order to design learning activities, to profile learners' current mathematical proficient levels in various assessments but also for teacher development. Wright (2013) has also argued that the one-on-one learner interview assessment tool from their Mathematics Recovery (MR) programme (of which the LFIN is a part) is useful for teacher development and understanding the developmental nature of numeracy learning.

Across our research projects however, we have found both the individual learner assessment interview and the LFIN difficult to use in practice. The primary limitations are: time for teachers to understand and learn the LFIN (including understanding of the levels and associated terminology), time to administer the individual assessment interviews and time to profile the learners using the framework following the assessment.

The second, referred to as the TAL Learning-Teaching Trajectory is the topic of the book "Children Learn Mathematics" (van den Heuvel-Panhuizen, 2008) and originates from the Netherlands. This learning and teaching trajectory for primary school mathematics covers whole number calculation from Grade R (kindergarten) up to Grade 6. In Grade R, the focus is on counting and elementary number sense, moving towards the ability to assign meaning to large numbers, insight into numerical patterns and rules, early knowledge of primes, square numbers and so on in Grade 6.

The third, has been used here in South Africa by Cranfield, Kühne, van den Heuvel-Panhuizen, Ensor et al. (2005) as a framework to describe a pathway for learning and how learners learn, understand and solve number problems from Grade R to Grade 4, progressing through four stages. Their framework drew on the TAL Learning-Teaching Trajectory as described above, the work of Steffe (1992) and the MR programme by Wright and colleagues mentioned above. As the combination of three models, their framework highlights that learning trajectories have similar phases of progression. What differs is the terminology and the way the trajectories deal with subject matter in the mathematical domains. I additionally note that none of the frameworks explicitly draw on any of the five strands as described by Kilpatrick et al. (2001).

VISUALISING PROGRESSION FOR INDIVIDUAL ASSESSMENT TASKS

In 2011, our experiences of using Kilpatrick et al.'s (2001) strands of mathematical proficiency as a framework for analysing learner responses to assessment tasks in a pilot maths club, led us to develop a procedural fluency spectrum, when overlap of learner methods/responses as both procedural fluency and conceptual understanding arose. Graven and Stott (2012) argue that following a procedure or method without evidence of understanding surely cannot be considered in the same way as adopting an appropriately chosen method flexibly and efficiently. When analysing student learning it was essential that the method of analysis records and reveals learner progress *over time* and allows one to compare learner responses.

Initially we created procedural fluency spectra in a visual form for each different assessment task using the methods the learners used (see Figure 1 below). These methods ranged from restricted / constrained procedural fluency towards more elaborated and fully flexible fluency. It became evident that as one moved to the upper end of the spectrum where flexibility and efficiency were high, conceptual understanding was increasingly intertwined with procedural fluency and the distinction between these strands became progressively blurred (Graven & Stott, 2012). This is an important point, as our subsequent work with spectra has shown that this is often the case, and I will talk about it further below.

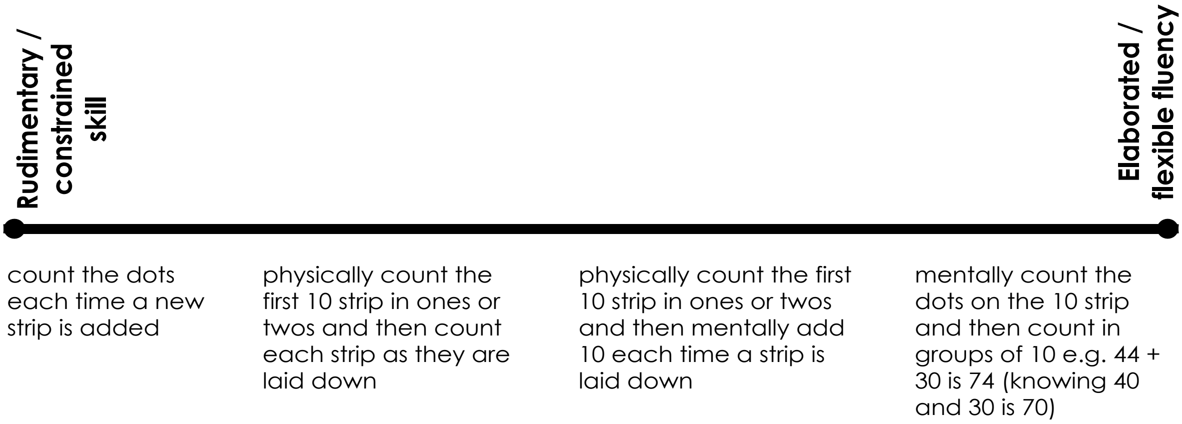

Figure 1 - “add 10 to 92” spectrum (from Graven & Stott, 2012a p.154)

Later, Graven and myself (2013) explored the viability of using some of these procedural fluency spectrums as a means of translating the methods recorded on the qualitative interview records into quantifiable, visual summaries of the learner data at given points in time for broader project reporting and for comparing my two clubs.

Since then a number of research studies emanating from the SANC project have used spectra to analyse learner responses in different ways in different mathematical domains. Mofu (2013) for example adapted the procedural fluency spectrums to create a spectrum representing progression in early multiplication, whilst Young (2016) and Wasserman (2015) both developed spectra for understanding progression particularly in Early Arithmetic Strategies which are part of the LFIN in the MR programme.

All these tools have proved useful for us as researchers. In spite of the limitations highlighted in the previous paragraphs, teachers need accessible tools such as these to enable them to identify where their learners are, and what they are struggling with.

PUSHING FOR PROGRESSION PROGRAMME

As one way to address this issue, in 2016, the SANC project developed and began implementation of a Pushing for Progression (PfP) teacher development programme based around running afterschool maths clubs. One important aspect of this PfP programme is for teachers to learn more about the development of Foundation and Intermediate Phase learner's early number skills along a learning pathway and to be able to identify when children are learning securely along this pathway through effective assessment and focused mathematics activities. The PfP teacher development programme includes a simplified progression framework for the four basic operations of addition, subtraction, multiplication and division with a specific focus on number sense and the two strands of conceptual understanding and procedural fluency. Drawing on the ideas from many of the above-mentioned learning trajectories and our earlier work with the procedural fluency spectrum, the PfP programme uses two spectrums to describe a learning trajectory for primary school learners in these 4 operations that range from constrained methods to increasingly flexible and fluent methods. The two spectra are shown in Figure 2 below.

The spectra highlight the learning trajectories that learners take in learning these four operations. For example, in Grade R, learners learn to count by using the more 'constrained' methods. As learners' progress through the FP and IP, we would expect them to progress further towards more fluent methods. It is important to note that the constrained and less constrained methods will work very efficiently for smaller numbers (in the range 0 to 20), but will become increasingly difficult when the number range gets bigger, especially in multiplication and division. Many of our learners are still stuck using these constrained methods, even in the IP grades.

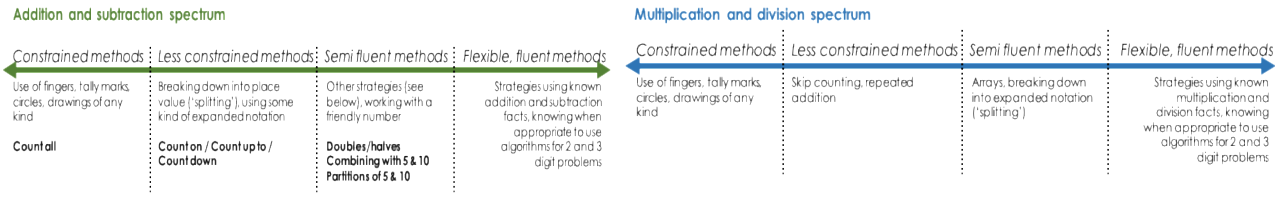

Figure 2 - Addition/subtraction and multiplication/division spectra

The notions of conceptual understanding and procedural fluency are integral in the above spectra. Looking at the descriptors for each strand, it is possible to see how both conceptual understanding and procedural fluency are intertwined and develop as one progresses along the spectrum.

WHAT DOES THIS 'LOOK LIKE'?

In my work with both teachers and after school maths club facilitators over the last 3 years, I have often been asked what these levels/stages '*look like*' in learner workings. Drawing on data collected from research in the SANC project classrooms and after school maths club since 2011 with over 1500 grade 3 and 4 learners a year, these examples illustrate what you can expect to see at different levels of the spectrum.

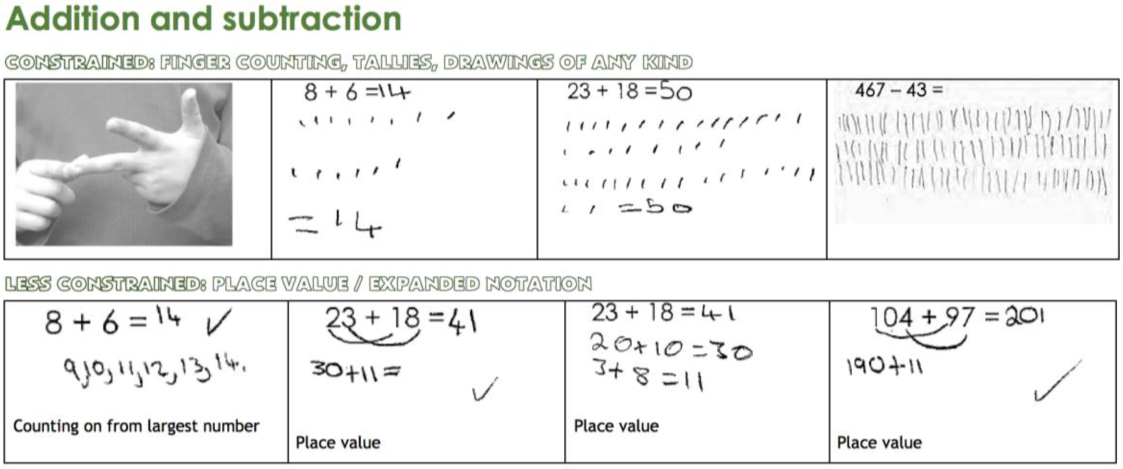

Figure 3 – Sample of addition and subtraction learner methods

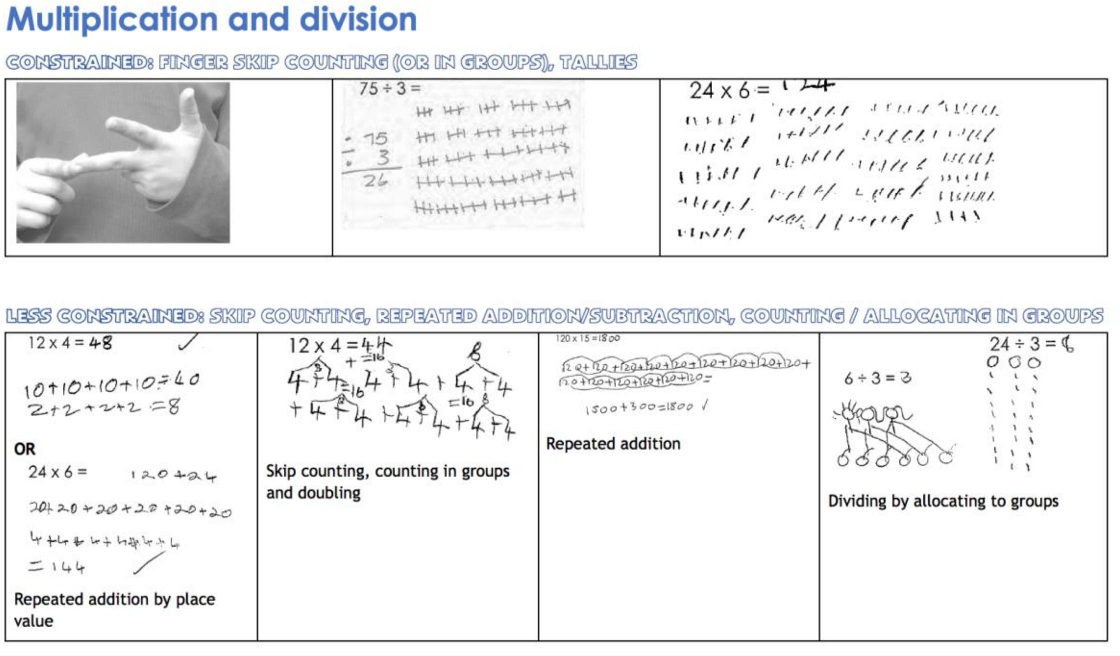

Figure 4 - Sample of multiplication and division learner methods

How are the spectra used by teachers?

In understanding how these spectrums reflect learner progression, teachers can use them in two ways. *Firstly*, they can be used to both profile learners after an assessment (and during classroom tasks) over time. For example, if a teacher gives learners an assessment at the beginning of the term and asks them to show their working, the teacher is able to determine where the learner is along the spectrum. If the same assessment is administered at the end of the term, the teacher will be able to see if the learner methods differ and will be able to compare them. *Secondly*, the spectra provide useful ideas of ways to move learners along the spectrum from using constrained methods to more fluent ones. For example, knowing that ‘count all’ is a constrained method, particularly with bigger numbers, teachers could suggest to learners that they 'count on' from the bigger number. This is a small but significant progression, as it can lead to understanding the commutative property in addition.

As van den Heuvel-Panhuizen (2008) pointed out, using the spectra allows the boundaries of the grade classes can be broken down and a language is provided for discussion between teachers. Additionally, the cohesion and connections between the mathematical work of the different grades is clearer.

What next?

In 2016, a few of our partners in the Eastern Cape Department of Education, implemented the PfP programme and the teachers who participated in the programmes were introduced to these ideas, along with a sample assessment and activities to help progress learners. They found the spectra a useful way to profile their learners and to understand the progress they made over time. They also found the spectra to be useful in their own understanding of how learners learn mathematics. I have also had feedback from our partners that the spectra have been piloted in error analysis sessions at district level.

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