# Using Arrays for Conceptual Understanding of Multiplication and Division

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## INTRODUCTION

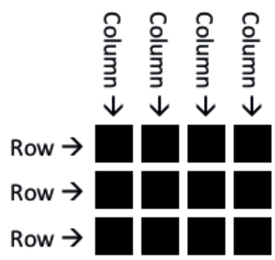
Research has shown that arrays are a useful tool for developing learners' conceptual understanding of multiplication and division across both the Foundation and Intermediate phases. Because arrays lend themselves to multiplicative understanding rather than additive understanding, they are a useful way of 'unitising' – i.e. seeing items in groups rather than as individual items. They are an important conceptual step between modelling multiplication with physical objects and using more algorithmic methods. Arrays also provide a way to make close connections between multiplication and division.

The Foundation Phase CAPS document for Mathematics (2011a) indicates that using arrays allows for (i) the building up and breaking down of numbers, (ii) linking multiplication to repeated addition, (iii) thinking of multiplication as an array, and (iv) laying the basis for the commutative law (p. 232). By Grade 3 it is expected that learners are able to represent multiplication using arrays. The CAPS document for Grades 4 to 6 (2011b) points to the use of arrays for teaching learners both the commutative and distributive properties of multiplication.

In this article I share some of the key ideas for using arrays to teach multiplication and division from Foundation through to Intermediate Phase, explore some contexts for working with arrays, and detail an activity that I have used in a number of after-school mathematics clubs. In a second article later this year I will share further activities.

## WHAT IS AN ARRAY?

An array is defined as a set of numbers or shapes laid out in a rectangle comprising rows and columns. For young learners, an array is a useful visual aid and typically consists of shapes rather than numbers. When describing an array we will use the convention of giving the number of rows first, followed by the number of columns. The array of squares shown alongside is a 3 × 4 array – i.e. it has 3 rows and 4 columns – and contains a total of 12 squares.




## EARLY CONTEXTS FOR INTRODUCING MULTIPLICATION ARRAYS

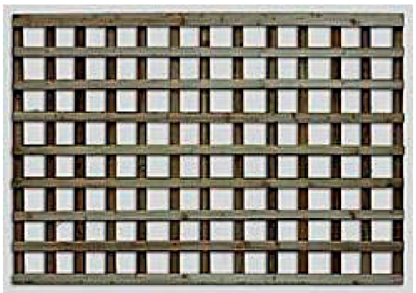
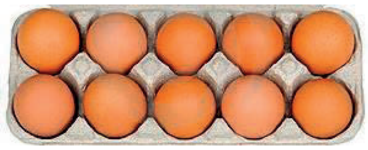
Mike Askew (2011), in his teacher-based article entitled "Signs of the Times", argues that it is beneficial to introduce the idea of multiplication to young learners using a familiar context that lends itself to arrays, rather than starting with a multiplication sentence. He uses an example of a baker putting pies on a tray which holds four rows of five pies as a context that lends itself to modelling a four by five array. From this context, discussions may arise with regard to quick ways to find the total number of pies on the tray without counting each pie individually, the different ways that the learners "see" the total number of pies, as well as the possibility of the observation arising that 5 × 4 gives the same answer as 4 × 5.

Askew (2011) indicates that other contexts that lend themselves to being modelled as arrays include rows of chairs, square tiles on a floor, and windows made up of small panes. Such contexts can provoke "rich conversations about 'shortcuts' to counting the total number of items" (p. 37). Over time, learners can move from modelling these contexts using counters to drawing a grid as a more conceptual array to represent the context.

### AN ARRAY SCAVENGER HUNT

I have used this scavenger hunt activity to provide a context for arrays with Grade 3 and 4 learners who attend my after-school maths clubs, especially when I come across learners who have not had any exposure to the idea of an array. Start by drawing a $3 \times 4$ array of squares and gradually introduce the vocabulary of 'row', 'column' and 'array'. Describe the array in terms of the number of rows and columns it contains. Next, ask learners to calculate how many individual squares there are in the array. Learners will typically do this in a variety of ways – either by counting individual squares one by one, by adding four multiples of three ($3 + 3 + 3 + 3$), by adding three multiples of four ($4 + 4 + 4$), by determining the product $3 \times 4$, or by determining the product $4 \times 3$. Ask different learners to show their calculation processes, and discuss multiplication as being a more economical approach to repeated addition.

Once learners have a sense of what an array is, move outside the classroom and identify an array in the close vicinity, for example a gate or a window. Point out to the learners that this is also an array, and physically indicate the rows and columns, asking the learners to determine the number of individual shapes in the array. Next begins the scavenger hunt. Get learners to look around the school and grounds trying to find as many arrays as possible. For each array get the learners to work out the number of rows, columns and individual shapes. As a final activity ask the learners to look for arrays in their own home and to return the next day with examples of any arrays that they found (a few examples are shown below).

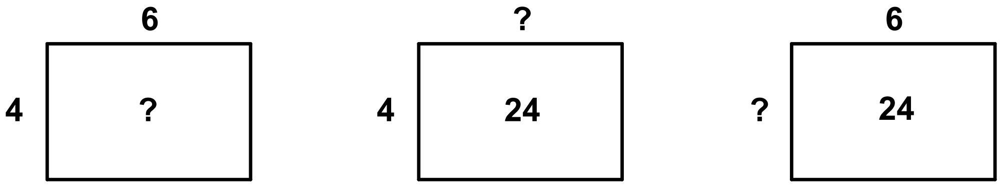





### CONTEXTS FOR EARLY DIVISION

At this point, learners should be encouraged to make connections between multiplication and division using an array. The following questions represent an example of a context that could be modelled using arrays and which provides a space for talking about the relationship between division and multiplication:

- *A vegetable garden has 4 rows of cabbages. Each row has 6 cabbages. How many cabbages are there in the garden?*
- *A vegetable garden has 4 rows of cabbages. Every row has the same number of cabbages. If there are a total of 24 cabbages, how many cabbages are in each row?*
- *A vegetable garden has 24 cabbages that are planted in rows. There are 6 cabbages in each row. How many rows are there?*

The above scenarios could be represented using the idea of an array as follows:

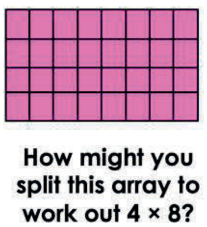
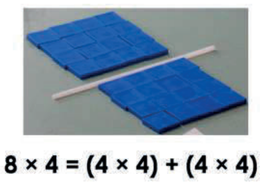
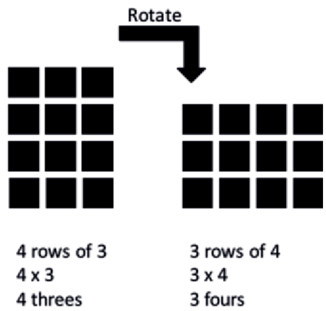



### ARRAYS IN THE INTERMEDIATE PHASE

If arrays are introduced in the Foundation Phase as contexts for carrying out multiplication and division, they can assist in deepening understanding in the intermediate grades with a specific focus on the distributive and commutative properties. Additionally, when used in a more abstract way, they can be used to model and practise multi-digit multiplication before the introduction of the long multiplication algorithm.

### THE DISTRIBUTIVE AND COMMUTATIVE PROPERTIES

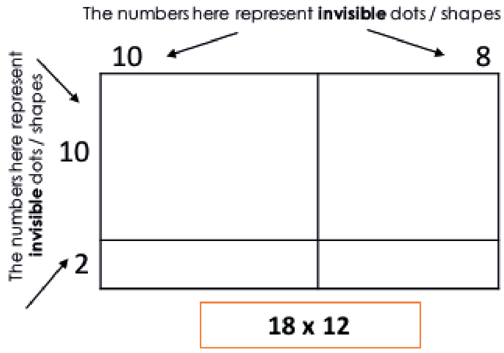
The Grade 4 to 6 Mathematics CAPS document indicates that learners need to understand the distributive and commutative properties, although it is not necessary for learners to know these actual terms. Both the distributive and commutative properties can be illustrated through the use of arrays:

| **The distributive property** | **The commutative property** |
| --- | --- |
| An array can be split into smaller parts based on place value or other subdivisions to represent the distributive property.     | Any array can be rotated through 90º in order to illustrate the commutative property.   |

### USING ARRAYS TO DO MULTI-DIGIT MULTIPLICATION

The area model provides a transition from concrete representations of arrays to a more abstract representation which discourages learners from counting the individual shapes in the array. It is a useful step between arrays and the more formal long multiplication algorithm. Instead of drawing all the individual shapes in an array, the area model represents the array in a more conceptual manner, breaking each number into its *place value components*. Learners need to make the transition from seeing the array with all its dots/shapes to a subdivided array containing "invisible" dots/shapes. This is illustrated over the page for 18 × 12.

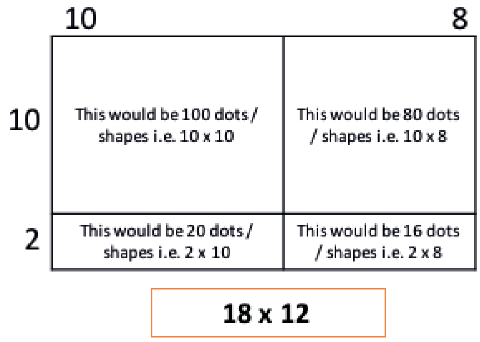
**1.** Break the numbers into parts by place value.



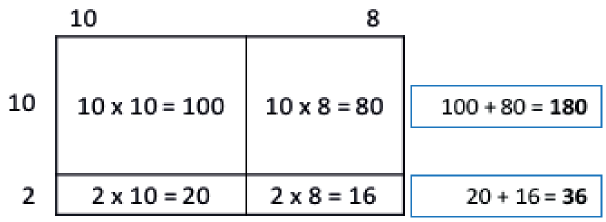

18 is broken into tens and units (10 and 8) across the top (representing columns).

12 is broken into tens and units (10 and 2) down the side (representing rows).

**2.** Multiply the rows and columns for each section of the array/grid.

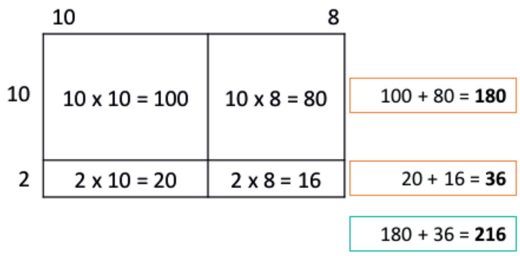



**3.** Add the values from the individual sections in each row.




The invisible dots/shapes in each section of the array are multiplied and added across the two rows to arrive at 180 and 36 respectively.

**4.** Add the totals from the two rows to arrive at the final answer.




The row values are added together to give a final answer of 216.

This method of multiplying has direct links to the long multiplication algorithm and to algebraic multiplication – see for example Samson and Kroon's article on methods of multiplication in LTM 18 (Samson & Kroon, 2015).

## REFERENCES

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