Brombacher
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Solving problems

An extract from the Number Sense Workbook Series
Teacher Guide

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Solving problems

The role of problems in learning mathematics

Mathematics is a tool for solving problems. Problems, however, also provide a way of supporting the learning of mathematics.

Children can add, subtract, multiply and divide long before they know these words. When a mother gives her children some sweets and asks them to share them equally between themselves they can do so.

Living organisms are natural problem solvers. Consider a plant growing in the ground. If the root meets a stone, it grows around the stone. When an animal senses danger, it will run away and hide or change colour or attack the perceived danger. When a young baby is hungry, he/she will cry to get attention. Children who come to school know how to solve problems – what they do not know are the labels that adults use to describe their natural responses to a problem. This is particularly true in mathematics.

When we present a young child with some toy animals and a pile of counters and ask the child to share the counters equally between the animals they will do so. Try it! You will be amazed. Young children have naturally efficient strategies for sharing the counters between the toy animals. Young children can solve this problem, and problems like it, long before they can count the counters in the pile, long before they know what it is to divide and long before they can write a number sentence to summarise the problem situation and its solution.

In a problem-driven approach to learning mathematics, we present children with problem situations that they are capable of solving and where the natural solution strategy they use is the

Illustration 1: Sharing in a ratio

Working with a group of Grade 3s (8 and 9 years olds) the teacher set the scene. He said, "In today's problem there are two dogs. A small dog and a large dog. Every time that the small dog gets one biscuit the large dog gets two. If there are twelve biscuits altogether how many biscuits will each dog get?"

The group of children set to work.

Ben drew a small circle and a large circle and then drew stripes, counting as he did so: 1 (under the small circle); 2, 3 (under the large circle); 4 (under the small circle); 5, 6, (under the large circle) and so on until he ended on 11, 12 (under the large circle). Next he counted the stripes under each circle and concluded that the small dog (small circle) would get 4 biscuits and the large dog would get 8 biscuits.

Fundi drew a small circle and a large circle. Instead of drawing stripes, she wrote the numeral 1 under the small circle and the numeral 2 under the large circle. On the next line she again wrote a 1 under the small circle and a 2 under the large circle and repeated this one more time. Then she paused and counted up as follows: starting under the large circle, she said, "two, four, six" and switching to the numbers under the small circle she continued, "seven, eight, nine". Realising that she had not reached twelve yet she wrote another 1 under the small circle, and a 2 under the large circle. She then counted again and realising that she had reached 12, she added up the numbers under each circle and concluded that the small dog (small circle) would get 4 biscuits and the large mathematics we want them to learn in a more formal way. In other words, we use a problem to provoke a natural response and that response is the mathematics we want to teach/develop. This approach is not limited to the basic operations; this approach applies throughout school mathematics.

The two illustrations make the point.

Problems in the context of a problem-driven approach to learning serve three key purposes:

1. They introduce children to the mathematics that we want them to learn.
2. They help children to develop efficient computational strategies.
3. They help children to experience mathematics as a meaningful sense-making activity.

Using problems to introduce children to the mathematics we want them to learn

Children come to school with an incredible capacity for solving problems in general. One only needs to watch children at play to realise how inventive and clever they are.

In a problem-driven mathematics classroom, we approach the teaching of mathematics as an ongoing sequence of problem solving activities. These problem activities are carefully structured to provoke anticipated/predictable responses from children. The responses of children are representations of "the mathematics" that we want children to develop.

To say all this in a different way, we have a choice when teaching mathematics – particularly so in the early grades. Either we start the lesson and say, "Today we are going to learn about ratio. Ratio is defined as …" Or, we pose a problem that provokes a

dog would get 8 biscuits.

The teacher observed the children working. After some time he invited Ben to show the other children how he had solved the problem. He asked Fundi to do so as well. Once they had finished explaining, the teacher asked the other children if they understood what Ben and Fundi had done.

After some discussion that included the children comparing their solutions to those of Ben and Fundi, the teacher posed a new problem. He said, "In my next problem there are two dogs. A small dog and a large dog. Every time that the small dog gets one biscuit the large dog gets three. If there are twenty biscuits altogether how many biscuits will each dog get?"

Again, the group of children set to work and the teacher observed them solving the problem. After some time the teacher asked two of the children to show their solutions to the others. He asked Sara to show her solution first. Sara had struggled on the first problem, but now, inspired by Ben's solution, she solved the problem using "Ben's method". Frank who had solved the first problem in a way that was similar to Ben's solution demonstrated his solution strategy. This time Frank used numbers instead of stripes – much like Fundi – and explained that he switched to "Fundi's method" to avoid having to draw twenty stripes. Again, the teacher asked the other children in the group if they had understood what Sara and Frank had done.

After some discussion, that included the children comparing their solutions to those of Sara and Frank, the teacher posed a new problem. He said, "In my next problem there are three dogs. A small dog, a medium dog and a large dog. Every time that the small dog gets one biscuit the medium dog gets two biscuits and the large dog gets four. If there are fifty-six biscuits altogether reaction and we then give a name to the reaction saying, “What we have been doing is described as sharing/dividing the biscuits in a ratio. A ratio is written as …”

The key difference in the approaches is that the problem-driven approach assumes that children have the capacity to make sense of situations. In addition, that by making sense of situations children will “do” the mathematics organically. Furthermore, by “generating” the mathematics themselves they will both experience it as more meaningful (less abstract) and with greater understanding.

The danger of this discussion is that the reader develops the impression that posing problems is enough to help children learn mathematics. This is quite incorrect! At the heart of the problem-driven approach to teaching/learning mathematics is the *deliberate* design of problems (the science of teaching). The teacher uses problems with purpose/intent.

how many biscuits will each dog get?”

The group of children set to work with different children solving the problem in different ways. Some continued to use “Ben’s method” even though it was proving a little less efficient with the larger number of biscuits. Some children used “Fundi’s method”. Some children noticed that after giving the small dog one biscuit, the medium dog two biscuits and the large dog four biscuits they had given away seven of the fifty-six biscuits. Instead of continuing to “hand out biscuits”, one, two and four at a time, they simply divided fifty-six by seven and concluded that the small dog would get eight biscuits. The medium dog would get eight times two – sixteen biscuits. The large dog would get thirty-two biscuits.

The teacher had, in effect, asked Grade 3 children to divide 56 in the ratio 1 : 2 : 4 – and they had done so with confidence.

In the early grades the teachers uses problems to introduce children to the four basic operations. The need for problems to provoke the basic operations reduces as children progress through the grades and as they are performing the basic operations with increasing confidence. The focus then shifts to problems that provoke the fraction concept. To problems

Illustration 2: Introducing the fraction concept

Working with a group of Grade 2s (7 and 8 years olds), the teacher wanted to start introducing the children to the fraction concept. The teacher chose a well-designed problem and started, “Today’s problem deals with chocolate. Do you like chocolate? Really, what is your favourite chocolate bar?” She led some discussion about chocolate bars to get the children involved in the story and to make it interesting/meaningful for the children. She also had the discussion to make sure that the idea of a chocolate bar (as a rectangular shape) was clear in the minds of the children. Next she said, “In today’s problem there are two children: Yusuf and Ben. They want to share three chocolate bars equally so that there is no chocolate left over. Can you show them how they can do that?”

The group of children set to work. Because children in this class were used to working on problems, they knew what they were expected to do. They tried to make sense of the problem and, because the problem was that provoke calculating with fractions. To problems that provoke the development and understanding of ratio, rate and proportion and so on.

Problem types that provoke the development of the basic operations

The challenge is to use problems purposefully. In order to do so teachers need to use well designed (purposeful) problems. When using problems to introduce the basic operations we do so by posing problems involving situations that provoke:

- Addition and subtraction like strategies through:
  - **Changing** the number of objects,
  - **Combining** two or more sets of objects, and
  - **Comparing** two or more sets of objects.
- Division-like strategies through:
  - **Sharing** objects,
  - **Grouping** objects, and
  - **Proportional sharing** of objects.
- Multiplication-like strategies through:
  - The **repeated addition** of objects/amounts, and
  - **Grid/array** like arrangements of objects.

We deliberately use these different problem types to provoke the different basic operations that we want children to learn. Each problem type can be posed in a range of different ways. The different problem types and the different ways in which they can be asked are illustrated below.

unfamiliar to them, many drew a picture of the situation.

Masixole drew two faces and three chocolate bars. Next, he drew a line from the first chocolate bar to the first face and then a line from the second bar to the second face. Then he paused for a while and thought about what to do with the remaining (third) chocolate bar. After a while he drew a line through the third bar of chocolate (as if to cross it out) and drew a small piece of chocolate next to each face.

After many of the children had grappled with the problem for a while the teacher selected three different children to show and describe their solution strategies to the others. When it was Masixole's turn, he said, pointing at his picture, "This is Yusuf and this is Ben. First I drew the three bars of chocolate and then" he said, pointing at the lines, "I gave one bar to Ben and one bar to Yusuf. Then there was one bar left over. I thought about this for a while and then decided to cut this bar into two pieces and to give Yusuf and Ben one piece each."

The teacher asked Ben, "Why did you cut the bar into two pieces?" and he responded "Because there were two children."

The teacher asked the other children in the group if they understood what Masixole and the others had done. After some discussion, that included the children comparing their solutions, she posed a two more problems:

Note: The number size in the problems that follow can be changed to suit the age and developmental state of the child.

Change problems used to provoke addition and subtraction like responses

- *Result unknown (✓± ✓= ?)*
  - *Ben has 8 marbles. His father gives him 6 more marbles. How many marbles does Ben have now?*
  - *Ben has 8 apples. He eats 2 of his apples. How many apples does he have left over?*
- *Change unknown (✓± ? = ✓)*
  - *Ben has 8 marbles. His father gives him some more marbles. If Ben now has 12 marbles, how many marbles did his father give him?*
  - *Ben has 8 apples. He eats some of his apples. If Ben now has 5 apples left over, how many*

"Jan, Sarah and Ben want to share four bars of chocolate equally so that there is no chocolate left over. Can you show them how they can do that?" and, "Fundi, Jan, Sarah and Ben want to share five bars of chocolate equally so that there is no chocolate left over. Can you show them how they can do that?"

The children worked on the problems and the teacher observed them and asked individual children to explain to her what they were doing. Occasionally she made a suggestion to an individual child, or asked some questions that forced the child to think about what the problem was asking. The teacher noticed that Verencia drew careful representations of the problems. In particular, she noticed that Verencia's pieces from the left over bar (which she gave to the children in each problem) were getting smaller from the first problem to the last. The teacher made a note to herself that she would ask Verencia to explain to the group why she had drawn these pieces as getting smaller and smaller.

After a while the teacher once again asked, carefully selected, children to explain their solutions to the group. The teacher led a discussion and when rounding off the discussion, asked the children to summarise what they had done. The group concluded that they have given each child in the problem a bar of chocolate and if there was a left over bar they would cut it up into pieces and give each child a piece. The teacher asked, "Do you just cut the bar up into pieces?" "No" said the children, "You cut the bar into as many pieces as there are children. If there are four children you cut the bar into four pieces." The teacher asked, "So what if there are six children and one left over bar of chocolate?" "Then" said one of the children, "you cut the left over bar into six pieces."

Finally, the teachers asked Verencia, "I noticed in your pictures that the pieces were getting smaller. Why was that?" Verencia responded, "As more and more children had to share the left over bar of chocolate the pieces got smaller. The more children you share a bar of chocolate with the less each one gets."

In one problem episode the teacher had, using a well-structured series of problems, established the notion that a whole can be broken into any number of pieces/parts – fractions. The structure of the problems had also enabled Verencia to realise, in effect, that one-half is larger than one-third and that one-third is larger than one-fourth.

The teacher posed a series of problems. The children responded to the problems in a completely natural way. The response of the children was the mathematics that the teacher wanted to introduce: the fraction concept.

apples did he eat?

- *Start unknown (? ± ✓ = ✓)*
    - *Ben has some marbles. His father gave him 4 more marbles. If Ben now has 12 marbles, how many marbles did he have to begin with?*
    - *Ben has some apples. He eats 2 of his apples. If Ben now has 5 apples left over, how many apples did start with?*

Combine problems used to provoke addition and subtraction like responses

- *Total unknown* (✓ + ✓ = ?)
    - There are 4 boys and 5 girls in the class. How many children are there in the class altogether?
    - Sara bakes 5 muffins. Tim bakes 8 muffins. They put the muffins together in a basket. How many muffins are there in the basket?
- *Part unknown* (✓ + ? = ✓ or ? + ✓ = ✓)
    - There are 14 children in the class. 5 are boys and the rest are girls. How many girls are there in the class?
    - Altogether Sara and Tim baked 12 muffins. If Sara baked 8 muffins, how many did Tim bake?

Compare problems used to provoke addition and subtraction like responses

- *Difference unknown* (✓ + ? = ✓ or ? + ✓ = ✓)
    - Fundi has 8 marbles. Sara has 3 marbles. How many extra marbles does Sara need to get to have the same number of marbles as Fundi?
    - Ben has 12 marbles and Frank has 7 marbles. How many marbles must Ben give away to have the same number of marbles as Frank?

Sharing problems used to provoke division-like responses

- Four friends share 12 sweets equally between them. How many sweets does each friend get?

Grouping problems used to provoke division-like responses

- A farmer has 12 apples. He puts 4 apples in a packet. How many packets can he fill?

A general note about sharing and grouping problems

The two problems used to illustrate sharing and grouping (above) both have the same mathematical structure: 12 ÷ 4 = __ . However, the way in which the problems have been stated provoke very different responses.

In the case of the **sharing** problem, the child solving the problem might think about (or draw) the 4 friends and start out by giving each friend 2 sweets – using up 8 sweets altogether. Noticing that she still has sweets left over the child might now give each friend another sweet and noticing that there are no more sweets to share among the friends she will count and establish that each friend got 3 sweets.

In the case of the **grouping** problem, the child solving the problem might think about (or draw) the 12 apples and then rearrange them in groups of four, one group at a time until there are no apples left over to put into packets.

The actions and thought processes of the child solving the sharing and the grouping problem are quite different. These differences contribute to what will one day be a richer understanding of what it means to divide. In the case of the sharing problem – the natural response is to divide by sharing out the objects a few at a time, stopping after each stage to see how many objects remain and then continuing until there are no objects left over. This thinking is what is at the heart of the so-called long division algorithm. In the case of the grouping problem – the natural response is to remove sets of objects until no objects remain and then to count the number of sets. This is to think about division as repeated subtraction (or addition). The point is that by posing problems to provoke division-like strategies, we help children develop a deeper sense of what it is to divide and, at the same time, they develop different ways of performing the calculation.

Proportional sharing problems used to provoke division-like responses

- Every time that the small dog gets one biscuit the large dog gets two biscuits. How many biscuits will each dog get if they share 12 biscuits in this way?

A note about remainders and problems that provoke division-like strategies

Some sharing and grouping situations will involve remainders. There are typically three such situations:

- Situations where the remainder does not have an impact on the answer.
    - Ben has 18 marbles. He puts 4 marbles in a bag. How many bags can he fill?

  In this case there are two remaining marbles – there is no impact on the number of bags of four that can be made. The class and teacher can decide what to do with the left over marbles, for example they may decide to “give them to the teacher”.

- Situations where the remainder impacts on the answer.
  - Mr Twala can load 15 bricks on his wheelbarrow. He has to move 85 bricks to the place where he is building. How many trips must he make with his wheelbarrow?

  After filling 5 wheelbarrows (5 ×15 = 75) there are 10 remaining bricks (85 – 75 = 10) that must also be transported. So, 5 + 1 = 6 trips are needed.

- Situations where the purpose of the remainder is to provoke the development of the fraction concept.
  - Fundi and Yusuf want to share three chocolate bars equally. Show them how to do it.

  This problem was used in the illustration at the start of this section. The development of the fraction concept needs a separate discussion.

Repeated addition problems used to provoke multiplication-like responses

- Mother buys 4 bags of apples that cost R3 per bag. How much must she pay altogether?

Grid/Array like situations in problems used to provoke multiplication-like responses

- A farmer plants tomato plants. There are 4 plants in every row and 3 rows of plants. How many tomato plants are there altogether?

A note about repeated addition and grid/array like situation problems

The two problem types used to illustrate situations that provoke multiplication-like responses have the same mathematical structure: 4 × 3 = __ . However, the way in which the problems have be stated provoke very different responses.

In the case of the **repeated addition** problem, the child solving the problem might think about (or draw) the 4 bags and work out the total cost by adding R3 + R3 + R3 + R 3 = R12.

In the case of the **grid/array** like situation, the child solving the problem might think about (or draw) the 3 rows of plants with 4 plants in every row and end up with an image of a neatly organised grid/array. The grid/array will help the child to realise that she can calculate the number of plants by counting either 3 + 3 + 3 + 3 = 12 or 4 + 4 + 4 = 12. This encourages the realisation that four lots of 3 is the same as 3 lots of four. Mathematically, that 3 × 4 = 4 × 3; the so-called commutative property of multiplication.

The actions and thought processes of the child solving these problems are quite different. These different processes contribute to what one-day will be a richer understanding of what it means to multiply.

A discussion, similar to this discussion on using problems to develop the basic operations, is possible for many/most other concepts in mathematics. For now, however, we turn our attention to the second reason for using problems: helping children to develop efficient computational strategies.

Using problems to help children develop efficient computational strategies

In the same way that we use problems to provoke children to 'do the mathematics' that we want to teach them, so we also use problems to help children develop their age appropriate/efficient computational (calculation) strategies.

To discuss how problems help children develop efficient computational strategies let us explore the case of sharing problems. We will then show how the teacher can change the details of the problem to help children develop more efficient and age appropriate computational strategies.

Physical modelling

Even before children know number names and symbols and are able to write these, children can solve problems. They do so by means of physical modelling.

The teacher, working with a group of children, uses toys and counters to pose a problem saying "Here are three teddy bears. Who can help me by sharing these counters between the bears so that each bear gets the same number of counters?" The children will take turns to physically share the counters between the bears. The teacher will ask the other children in the group to comment on whether the child doing the sharing of the counters has done so properly.

On the one hand, physical modelling is a primitive way of solving problems – counting out counters and moving them about is not efficient, especially as the numbers involved get larger. On the other hand, it is important that children appreciate the value of physical modelling – too many children do not realise that they can draw a picture of a situation or create a model of a situation in order to help them understand the situation (the first step to solving a problem).

Physical modelling and drawings

As children develop their fine motor skills and learn to write and draw, so the teacher continues to pose problems using physical objects (teddy bears and counters). The teacher continues to ask children to solve the problem by means of modelling it physically using the objects. After the children have solved the problem by means of physical modelling the teacher will ask the children to draw a record of what they did in their books.

The shift to asking children to draw a record of the problem situation and its solution is an important first step to the child recording their thinking. At first children are likely to draw elaborate and detailed drawings. Over time and because the teacher encourages the children to work quickly, the drawing will become less elaborate and more focussed. Children will typically start to draw a stick figure instead of a detailed drawing of a person and eventually just draw a face to represent a person. Drawing pictures of the problem situation and solution, developed through physical modelling, is an important step in helping children make the shift to using drawings to represent a problem situation and then solve it.

Drawings

As children gain confidence the teacher will be able to pose a problem and ask children to "make a plan" in their books to solve the problem. Using drawings to represent and solve problems is not only a shift away from physically modelling the problem, but an important problem-solving strategy in its own right. Generally, good problem solvers will often draw a sketch of the problem situation before attempting to solve the problem. That said, it is important that the teacher does not ask children to draw a picture, but rather to "make a plan". We do not want children to get the impression that drawing pictures is what we always want.

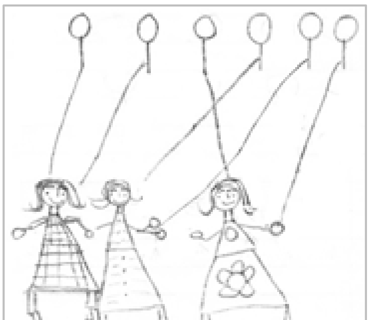

Figure 1

The ways in which children draw their solutions when solving a problem will change as the problems change. Teachers support this transition by the problems that they pose. By increasing the sizes of the numbers in the problem, the teacher provokes children to be more efficient.

Figure 1 is a drawing made by a child solving the problem "*Three friends share 6 sweets equally between themselves. How many sweets does each friend get?*" In the drawing, the child has drawn the 3 friends and the 6 sweets. The drawing shows quite clearly how the child has shared the sweets out one by one using lines to record the movement of the sweets.

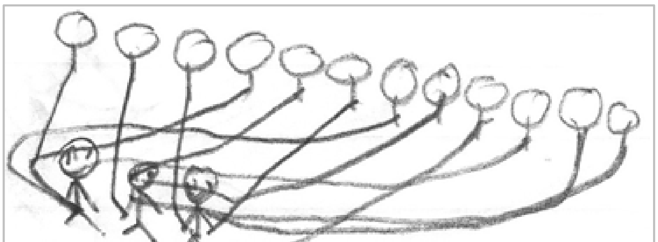

Figure 2

Figure 2 is a drawing made by a child solving the problem "*Three friends share 15 sweets equally between themselves. How many sweets does each friend get?*" This child has used a similar drawing to the one used by the child in figure 1, however because the number of sweets has increased the drawing that was appropriate for the easier problem is no longer appropriate and the child's work becomes messy and illegible. Typically, children realise that the drawing is no longer appropriate because they are starting to make errors and then they begin to use more efficient drawings.

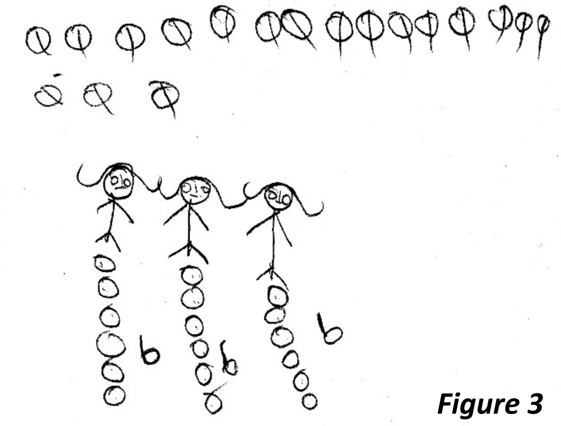

Figure 3

Figure 3 is a drawing made by a different child solving the problem "*Three friends share 18 sweets equally between themselves. How many sweets does each friend get?*" Notice how this child has drawn the friends and the collection of sweets above the friends. Instead of drawing lines to "move" the sweets as the earlier child did, this child has systematically crossed out the sweets one by one and recorded the allocation of the sweets to each friend by drawing the sweets below the friends.

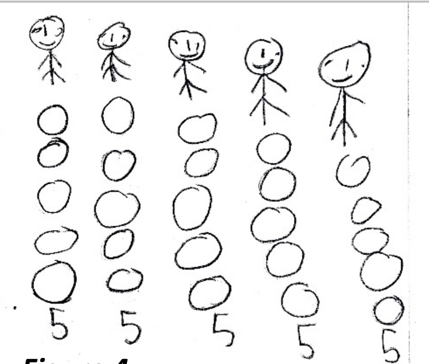

Figure 4

Figure 4 is a drawing made by another child solving the problem "*Five friends share 25 sweets equally between themselves. How many sweets does each friend get?*" Notice how this child has drawn the friends but not the collection of sweets. This child simply placed the sweets below each friend counting as she did so "one, two, three" and so on until she reached "twenty-five". This representation is more sophisticated than the previous one since the child did not first have to draw each sweet – she was able to share them out straight away.

What is evident in figures 1 to 4 is an increasing sophistication and efficiency in the drawings used by children to solve a problem. What is important to realise is that in the same way that teachers ask problems to provoke specific responses from children, so teachers also vary the sizes of the numbers in the problems to provoke children to become more efficient in their representation of the problem situation and its solution. The teacher deliberately changed the question from "3 friends and 6 sweets" to "3 friends and 12 sweets" to make the drawing used in figure 1 inefficient and in so doing to encourage children to use a more efficient drawing.

Primitive number strategies

In the same way that the type of drawing used to represent a situation and its solution became inefficient over time, so drawings also become inefficient, as the numbers in the problems get larger.

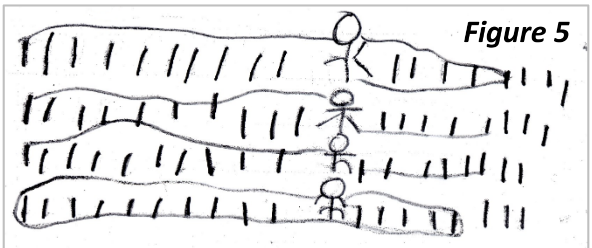
Figure 5

Figure 5 is a drawing made by a child solving the problem "*Four friends share 72 sweets equally between themselves. How many sweets does each friend get?*" The drawing does show some sophistication in that the child first drew 10 sweets for each friend (on the left) and then

5 sweets for each friend (on the right) and finally counted on from 60, allocating one sweet to each child until she reached seventy-two. However, it should be obvious that this approach is no longer appropriate for the number range of the problem.

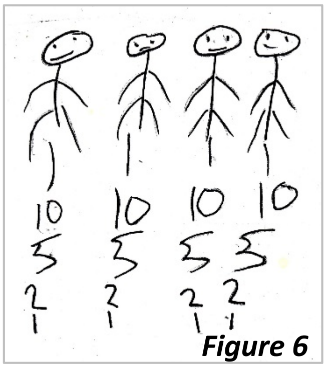

Figure 6

Figure 6 is a more appropriate response to the problem "*Four friends share 72 sweets equally between themselves. How many sweets does each friend get?*" This child has done exactly what the child in the previous problem did: he first gave each friend 10 sweets, then 5 sweets and then 2 sweets and finally one sweet each. The key difference is that this child is no longer drawing all of the sweets and is instead representing collections of them by means of numbers. The reason that the child is using numbers to represent collections of sweets is because the number of sweets in the problem is now so large that drawing individual sweets is no longer efficient or appropriate.

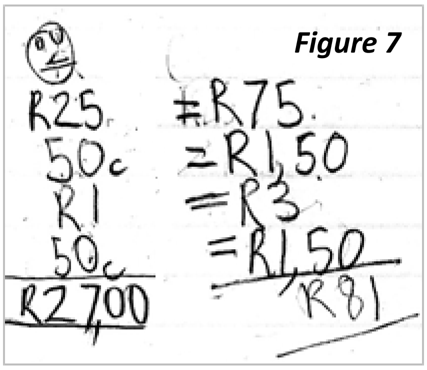

Figure 7

Efficient number strategies

As the complexity (in particular the number size) of the problems increases so children start to develop solution strategies that increasingly involve manipulating with numbers only and rely less on drawings.

Figure 7 is a response by a child to the problem "*Three friends share R81 equally between themselves. How much does each friend get?*" The child does not draw each friend in the problem. Instead, he draws one friend only and gives the friend R25, he then records how giving one friend R25 amounts to giving the three friends getting R75 of the R81. He continues in this way until the R81 has been shared. In the end, he can account for the R81 that was shared out and can tell how much money each friend got. This method is largely numerical, is more efficient than using drawings nd shows an ability to manipulate with numbers.

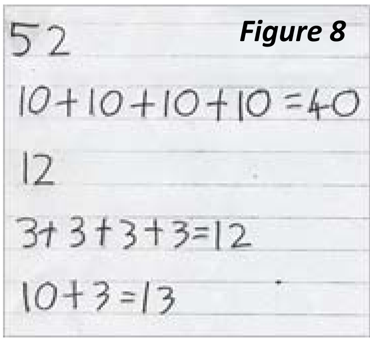

Figure 8

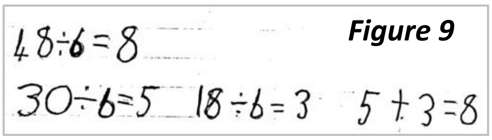

Figure 9

Figure 8 is a response by a child to a problem with the structure 52 ÷ 4. ***Figure 9*** is a response by a child to a problem with the structure 48 ÷ 6 and ***Figure 10*** is a response by a child to a problem with the structure 91 ÷ 7. These children are all solving the problem by numerical methods alone. The child solving the problem with the structure 52 ÷ 4 first adds four tens, then realises that she still needs another twelve and knowing that four threes is equal to twelve concludes that 52 ÷ 4 = 13. This child has calculated 52 ÷ 4 by building up the number. The child solving the problem 48

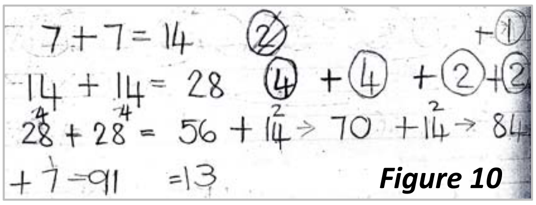

Figure 10

÷ 6 breaks up 48 into two numbers that are divisible by 6: thirty and eighteen. She divides each by 6 and concludes that 48 ÷ 6 = 8. The child solving the problem 91 ÷ 7 builds the number up through doubling to a point (56) and then building up in multiples of 7 till he gets to 91. Finally, he counts up the number of 7s used and concludes that 91 ÷ 7 = 13.

The point made in this discussion is that teachers, by asking appropriate problems, can provoke children to become increasingly efficient in their computational strategies. As the numbers used in the problems become increasingly larger and the problem increasingly complex so the early primitive strategies are no longer useful and children begin to think about more efficient solution strategies.

The role of the teacher in helping children develop more efficient calculation strategies

As has already been discussed, teachers contribute to children developing more efficient calculation strategies by managing the size of the numbers in the problem. There is, however, another very important role played by the teacher, it is: managing discussions.

When teachers set a problem for children to solve, it is important that the teacher allows enough time for children to work on the problem independently. While the children are working, the teacher observes them and, in her mind, classifies each child's approach in terms of its level of sophistication. After the children have had enough time and most of them have solved the problem the teacher then manages a discussion of the solution strategies. Typically, she will ask a child whose method is not very sophisticated (in terms of the age of the child) to explain their method so that those children who were struggling to make sense of the problem can be inspired. She will also select one or more children with more sophisticated approaches to explain their methods to the other children so that in turn those using less sophisticated methods will be inspired. The teacher may also ask a child who made a mistake to explain their thinking so that the group can discuss why that approach did not work – there is much to be gained from a discussion of mistakes.

By asking different children who used methods varying in sophistication to explain their solution strategies to the others, the teacher exposes children to increasingly sophisticated computational approaches. This exposure to more sophisticated approaches coupled with the increasing demand of the problems (resulting from the increased number ranges, etc.) encourages children to develop increasingly more efficient computational strategies.

Using problems to help children experience mathematics as a meaningful sense-making activity

Problems give purpose to the mathematics that children learn. Children who learn mathematics through solving problems experience mathematics as a purposeful, meaningful activity. They can see the value in the mathematics that they learn. They experience mathematics as a tool – a tool for solving problems.

By contrast, children who experience mathematics only as the memorisation of facts, rules and procedures used to determine the answers to questions that make no sense, do not see the purpose in what they are doing. They then experience mathematics as confusing, frustrating and mysterious.

Managing problem solving activities in class

Solving a problem, or series of problems, is part of the everyday routine of the early grade mathematics lesson. For that matter, problem solving should be at the heart of mathematics lessons in all grades.

The problem solving activity typically involves the following stages:

- The teacher poses the problem,
- Children work on the problem while the teacher monitors their progress, and
- The teacher manages a discussion of the solutions (including mistakes) made by the children.

The teacher poses the problem

- The teacher chooses a problem mindful of the mathematics that she wants the problem to provoke (see problem types above).
- The teacher makes sure that the number range of the problem is appropriate for the developmental state of the children. With younger children, she will use smaller numbers and with children whose confidence is greater she will use larger numbers. The teacher will also adjust the number range of the problem to be in line with the computational strategies that the children are using.
- When posing the problem to the children she will say "*Today I have another problem that I want you to solve. I want you to solve it in a way that makes sense to you and I want you to be ready to explain what you did as you attempted to solve the problem. Are you ready?*" Next, she sets the scene and explains the problem, making sure that each of the children in the group understands the question being asked. Once she has finished posing the problem she encourages the children to work on the problem.

Children work on the problem while the teacher monitors their progress

- In a classroom with a healthy problem solving culture children know that they must:
    1. Think about/understand the problem,
    2. Make a plan to solve the problem,
    3. Solve the problem, and finally
    4. Think about whether their solution makes sense in the context of the problem.
- As children work on the problem the teacher will monitor what they are doing. She will:
    - Encourage those who are stuck by asking them questions such as:
        - "Tell me what the problem is in your own words."
        - "Have you solved a problem like this before? What did you do then?"

        - "What do you know? What do you want to know? How can what you know help you to solve the problem?"
    - Ask children, who are working, questions such as:
        - "Why did you do this?"
        - "How do you know that you can do what you have done?"
        - "Does your answer make sense? Can you check your answer?"
    - Make sure that the children have access to the resources they may need for the problem solving strategy they are likely to use. For example, counters for those using counters, etc.
- Throughout the problem solving time the teacher observes each child as they are working and she classifies each child's approach in terms of:
    - Whether it is likely to work in determining a solution or not.
    - How sophisticated it is in terms of the age and developmental stage of the child.
- As the children are working, the teacher thinks about who she will ask to explain their solution method to the other children in the group. Typically she will choose one or more children whose method are not very sophisticated (in terms of the age of the child) to explain their method so that those children who were struggling to make sense of the problem can be inspired. She will also select one, or more, children with more sophisticated approaches to explain their methods to the other children so that they will inspire those using less sophisticated methods. The teacher will also ask children who made mistakes to explain their thinking so that the group can discuss why that approach did not work – there is much to be gained from a discussion of mistakes.

Children discuss their solutions (including mistakes)

- After the children have had enough time to work on the problem, the teacher will ask them all to stop working and to get ready to listen to each other's solutions.
- The classroom culture needs to be such that the children know to listen carefully to each other making an effort to understand each other's explanation.
- The teacher then asks the children, she has identified, to explain their methods to the other children. As the children explain what they did she encourages the engagement of the other children by asking questions such as:
    - "So do you understand what he did?
        - If yes: "Please explain what he said in your own words?"
        - If no: "Listen carefully, we will ask him to explain again."
    - "How is what he did similar to and different from what you did?"
- Depending on the time available the teacher will pose an additional problem – the additional problem will often be of a similar structure so that children can consolidate their understanding by doing a similar/related problem.