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The Role of the Story in Enabling Meaningful Mathematical Engagement in the Classroom

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We draw on our work with designed instructional sequence on Fractions as Measures across three international contexts and explore the functions of the design features related to the stories embedded within the sequence. We discuss functions that relate to how stories support students' meaningful engagement in classroom mathematical activities, focusing, in particular, on students from under-resourced environments. While doing so, we highlight the mediating role of teachers who adapt designed resources to the specific needs of their classrooms. We clarify how instructional sequences can provide guidance for teachers' adaptations by making design rationales for mathematics and functions of the story explicit.

Educators and education researchers across subject areas recognise the learning potential of stories and storytelling experiences (Phillips, 2000). They found children to be drawn to narrative and demonstrated that storytelling, among other benefits, enhances children's imagination as well as their cognitive skills, contributes significantly to all aspects of language development, supports and extends their social lives (Britsch, 1992; Cooper, Collins, & Saxby, 1992; Raines & Isbell, 1994). Furthermore, stories can be used to explore complex notions such as social justice and active citizenship even with very young children (Phillips, 2012a, 2012b). We link to this literature and demonstrate how stories and storytelling can productively support classroom *mathematical* learning experiences, and help to guide children's reinvention of specific mathematical ideas.

Drawing on Benjamin (1955/1999), Nussbaum (1997), and Greene's (1995) work, Phillips (2012b) highlights how storytelling enables listeners to "connect with the characters and accompany the teller on the journey of experience" (p. 142). She posits that as a result, two things become possible. First, children can be led to *reach the understandings of humanity* "via the cultivation of sympathetic imagination that storytelling fosters" (ibid). Second, by having the capacity to "captivate people to see and feel the perspective of another" (ibid), stories *motivate relations, possibilities, and actions*. Part of our intent in this paper is to illustrate how these very issues matter, and can be leveraged, in teaching and learning mathematics in classrooms, and in designing *mathematics teachers' resources* (Visnovska & Cortina, 2018).

We pursue this intent by presenting what, at the moment, is a web of ideas and theoretical connections closely related to the various functions that stories served across our experiences of using a designed instructional sequence on Fractions as Measures (Cortina, Visnovska, & Zuniga, 2014) in different classroom and professional development design experiments in Mexico, Australia, and South Africa. We focus on the students, in

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particular from under-resourced environments, and document functions related to how stories support students' meaningful engagement in classroom mathematical activities. In doing so, we are mindful of the mediating role of teachers and consider supports that the sequence provides for their work of making reasoned adaptations, specifically in the space of stories. We now open the space of stories in our design work to closer scrutiny.

Stories and the Theory of Realistic Mathematics Education

The perspectives from research on storytelling deeply resonate with authors' experiences from mathematics classrooms and our own conceptualisations of how students' learning can be supported. Partly, this resonance stems from the shared grounding in humanistic philosophies, from foregrounding the students' active role in learning, and the expectation that effective pedagogies are inevitably those that are responsive to students' actual needs.

Our work as instructional designers is guided by the theory of Realistic Mathematics Education (RME; Gravemeijer, 1994), and rooted in Freudenthal's (1973) interpretation of mathematics as a human activity. In Freudenthal's view, mathematics is highly relevant to many human endeavours, and students should be given the opportunity to *reinvent* mathematics by organising or mathematising either real world situations or mathematical relationships and processes. In developing this position, Freudenthal emphasised that students should encounter the material they are to mathematise as being *experientially real* for them.

Problem situations and tools that are experientially real are those with which students can immediately engage in personally meaningful mathematical activity (Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997). In psychological terms, these would be problems and representations with which students can readily become imagistically involved (Thompson, 1996). Consequently, for a problem situation or a tool to be regarded as experientially real, it does not necessarily need to come from or be relevant to all students' everyday experiences. However, it has to be possible for all students to construe the problem and representations as personally meaningful and mathematically engaging, *with teacher guidance and support*. We would like to point out that we find it is unreasonable to expect students to construe in this way the problems that, in their view, can only be considered meaningful in school.

This is where we see the tenets of RME and the research on benefits of stories to children's learning overlap. Stories are spaces in which children can come to relate to problems that can be addressed by mathematical means. When children engage with the virtual world of stories (Gee, 2007), identify and connect with the characters, and come to see the world from their perspective, the storyteller can motivate the classroom to the actions of mathematising and problem solving. In this way, the story becomes a means of presenting the listeners with the purpose and need for mathematical actions.

Purposes and Functions of the Story

We start our conceptualisation of the purposes and functions of story and storytelling in mathematics teaching and learning by discussing the initial design work on the instructional sequence on Fractions as Measures. We then address how attending explicitly to the purposes and functions of the story within the designed sequence allowed for successful adaptations of the sequence to new settings, and new instructional purposes.

The Initial Story

An important aspect of the instructional sequence on Fractions as Measures involves having students use an unconventional unit to measure lengths, namely an unmarked measuring *stick*, which later comes to play the role of a reference unit. The stick is used to engage the students in reasoning about the function of such a measurement unit (its utilitarian purpose) and the relative size of subunits of measure, which are each the size of a unit fraction, and physically separate from the stick (e.g., $1/5 > 1/6$). It is also used to help students gauge, and reason about, the size of a measure as being shorter than, as long as, or longer than the length of the stick (e.g., $5/6 < 1$; $4/4 = 1$; $6/5 > 1$).

The design decision of basing the instructional sequence in measurement activities with informal units of measure was based on conceptual and phenomenological analyses of fractions that we reported in detail elsewhere (Cortina, Visnovska, & Zuniga, 2015). Pursuit of mathematical learning goals necessitated that we, the designers, (a) consider how teachers could make it *reasonable* for the students to use the stick as a measurement tool, and (b) support teachers in this endeavour (Cobb, Zhao, & Visnovska, 2008). Without a good story, students could rightly wonder why they should engage with such activities, given that outside of the classroom, people use a ruler or a measuring tape to accomplish what the teacher would be asking them to do.

When first trialling the sequence in Mexico, we thus aimed to engage students in an *interactive legend* about how people measured before the metric system was invented. We capitalised on the rich historical heritage of the region, in which the school was situated. The main characters of the story we created were the Acahay, a group of wise elders, women and men, who lived in a legendary Mayan city that has been long lost: Napiniaca. Our aim was that the students would experience the instructional sequence as an inquiry journey into the challenges that the ancient Acahay faced, as they struggled to come up with better ways of accounting for the lengths of things (what tools to use, how to name and symbolise them).

This example illustrates how RME design heuristics oriented us to develop meaningful narratives, within the broader storyline, so that each instructional activity could then be introduced as a genuine problem that story characters faced. Students' identification with characters in the story was, in turn, intended to motivate and warrant students' effort and persistence while advising the characters on the resolution of challenging problems.

We contrast the outlined approach with that of using a story as a 'hook' to encourage students' 'buy in' to subsequent mathematics activities. Here students are enticed to engage with fun or enjoyable, familiar contexts, which then abruptly transition to mathematics activities with only a vague connection to the 'hook' and where the initial 'hook' is no longer of consequence. From the students' perspective, the mathematics they are asked to do, while connected to some aspect of the world beyond school, is essentially detached from real life considerations. Doing school mathematics and solving a problem that requires mathematisation in the real world (outside of the classroom) are then essentially two different practices for the children.

Story and Mathematics

While the broader storyline can be used to situate day-to-day classroom mathematical activities and provide connections across them, story and storytelling can also play an important part in supporting meaning-making and organising activities when teachers pursue specific mathematical goals and connections within a lesson (cf. Sleep, 2012). We

return to RME and the guidance the theory provides for designing viable instructional starting points - or how students can be productively introduced to new problem situations. In addition to the requirement we mentioned earlier, that problem situations would become *experientially real* to students during their introductory classroom discussions, an additional characteristic is key to our present purpose. The problem situations need to, when introduced by the teacher in her classroom, *trigger students' informal ways of reasoning* that can become a basis for developing increasingly sophisticated mathematical ways of knowing in a particular domain (Cobb et al., 1997). In other words, the problems and inscriptions need to be a means of achieving learning goals in the lessons.

We illustrate how, in the Fractions as Measures sequence, the story becomes a key means for the teacher to elicit informal ways of reasoning *before* more formal mathematical ideas and innovations that present the short-term learning goals would be introduced. In a number of initial activities in this sequence, the focus is on supporting students' realisation of how the tools (or inscriptions) that were previously legitimately used for measuring (or symbolising) are, in new situations, no longer suitable. Such realisation is intended to provide students with the *need* for innovations, which can then be either devised by the students or introduced by the teacher.

Within the instructional sequence, guiding the reinvention of fractions is expected to start with students recognising a need for a standardised unit of measure. Students are first asked to measure objects in the classroom using parts of their bodies (e.g., their hands) so that they come to experience and recognise that measuring in this way can be problematic. To support this recognition, a story is told in which one Acahay daughter took a measure for a clay pot, ordered by a villager, with her hand, which led to her mother making the pot (using her own hand to measure) the wrong size. This is presented as a puzzle for students to figure out why the pot was not the right size. Once such recognition is accomplished, the standard unit of measurement (the stick) is introduced as a resource that allows consistent measurement of the lengths of things, by different people and at different times.

As a next step, the students are supported to become mindful of the limitations of solely using the stick for 'accurate' measurements. This time, a story can be used in which the villagers (students) measured their height with the stick and were all claimed to be the same height – five sticks and a bit tall. Students typically vehemently disagree with such a conclusion and recognise the limitation of their measurement tool. At this point, they learn that Acahay elders solved this problem by introducing smaller length measures, *smalls*, which are created by using the stick in the pattern: *small of two* (*small of three,* etc.) is a rod of such a length that, when used to measure the stick, it measures exactly two (three, etc.) rods (Figure 1). Each small thus represents a unit fraction of the length of the stick.

Figure 1. Small of three rod with such a length that three iterations of the rod cover the same length as the *stick* (reference unit).

We would like to highlight that the situations in the story are intended to support students to reason intuitively, based on the experiences generated through in-class activities, about why measurement tools used by Acahay were insufficient for certain purposes. While intuitive, this reasoning is inherently mathematical and helps to orient students' attention to aspects of the situation on which further mathematics will be built. In

addition, any innovation - and fractions in particular - are then introduced as a solution to a problem which all students have already recognised. Importantly, we learned from the teachers with whom we worked in Mexico and Australia that once they saw some of their students produce the initial intuitive arguments (e.g., why measuring with hands can be problematic), they found they wanted to support all their students to reason in these ways. Seeing how a story functioned in their classroom, they keenly developed similar stories to provide their students with additional opportunities for mathematical reasoning, thus supporting them in accomplishing specific learning goals (Visnovska & Cortina, 2017, 2018).

It is important to clarify that it was not our intention to design a story that would 'work' irrespective of local aspects of the classroom context. Creating characters with which students can identify and purposes for which they would be keen to engage in problem solving would indeed need to be informed by local knowledge. However, the Fractions as Measures sequence provides a strong mathematically driven frame within which to develop locally adapted stories to support student learning of the intended mathematics. As a consequence, only some of this work can be done by designers, much is left for teachers. Indeed, the notion that underlines our design work is that of implementation as a conjecture-driven adaptation. We thus aim to design resources that make local adaptations possible. This necessitates provision of guidance for resource users in terms of the sequence rationale, so that in reasoned adaptations, the story can still be used to drive both the initial and the sustained mathematical engagement. We now discuss the kinds of adaptations that were made when the sequence was used in South Africa, the purposes that necessitated these adaptations, and how the sequence facilitated this work.

Sequence Adaptations

The South African Numeracy Chair Project is mandated to research innovative, sustainable and practical solutions to the challenges of numeracy education in South Africa, and particularly in low socioeconomic status schools in the Eastern Cape. As part of this mandate we (the latter two authors) trialled the Fractions as Measures sequence in three Grade 3 classes in a local school as a possible approach to help students to better comprehend the multiple meanings of fractions. In the research reported here, we sought to find out specifically whether the sequence promotes an understanding of the relative sizes of unit fractions.

In South Africa, language proficiency is acknowledged to be a key contributing factor to students' continued poor performance in mathematics (Graven & Venkat, 2017). The issue of the language of instruction affects access to mathematical knowledge for the majority of students. Policy advocates for mother tongue instruction, particularly in the early years, and allows schools to select which of the 11 official languages to use as the language of learning and teaching (LoLT, Department of Basic Education, 2010). English, however, remains overwhelmingly the preferred LoLT, with Afrikaans a distant second, while the remaining 9 indigenous languages together account for the LoLT of less than 10% of students, despite being the native languages of 82.8% of all students (DBE, 2010; Robertson & Graven, 2015). These young students are thus faced with the "dual burden … [of] mastering their LoLT while at the same time gaining epistemological access to mathematics through the LoLT" (Robertson & Graven, 2015, p. 286).

This was the reality of the students participating in this research in the Eastern Cape. The vast majority of the 105 Grade 3 students who participated in the implementation of the Fractions as Measure instructional sequence across five lessons were isiXhosa-

speakers, who were learning in either English (two classes of 35 students) or Afrikaans (one class of 35). We were mindful of this 'dual burden' when adapting the Fractions as Measures sequence to this context. Vygotsky (1978) speaks of the transition learners must make from spontaneous everyday language to scientific language and the importance of the role of a mediator in moving from spontaneous to scientific concepts. The story, the activity sequence, and the teacher serve as mediators in supporting this transition. In our experience we saw the support of a story driving mathematical activities, as especially important in our second language learning context because it supported classroom talk and the use of exploratory language. This was important for enabling students to develop meaningful understanding of scientific concepts that students were learning about in a second language. This paved the way for better access to the meaning of the dense and structurally complex language of mathematics (Hammill, 2010). The Fractions as Measures instructional sequence was well suited to providing resources for addressing these complexities meaningfully in the classroom. The story through which mathematics was to be introduced allowed for adaptations that leveraged and increased student talk, while maintaining the coherent storyline of students' mathematical learning. This supported later development of student fluency in formulating explanations of generalised mathematical relationships by working through the imagery in the story, and using the increasingly abstract ways to symbolise fraction quantities (concluding with conventional $5/6 < 1 < 6/5$).

The students were all familiar with the well-established social practice of engaging with stories in the context of literacy and language teaching. They all demonstrated an appropriate everyday command of the LoLT, but, concurring with national data, teachers indicated that many did not read at a grade-appropriate level. In these classrooms, it was therefore important to work on mathematical ideas without reliance on students reading a text or working from written worksheets. Storytelling in the context of the Fractions as Measures sequence provided a suitable means of conveying and negotiating the mathematical questions and ideas that we wanted students to consider.

While discussing stories is a common literacy teaching practice in South Africa, using stories as instructional starting points for teaching specific mathematical ideas in a classroom setting is certainly not part of typical mathematics teaching (see Hoadley, 2007 for discussion of primary mathematics pedagogy in South Africa). Bringing these two together provided a space where students could engage in a familiar story discussion space with everyday language around complex mathematical ideas including the relative size of unit fractions and the concept of fraction as measure.

To illustrate the reasoning this discussion space made available, we provide examples of students reasoning about the relative sizes of the 'smalls' (unit fractions of the stick) that were typical during whole class discussions in late classroom sessions. Students used colourful straws and cut these to create smalls up to small of ten, in a process of trial and error. When asked to compare lengths of the smalls they had created, they were able to respond with explanations. Two examples of student explanations are given below:

Student 1:	Small of nine is smaller [than small of three] because it [nine] is a bigger number and it [small of nine] must fit nine times [that is, more times onto the stick].
Student 2:	A small of two is bigger [than small of five] because a small of two fits in [the stick] two times and a small of five fits in [the stick] five times.

Students had many opportunities to informally construct such explanations. They were consistently asked to predict the size of the next small they were to create (i.e., *Will small of 3 be longer or shorter than small of 2 that you just made?*). They were also asked to

collaborate on formulating and writing down explanations for such comparisons in small groups. As illustrated above, students did not base their reasoning on comparing the lengths of physical rods. Instead, they called on the imagery of iterating specific smalls along the stick when measuring it (e.g., the image that "small of two fits in two times"). In this way, they came to think about unit fractions as quantities that can be compared and ordered meaningfully. Importantly, they linked these discussions to the kinds of thinking with which the story characters engaged in addressing genuine problems they faced.

To support the students' access to these mathematical ideas, we needed to construct the story so that the language would not present a barrier to students' understanding and active participation. We first adapted the context of the story so that it happened in an African village. We replaced all names and native words in the original story with more familiar sounding words and isiXhosa names to support students' ease of comprehension. Such modifications were possible, and indeed expected by initial designers. They maintained coherence with the designed sequence in that (a) the time and place in which the story happened provided a rationale for use of informal units of measure, (b) familiar names were used to support students' identification with the story characters, and (c) adaptations were made to avoid break-downs in the story that use of new or incomprehensible names, words, or situations could generate.

During storytelling episodes, we intentionally used voice, gesture, and visual aids to support students in making sense of spoken language. The story was told using props, miming the actions of the main characters, and including a student to act as the child character in the story. We highlight the dialogic nature of this practice and contrast it with 'story-reading'. While not a focus of this paper, students showed impressive gains in their understanding of the inverse order relation of unit fractions in a post assessment.

Our experiences of the Fraction as Measure sequence, combined with the student improvements on post assessments led us to conclude that student meaning making showed strong coherence with: everyday experiences of measuring and the relative size of objects, the story and its problem, and mathematical coherence in understanding the relative size of unit fractions. Through the story, the teacher supported students in engaging meaningfully with the mathematical ideas that were the aim of the sequence. This resulted in increased overall classroom communication, richer responses from more students, and deeper engagement with the mathematical ideas.

Summary and Conclusions

We opened the conversation about functions of stories within instructional sequences that are aimed at guiding students' reinvention of key mathematical ideas. We documented how several of these functions relate to RME heuristics for setting viable instructional starting points. We stress that instructional resources should be designed as *teachers' resources* (Visnovska & Cortina, 2018), and thus allow for and support reasoned adaptations by teachers and others who use them. This paper illustrates how stories can be designed in this way and accompanied by the rationale for specific design decisions.

Sleep (2012) refers to the coherence and connectedness in how mathematics is being taught, or sequenced in curricular documents, as a mathematical storyline. We fully agree that such coherence in design is essential if our goal is for the students to experience the mathematics they learn as coherent. We would also like to extend this argument and point out that when a coherent mathematical storyline is combined with a coherent take on experiences that generate the need for intended mathematics, the classroom-based reinvention of mathematics can become a mesmerising storytelling journey.

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