



eNICLE Grade 1 and 2 Teacher Development Programme

Session Seven Teacher Handbook

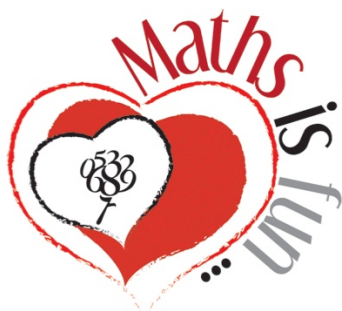
Name

School

DECLARATION

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Re-focus: eNICLE key ideas

This eNICLE programme is based on a number of key ideas (presented in Session Two). We are now more than halfway through the programme. It is useful to re-focus on which ideas we have covered.

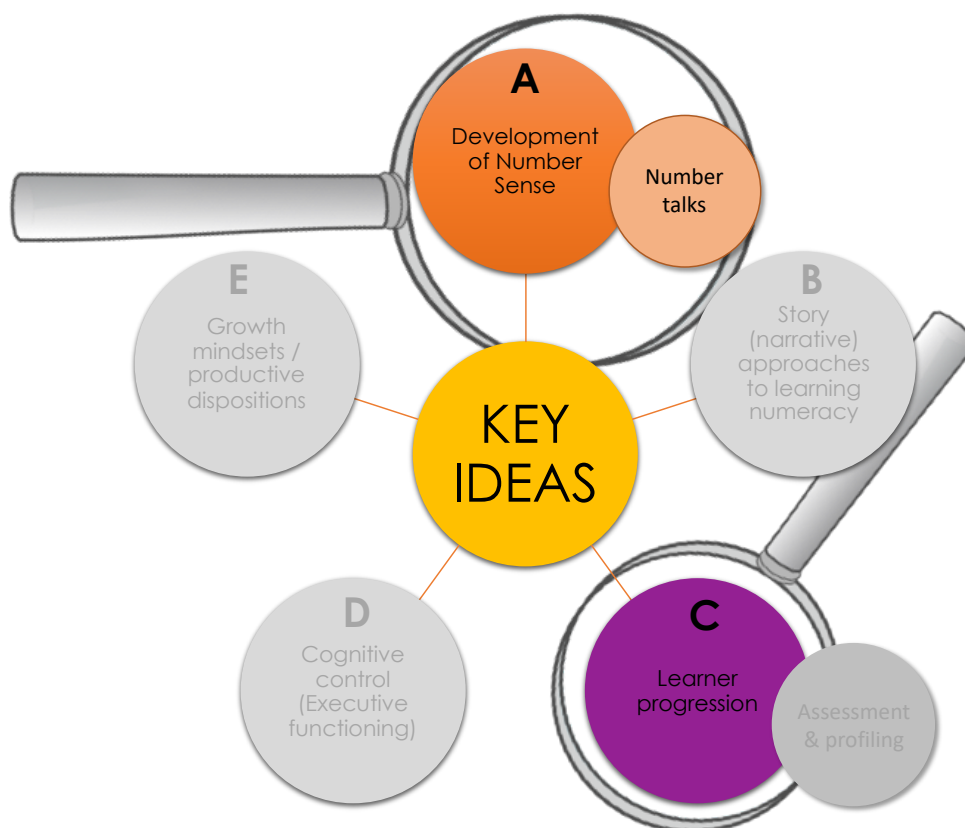


Figure 1: eNICLE programme – re-focussing on key ideas

So far in the programme, we have had sessions and provided resources for:

Key Idea	Description of activities	Session Number
A	Part-part-whole model Subitising (dot patterns) 5 and 10-frames	4 5 6
B	Monkeys in the Trees / Fire & Sticks stories	3 and 4
C	Progression and assessment ideas	2 and 3
D	Cognitive control activities	1 to 4
E	Growth Mindset posters and related activities	1 to 4

Re-focus: Counting and early arithmetic strategies



As discussed in Session Two, this eNICLE programme highlights progression in the four elements listed below but with a key focus on counting and early arithmetic strategies:

- Number word sequencing
- Number identification
- **Counting and early arithmetic strategies**
- Conceptual place value

We focus on these elements when adding numbers up to 20, because children progress through a well-researched sequence of:

- count all
- count on from the first number
- count on from the larger number
- using known facts and deriving number facts.

Children can be taught to progress through this sequence.

Re-focus: Moving from 'Calculating by Counting' to 'Calculating by Structuring'



Our focus since session four has been on progressing learners to **Level 3** (*Calculation by counting*) and **Level 4** (*Calculation by structuring*).

The diagram below shows how this fits into counting learning and what you can use to support learning.

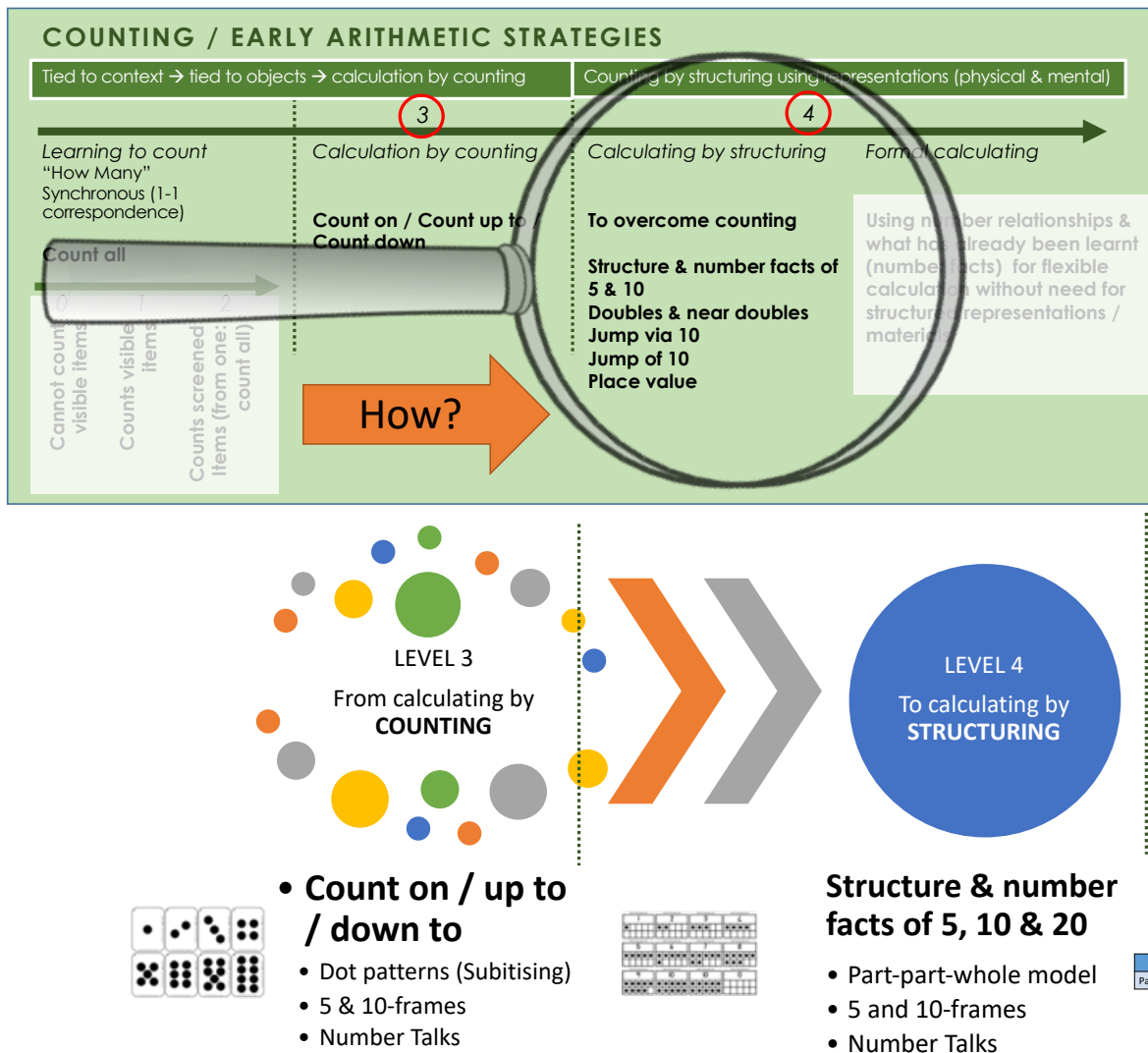


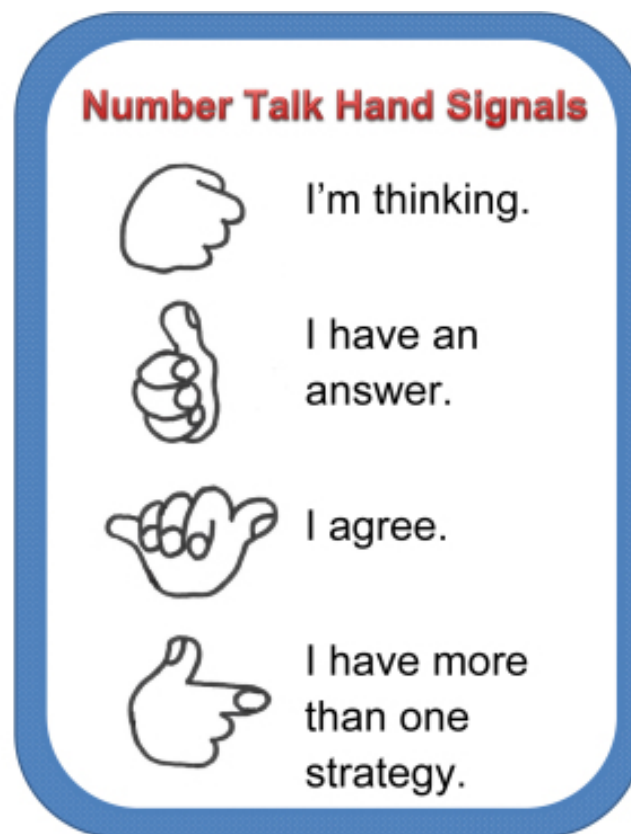
Figure 2: eNICLE programme - learner progression

To support learning at Level 3, we used dot patterns and 5 and 10-frames.

At level 4, the child begins to use strategies that work with the structure of numbers. These are strategies that **break down numbers (decomposing)**, **reorganise them and then build them up again (recomposing)** in some way that is not related to counting up or down. To support learning here we have used the part-part-whole model and 5 and 10-frames.

Expanding on Number talks: Hand Gestures / Signals

In our Number Talk sessions, we have been using these hand signals as an alternative to the common practice of putting up a hand.



	A closed fist indicates that the child is thinking.
	A single thumb indicates that the child has one method or strategy to share.
	Shaking the thumb and small finger, shows agreement with a method that someone else has shared.
	A thumb and another finger show that the child has more than one method or strategy to share.

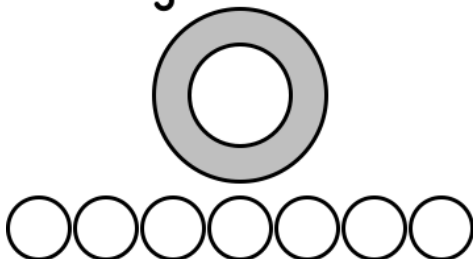
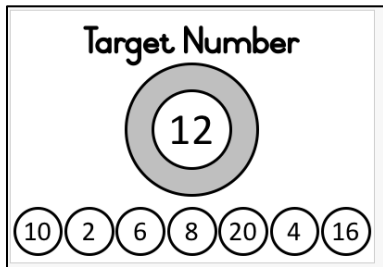
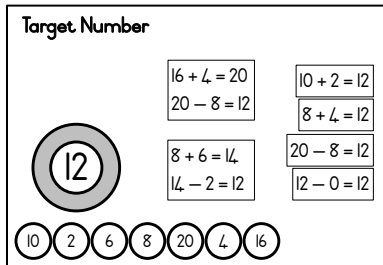
Benefits of using hand signals in Number Talk sessions

- They are quieter than putting up a hand and clicking fingers. This contributes to a quiet space for others to think.
- If used exclusively for Number Talks, they are something special that are not used in other lessons.
- The teacher can clearly see what a learner is ready to offer.

In today's Number Talk session, we offer some prompts that add variety to your Number Talk sessions. They are not based on any particular model or representation.

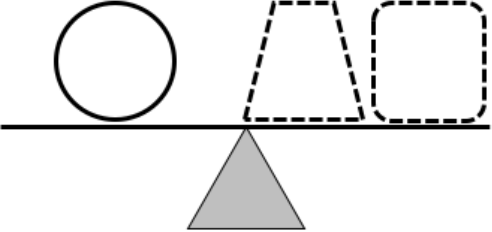
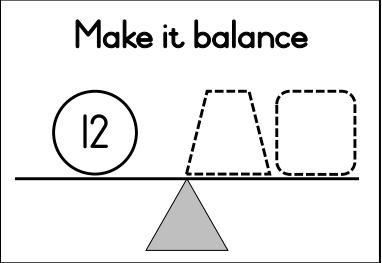
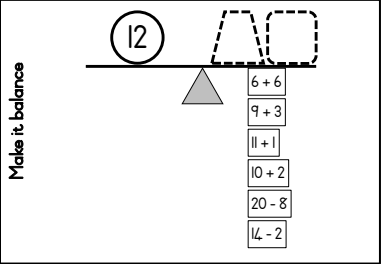
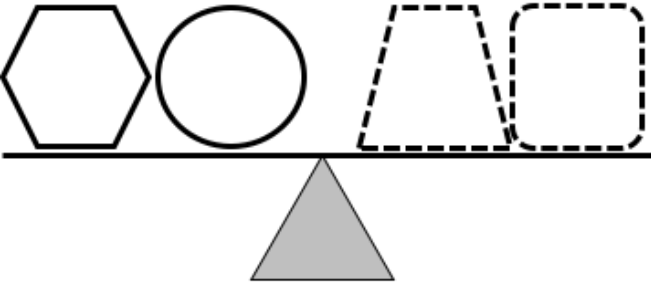
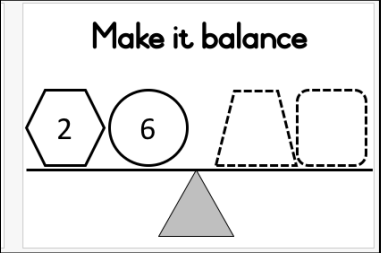
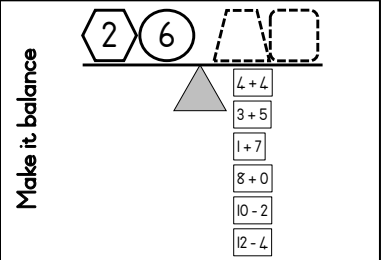
To use these prompts:

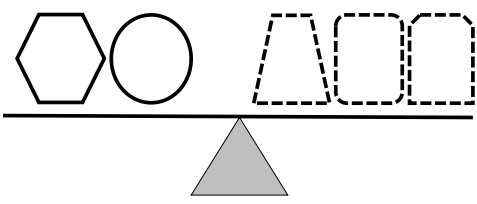
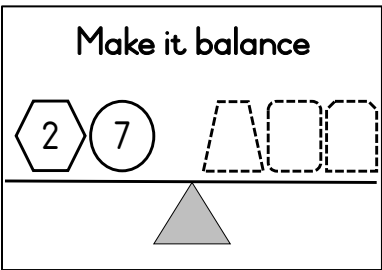
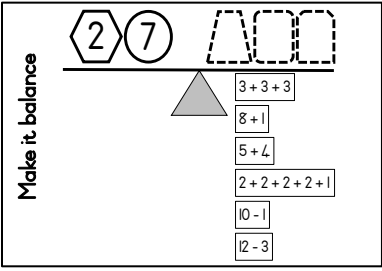
- Select a number talk prompt from the templates below.
- Follow the instructions in the second column to prepare for the talk.
- Draw the template on the board or on flipchart paper.

	Template and suggested mathematical focus	How to use for a Number Talk																																													
1	Target Number  Choose a target number for the number range you are working with and 7 numbers that combine to make the target number. Mathematical focus: • Addition and / or subtraction in any number range you are working with • Multiplication by 2, 3, 5 and 10 if the numbers are carefully selected (see 22 in the Other Examples to the right)	• Write the target number in the centre circle. • Write in 7 numbers that the learners can use to make the target number. • Include some larger numbers so that subtraction options are possible. • Record learners' contributions. Example Target Number  Sample contributions Target Number  Other examples<table><tr><th>Target</th><th colspan="7">Numbers</th><th>Operation</th></tr><tr><td>11</td><td>9</td><td>2</td><td>10</td><td>1</td><td>13</td><td>12</td><td>20</td><td>+ -</td></tr><tr><td>13</td><td>10</td><td>3</td><td>20</td><td>7</td><td>6</td><td>16</td><td>19</td><td>+ -</td></tr><tr><td>17</td><td>9</td><td>10</td><td>6</td><td>7</td><td>23</td><td>20</td><td>3</td><td>+ -</td></tr><tr><td>22</td><td>11</td><td>2</td><td>24</td><td>27</td><td>5</td><td>17</td><td>10</td><td>+ - x</td></tr></table>	Target	Numbers							Operation	11	9	2	10	1	13	12	20	+ -	13	10	3	20	7	6	16	19	+ -	17	9	10	6	7	23	20	3	+ -	22	11	2	24	27	5	17	10	+ - x
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22	11	2	24	27	5	17	10	+ - x																																							

	Template and suggested mathematical focus	How to use for a Number Talk																																																												
2	Make Choose a target number for the number range you are working with and 10 numbers that combine to make the target number. The numbers are in an ordered pattern and are easier to choose from. Mathematical focus: • Addition and / or subtraction in any number range you are working with • Multiplication by 2, 3, 5 and 10 if the numbers are carefully selected (see the Other Examples to the right)	• Write the target number in the dotted box. • Write in 10 numbers that the learners can use to make the target number. • Include some larger numbers so that subtraction options are possible. • Record learners' contributions. Example Make 10 Sample contributions Make 10 8 + 2 = 10 6 + 4 = 10 7 + 1 + 2 = 10 6 + 3 + 1 = 10 5 + 3 + 2 = 10 8 + 4 = 12 12 - 2 = 10 12 - 2 = 10 Other examples <table><tr><th>Make</th><th colspan="10">Numbers</th><th>Operation</th></tr><tr><td>12</td><td>3</td><td>4</td><td>9</td><td>8</td><td>15</td><td>16</td><td>20</td><td>1</td><td>11</td><td>23</td><td>+ - x</td></tr><tr><td>14</td><td>24</td><td>3</td><td>4</td><td>7</td><td>2</td><td>10</td><td>21</td><td>20</td><td>6</td><td>8</td><td>+ - x</td></tr><tr><td>16</td><td>8</td><td>2</td><td>9</td><td>7</td><td>20</td><td>4</td><td>6</td><td>10</td><td>26</td><td>24</td><td>+ - x</td></tr><tr><td>24</td><td>12</td><td>2</td><td>4</td><td>10</td><td>20</td><td>26</td><td>28</td><td>8</td><td>30</td><td>6</td><td>+ - x</td></tr></table>	Make	Numbers										Operation	12	3	4	9	8	15	16	20	1	11	23	+ - x	14	24	3	4	7	2	10	21	20	6	8	+ - x	16	8	2	9	7	20	4	6	10	26	24	+ - x	24	12	2	4	10	20	26	28	8	30	6	+ - x
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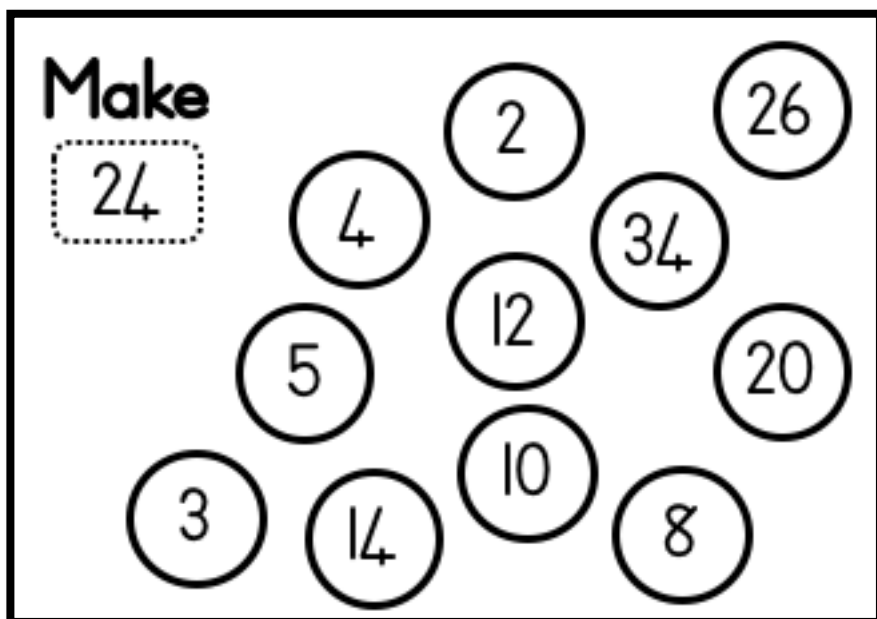
	Template and suggested mathematical focus	How to use for a Number Talk																																																				
3	Make Choose a target number for the number range you are working with and 11 numbers that combine to make the target number. The numbers are in a scattered pattern, making them slightly harder to choose from. Mathematical focus: • Addition and / or subtraction in any number range you are working with • Multiplication by 2, 3, 5 and 10 if the numbers are carefully selected (see the Other Examples to the right)	• Write the target number in the dotted box. • Write in 11 numbers that the learners can use to make the target number. • Include some larger numbers so that subtraction options are possible. • Record learners' contributions. Example Make 15 Sample contributions Make 15 9 + 6 = 15 10 + 5 = 15 7 + 3 + 5 = 15 8 + 6 + 1 = 15 20 - 5 = 15 25 - 10 = 15 Other examples <table><tr><th>Make</th><th colspan="11">Numbers</th><th>Operation</th></tr><tr><td>18</td><td>9</td><td>2</td><td>10</td><td>8</td><td>14</td><td>4</td><td>12</td><td>6</td><td>1</td><td>27</td><td>5</td><td>+ - x</td></tr><tr><td>19</td><td>10</td><td>21</td><td>23</td><td>9</td><td>20</td><td>8</td><td>5</td><td>11</td><td>4</td><td>1</td><td>2</td><td>+ - x</td></tr><tr><td>24</td><td>12</td><td>2</td><td>4</td><td>10</td><td>5</td><td>20</td><td>8</td><td>3</td><td>14</td><td>34</td><td>26</td><td>+ - x</td></tr></table>	Make	Numbers											Operation	18	9	2	10	8	14	4	12	6	1	27	5	+ - x	19	10	21	23	9	20	8	5	11	4	1	2	+ - x	24	12	2	4	10	5	20	8	3	14	34	26	+ - x
Make	Numbers											Operation																																										
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24	12	2	4	10	5	20	8	3	14	34	26	+ - x																																										

	Template and suggested mathematical focus	How to use for a Number Talk
4	Make it balance  Choose one number for the left. Learners find 2 numbers on the right that will make the scale balance. Mathematical focus: • Work with one number and find 2 other numbers that equal the number on the left • Addition and / or subtraction in any number range you are working with • Multiplication by 2, 3, 5 and 10 if the numbers are carefully selected	• Write a target number in the shape on the left e.g. 12. • Record learners' contributions for the two dotted shapes on the right. Example  Sample contributions 
5	Make it balance  Choose two numbers for the left. Learners find 2 numbers on the right that will make the scale balance. Mathematical focus: • Work with two numbers and find 2 other numbers that equal the two numbers on the left • Addition and / or subtraction in any number range you are working with • Multiplication by 2, 3, 5 and 10 if the numbers are carefully selected	• Write two numbers in the shapes on the left e.g. 2 and 6. • Record learners' contributions for the two dotted shapes on the right. Example  Sample contributions 

	Template and suggested mathematical focus	How to use for a Number Talk
6	Make it balance  Choose two numbers for the left. Learners find 3 numbers on the right that will make the scale balance. Mathematical focus: • Work with two numbers and find 3 other numbers that equal the two numbers on the left • Addition and / or subtraction in any number range you are working with • Multiplication by 2, 3, 5 and 10 if the numbers are carefully selected	• Write two numbers in the shapes on the left e.g. 2 and 7. • Record learners' contributions for the two dotted shapes on the right. Example  Sample contributions 

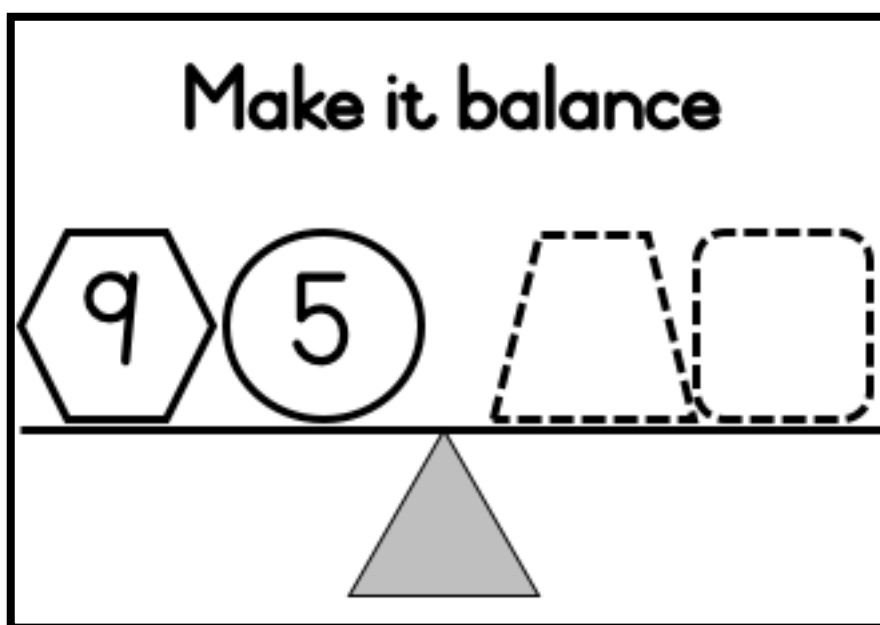
Today's Number Talk(s)

Talk One



Use any combination of addition, subtraction and multiplication to make 24. You may combine 2 or more numbers.

Talk Two



Work out the value of the numbers on the left. Then find other combinations of two numbers that are equal to this number, using any operation you like.

Linear representations of number: Bead strings to structured number lines

The regular use of a number line can help learners to form a *mental number line*. This can help learners to calculate mentally.

Research has found that many learners cannot use a number line effectively. This is why it is important to introduce number lines as early as possible.

One way to introduce number line work is to start with **bead strings**. Then connect these to a **counting line** and then move on to a **number line** (see Figure 3 below).

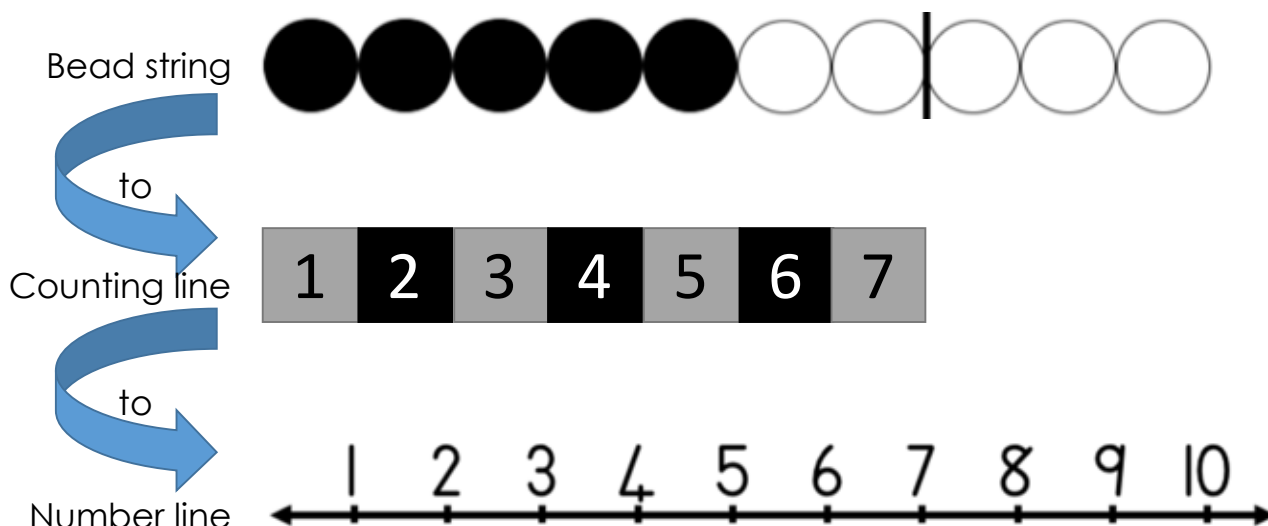


Figure 3: Introducing linear representations

Bead strings

Visual and concrete items that children can touch, such as a bead string, help children to link counting to movement. The beads are blocked in groups of fives or tens. They use colour and groupings of beads to make the structure of number more visual for children.

Resources like this can help children to develop a sense of number order and number pattern. Using the string helps children to picture a number line in their minds.

Bead strings can be used for:

- Making / structuring numbers up to 10 and 20
- Skip counting in 5s and 10s
- Early addition and subtraction
- Counting on / counting back
- 1 more / 1 less (2 more / 2 less etc)
- Conceptual place value

NOTES

Counting lines or number tracks

The first way that children come across “number lines” is as a line or row of numbers or objects for counting. This is known as a “counting line”. This is an excellent way to support learning to count. It can also be used for learning to add and subtract small numbers.

Figure 4 is an example of a counting line next to a bead string. The counting line is made of numbered blocks. The first block is labelled with 1 which indicates that objects are being counted.



Figure 4: Bead string and counting line

Over time, you can start to use a more abstract counting line. This is also known as a *numeral (number) track*. This is a printed set of objects that can be counted (Figure 5).



Figure 5: Numeral (number) track

From counting lines to structured number lines

The counting lines above are very useful, but they are **NOT** number lines. You need to move carefully from bead strings or counting lines to number lines, because learners do find them challenging.

Number lines can go beyond counting individual objects. They can be used to **measure** from a fixed point. **Fractions** can be shown on the number line but cannot be shown on a counting line.

How to progress from a counting line to a number line

It is important to help the learners to understand that the number marker on a number line indicates where one object finishes (in this case one bead from the string).

For example, the numeral 1 on the number line (Figure 6 below), shows where bead one finishes.

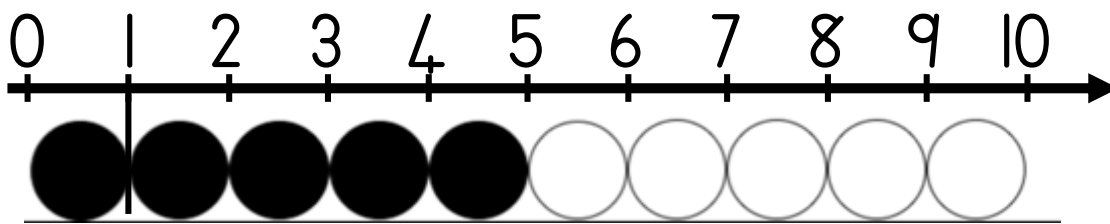


Figure 6: Example of connection between number line and bead string

Moving onto measuring

The number line in Figure 7 below shows the length of the pencil is $4\frac{1}{2}$ beads (in this case).

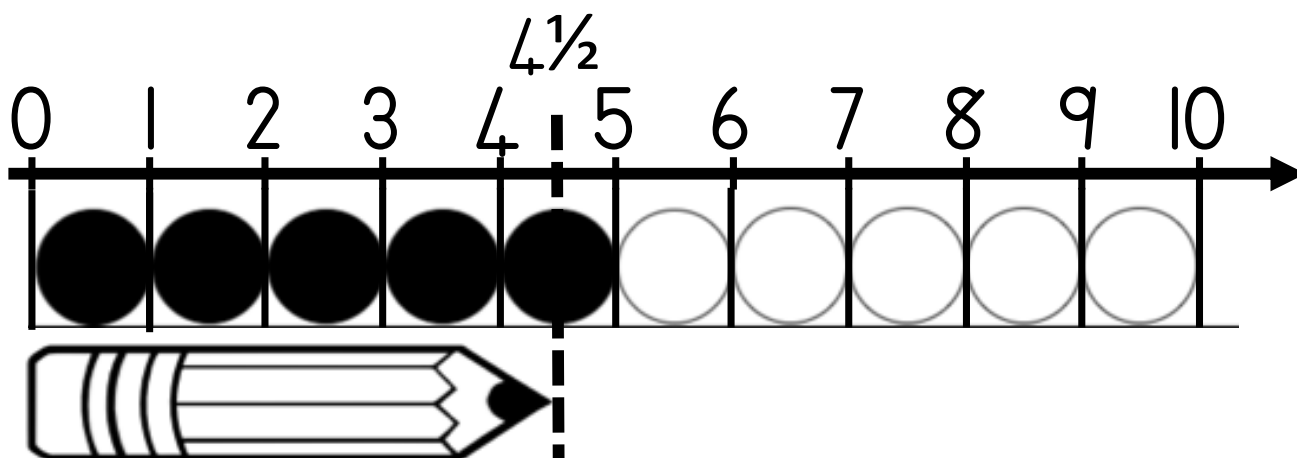


Figure 7: Measuring with a bead string

The number line can be connected to measuring tapes and rulers later. This can be seen in the number line in Figure 8 below, where the pencil is $4\frac{1}{2}$ units.

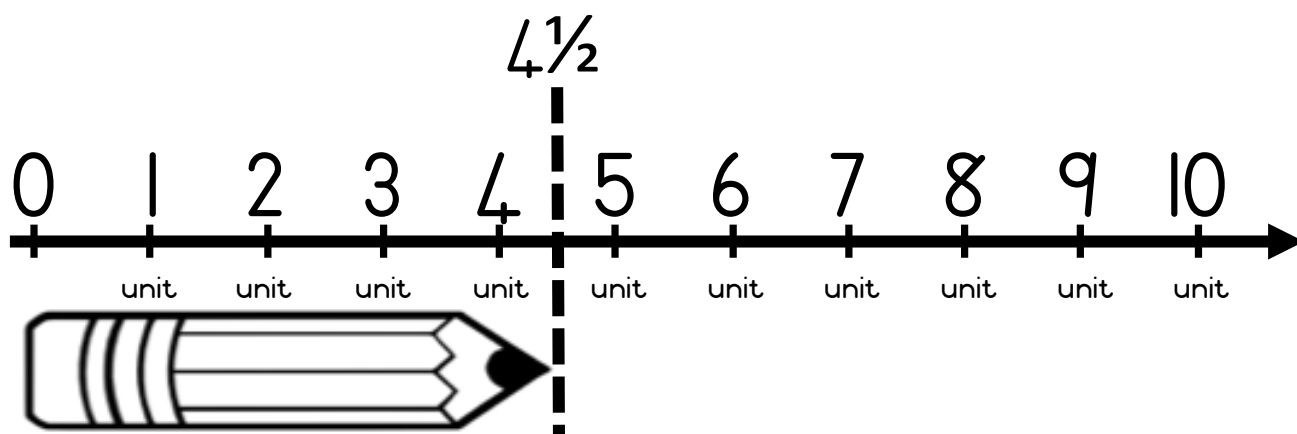


Figure 8: Measuring using a number line

Reflection Activity



Get into groups of 3-5 or work with the teachers at your table.
Reflect on your use of the following activities from the last session.

Thinking about Number Talks and 5/10-frames

1. Did you do any **Number Talks** with your class this month?
If so, what did you do?
How did you find the experience?
Did you learn anything about how your learners think?
2. Reflect on your experiences of using **5 and 10-frames**.
3. Did you make any adaptations to the activities? If so, show / explain to the members of your group.
4. What were the learner experiences of the activities?

NOTES:

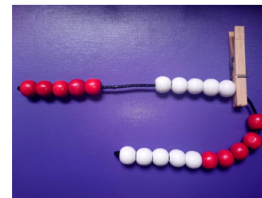
This section provides details of the activities that are presented in this workshop.
Every workshop will have a similar section so you know where to look in the handbook.

Resources

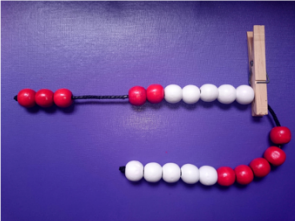
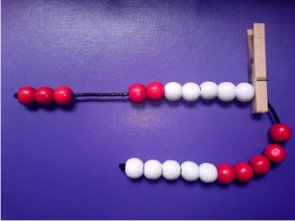
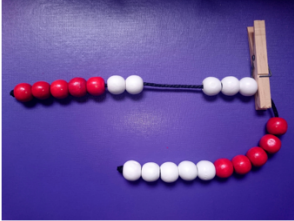
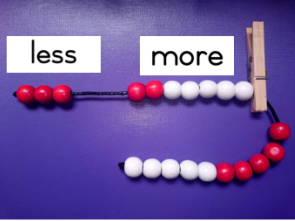
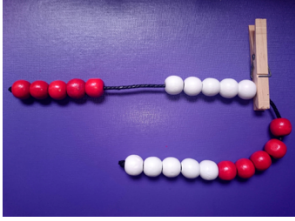
Bead string activities

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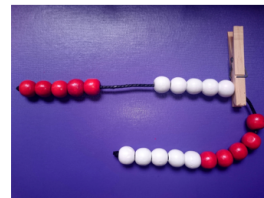
Using bead strings for Number Sense



Bead strings can be used for developing number sense in different ways. Work in the range 1 to 10 first and then extend it to 20.

Number recognition Move the beads to show a chosen number or show the beadstring and ask what the number it shows. Dice can be thrown to choose a random number.	
What's the same, what's different? Look at two bead strings and say what is the same and what is different about the two strings. In this example we can see 3 and 7 on the left string and 7 and 3 on the other. They both show 10. One shows 10 as 3 and 7; the other shows 10 as 7 and 3.	 and 
More and less Use the more/less cards from the story books. Arrange the beads so that there are more on one side. Ask learners to place the correct cards under the beads and explain why they think this.	
1 more or 1 less The first string shows 4, the second shows 5. The movement associated with moving the beads helps learners to see and feel the effect of adding/making more or of subtracting/making less.	 to 
Counting vertically Turn the bead string and use it vertically. This will prepare learners for reading scales on a measuring jug or values on a thermometer. Showing learners, the same resource in a different position can help them to understand that counting vertically is the same as counting horizontally.	

Using bead strings for Number Sense continued



Bigger and smaller than

Use one bead string to represent a value. Ask children what value is shown. This example shows 9.

Ask children to show a number bigger or smaller than the one shown on your string.

You could ask them to show 1 more, 5 more, 1 less, 5 less etc.

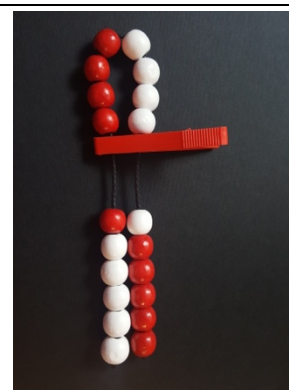


Doubling

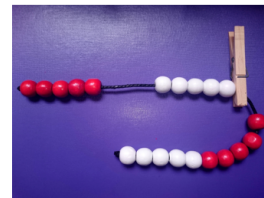
Hold the two ends together with the same number showing. This image shows double 6 at the bottom.

Learners can explain what they see by looking at the number of different coloured beads and working with the 5.

e.g. "I see 5 whites and 5 reds, that's 10. Add one more red and one more white. That's 12 altogether".



Using bead strings for Number Facts



When learners are used to working with bead strings, they can move onto using them to learn the number facts to 10 and 20.

Number facts for numbers up to 10

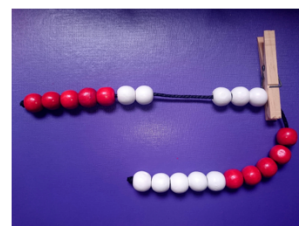
Use the first 10 beads of the string to show ways of making any number up to 10 using a mixture of red and white beads.

This first string shows 7 (5 red and 2 white) beads to represent $7 + 3 = 10$.

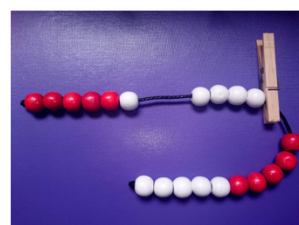
Moving one white bead to the right would then show $6 + 4 = 10$.

Children can explore all the ways of making 10 or any number up to 10 in this way.

$$7 + 3$$



$$6 + 4$$



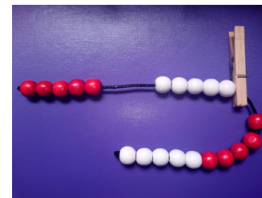
Number facts for numbers up to 20

Use the full bead string to work with numbers up to 20.

This example shows 11 (5 white, 5 red, 1 white) and 9 (5 red and 4 white).



Bead Strings and Ordinal Numbers



We suggest that activities 1, 2 and 3 should be done a few times over many days.

Activity 1: Bead strings

Mathematical object of learning: Working with ordinal numbers on the bead string	You need: Teacher and learner bead strings	Work with: We suggest working with smaller groups on the mat where learners show you the following on their own bead strings.
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- How many red beads are there here? [show the first red group of 5]
- How many white beads are there? [show the first white group of 5]
- Hold your bead string with the red beads starting in your left hand.
- Using a pattern of prompts, first ask the group a question, then ask individual learners “**how they know**” type questions (see more about this on the next page).
- We suggest working from left to right on the string.

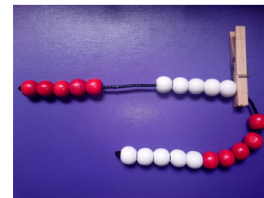
Here are some possible prompts.

• <i>T to group:</i> Show me the 2nd bead. • <i>T to a learner:</i> How do you know that's the 2nd bead?	• <i>T to group:</i> Show me the 5th bead. • <i>T to a learner:</i> How do you know that's the 5th bead? • <i>T to group:</i> Any other ways you know that is the 5th?
• <i>T to group:</i> Show me the 3rd bead. • <i>T to a learner:</i> How do you know that's the 3rd bead?	• <i>T to group:</i> Show me the 6th bead. • <i>T to a learner:</i> How do you know that's the 6th bead? • <i>T to group:</i> Any other ways you know that is the 6th?
• <i>T to group:</i> Show me the 4th bead. • <i>T to a learner:</i> How do you know that's the 4th bead? • <i>T to group:</i> Any other ways you know that is the 4th?	• <i>T to group:</i> Show me the 7th bead. • <i>T to a learner:</i> How do you know that's the 7th bead? • <i>T to group:</i> Any other ways you know?
• <i>T to group:</i> Show me the 10th bead. • <i>T to a learner:</i> How do you know that's the 10th bead? • <i>T to group:</i> Any other ways you know? (can relate 5 & 5 fingers on our hands being 10 fingers)	• <i>T to group:</i> Show me the 9th bead. • <i>T to a learner:</i> How do you know that's the 9th bead? • <i>T to group:</i> Any other ways you know?
• <i>T to group:</i> Show me the 8th bead. • <i>T to a learner:</i> How do you know that's the 8th bead? • <i>T to group:</i> Any other ways you know?	• <i>T to group:</i> Show me the 11th bead. • <i>T to a learner:</i> How do you know that's the 11th bead? • <i>T to group:</i> Any other ways you know?

Moving beyond 10

For those learners who have started using the structure of 5 and 10 in answering these questions shift to the 10 to 20 range e.g. show me 15; 20; 14; 16; 19 etc.

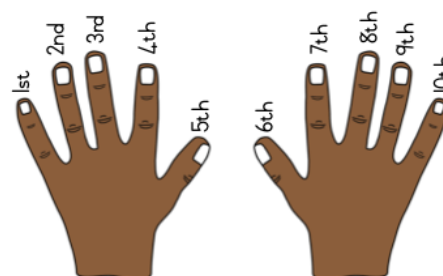
Bead Strings and Ordinal Numbers continued



Activity 2: Relating the above activity to our hands

Mathematical object of learning: - Working with ordinal numbers - Asking learners to explain their thinking with "how do you know?" questions	You need: - Just learners' hands - Coloured stickers for struggling learners	Work with: We suggest working with smaller groups on the mat.
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- Learners place both hands in front of them as shown here.
- Get children to read the order of the fingers from left to right as the 1st finger; 2nd finger; 3rd finger; 4th finger etc. up to the 10th finger.
- Use the same questioning sequence from the previous page but end at 10.
- If you want to go on to 11, 15 etc. they could include their toes.



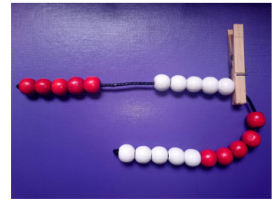
Example of questioning sequence

- T to group:* Wiggle your 2nd finger.
T to a learner: How do you know that's your 2nd finger?
- T to group:* Wiggle your 5th finger (yes, it's a thumb but a thumb is also a finger).
T to a learner: How do you know that's your 5th finger?
- T to group:* Any other ways you know that's the 5th finger? (e.g. answer because there are 5 fingers on each hand and this is the last finger on my left hand)
- And so on...

For learners that really struggle:

- To start with, write the numbers 1 to 10 on small coloured stickers.
- Stick these onto the learners' fingers.
- Try to progress them from relying on the numbers after a time.

Why ask “How do you know?” questions?



The question “*how do you know?*” focuses on the learners' development of mathematical and reasoning language. Learners must describe why they think that it is, for example, the 6th bead or finger.

Many learners might just point to their strings by counting in ones and say: ‘*Because look... one, two, three, four, five, six*’.

Other learners might begin to see the structure of 6 as $5 + 1$.

For example: ‘*Because 6 is 5 red and 1 white*’ or, ‘*Because the 6th is the one after the 5th and I know the last red one is the 5th*’.

Learners don’t always explain their thinking this clearly. You can rephrase their explanations and share what they ‘notice’ with other learners. This shows other learners how seeing and using the structure is quicker than counting by ones.

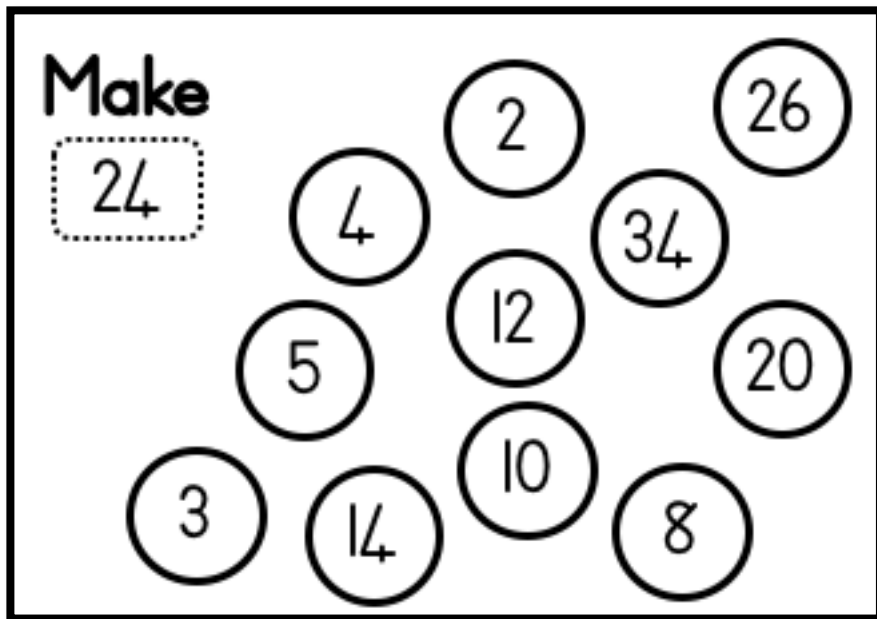
If learners do not come up with a ‘*non-counting by ones*’ way of finding the 6th; 7th; 4th; 10th etc. you can encourage them to find quicker ways to know without counting, for example by noticing and emphasising which beads are the 5th and 10th.

NOTES:

Today's number talk discussion

Talk One

Today, we talked about using the numbers in circles to make 24.



Here are some possible combinations that make 24:

Addition	Subtraction	Multiplication	Combinations
$10 + 14$	$34 - 10$	12×2	$(10 \times 2) + 4$
$10 + 12 + 2$	$26 - 2$	8×3	$(20 - 8) + 12$
$8 + 8 + 8$			$(14 \times 2) - 4$
$20 + 4$			
$10 + 10 + 4$			
$12 + 12$			

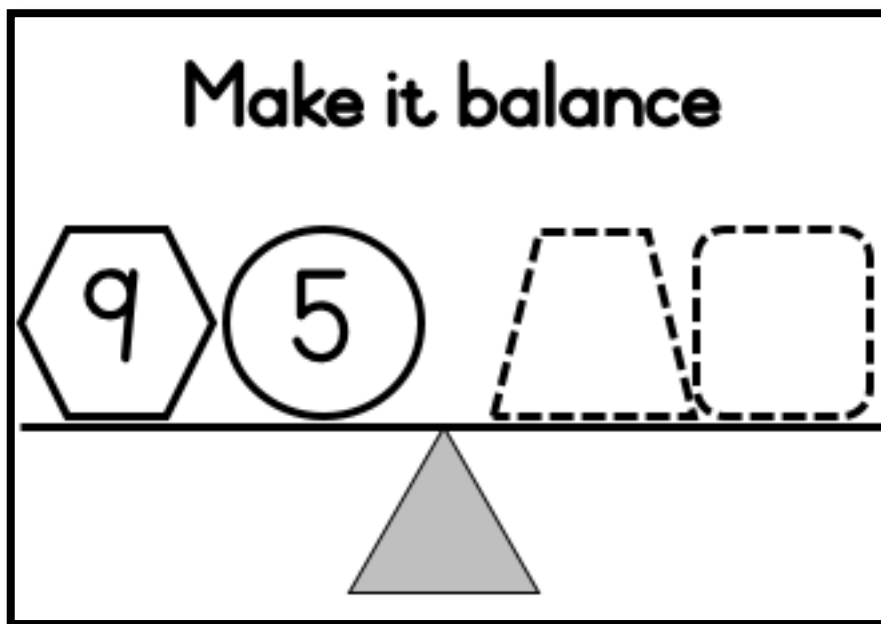
Can you think of any others?

Reflection questions:

- Do any of the addition solutions relate to a multiplication solution?
- How could you explain to learners the use of brackets?

Talk Two

Today, we talked about using the sum of $9 + 5$ to make other sums that equal 14.



Here are some possible combinations that make 14 for addition and subtraction:

Addition	Subtraction
$10 + 4 / 4 + 10$	$24 - 10$
$8 + 3 + 3$	$16 - 2$
$4 + 4 + 4 + 2$	$18 - 4$
$10 + 2 + 2$	$20 - 6$
$8 + 6 / 6 + 8$	$34 - 20$
$7 + 7$	
$13 + 1 / 1 + 13$	
$12 + 2 / 2 + 12$	
and so on...	and so on...

Can you think of any that might use multiplication?

Reflection questions:

- How could you encourage the learners to find all the combinations that add to 14? Think about helping them to see the pattern.
- How could you do the same thing with subtraction?