

eacher Handbook

Session Ter

eNICLE Grade 1 and 2 Teacher Development Programme

Name

School

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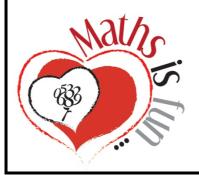
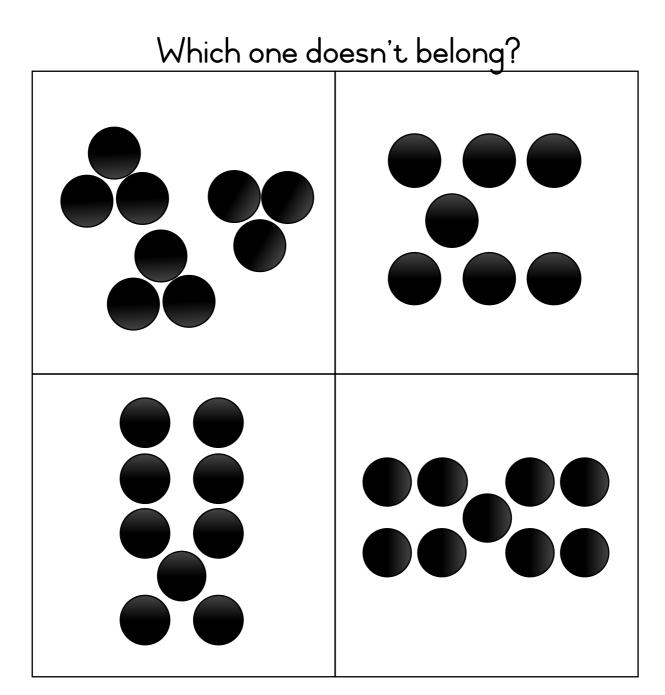


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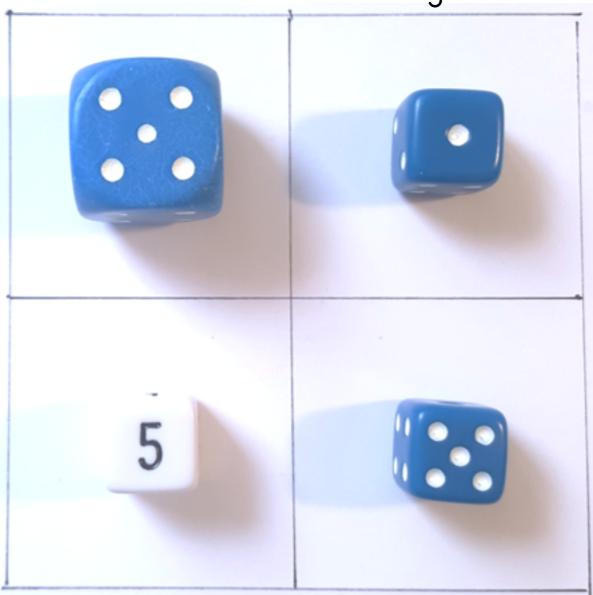
Today's Number Talk(s)

One



- Why?
- Try to think of at least **two ways** to justify which one doesn't belong.
- Think of more if you can.

Which one doesn't belong?



- Why?
- Try to think of at least **two ways** to justify which one doesn't belong.
- Think of more if you can.

Re-focus: Counting and early arithmetic strategies



As discussed in Session Two, this eNICLE programme highlights progression in the four elements listed below but with a key focus on counting and early arithmetic strategies:

- Number word sequencing
- Number identification
- Counting and early arithmetic strategies
- Conceptual place value

We focus on these elements when adding numbers up to 20, because children progress through a well-researched sequence of:

- count all
- count on from the first number
- count on from the larger number
- using known facts and deriving number facts.

Children can be taught to progress through this sequence.

Re-focus: Moving from 'Calculating by Counting' to 'Calculating by Structuring'



Our focus since session four has been on progressing learners to **Level 3** (Calculation by counting) and **Level 4** (Calculation by structuring).

The diagram below shows how this fits into counting learning and what you can use to support learning.

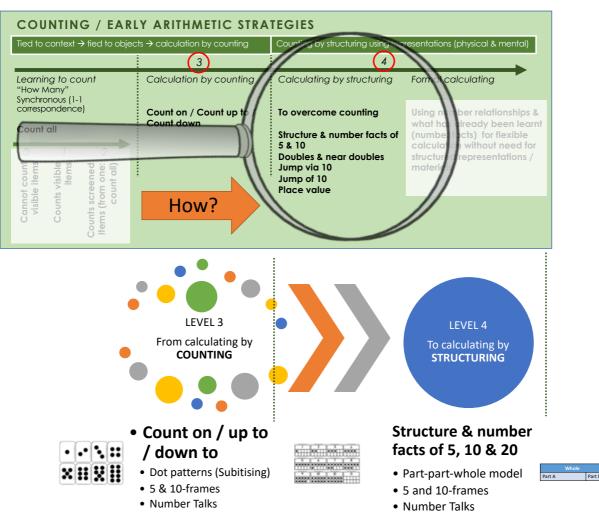


Figure 1: eNICLE programme - learner progression for counting / early arithmetic strategies

To support learning at Level 3, we use dot patterns and 5 and 10-frames.

At level 4, the child begins to use strategies that work with the structure of numbers. These are strategies that **break down numbers (decomposing)**, **reorganise them and then build them up again** (**recomposing)**. To support learning here we have used the part-part-whole model and 5 and 10-frames.

In the last three sessions, we have focused on the use of different types of **number lines** to calculate by structuring. We began with structured number lines, then moved to semi-structured number lines and finally, in this session to empty number lines.

Empty number lines for <u>calculating by structuring</u>

We have seen that there are many types of number lines:

- Empty (with no beginning or end points or other marks)
- Closed (with beginning and end points)
- **Open** (with a beginning point but no end point or vice versa)
- Structured (with well-defined partitions and labelled marks)
- Semi-structured (with some partitions and some missing labelled marks)

We have worked with **structured number lines** and semi-structured number lines. In this session we move on to working with **empty number lines**. An empty number line is a tool for **calculating by structuring**. It helps children to show their thinking when adding and subtracting numbers. It helps to develop their number sense and to help them record and explain their thinking.

An empty number line can be drawn by hand as an aid to calculating as shown in Figure 2 below.

- It is a single empty line to start with (step 1)
- As children solve a problem, they add numbers to it
- They can also show their thinking on the empty number line

Example

Here is a step-by-step example for 16 + 9

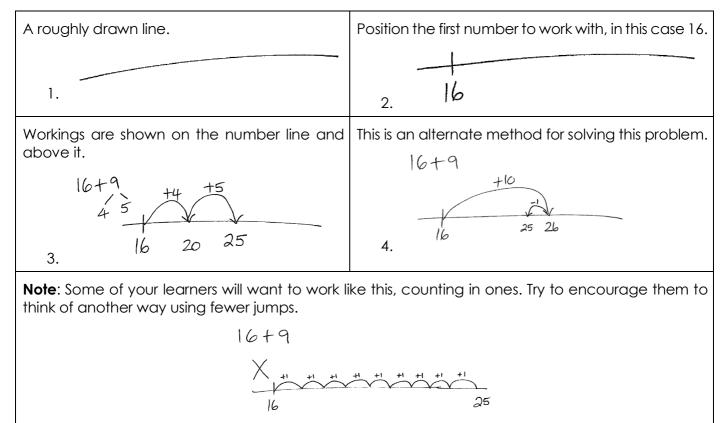


Figure 2: Solving 16 + 9 using an empty number line

Before you work with empty number lines

Make sure that you have done plenty of work with both structured and semi-structured number lines. The learners should be comfortable with working with these before doing any empty number line work. (See Sessions Eight and Nine Teachers Handbooks for activities.)

Learners need to have these in place before using empty number lines to solve problems:

- Know that they can put the larger number (as they do with counting on) first if it helps solve a problem
- Add / subtract a multiple of 10 to or from any two-digit number up to 100 e.g.
 Add 20 to 16 by counting on 2 tens
- Recall addition and subtraction facts for all numbers to at least 10 e.g.
 - Knowing that 7 can be: 6 and 1; 5 and 2; 4 and 3 and so on
- Know how many to get to the nearest 10 using their knowledge of bonds to 10 e.g.
 Know that 6 must be added to 34 to make 40

What else do children need to know?

Children need to practise these skills and become confident with them before adding or subtracting using an empty number line.

- Position a number on a semi-structured number line and explain their thinking as we did in Session 9.
- Jump to a number from zero in different ways e.g.
 - To jump to number 29, you could do two jumps of 10 and nine jumps of 1 OR
 - a jump of 20 and a jump of 9 OR
 - three jumps of 10 and a jump back of 1 OR
 - o a jump of 30 and a jump back of 1

Why work with empty number lines?

Working with empty number lines helps learners to:

- Develop good number sense
- Visualise a mental number line in their heads for calculating with 2-digit numbers (addition and subtraction, and later multiplication and division)
- Use and develop strategies such as:
 - Next friendly number (bridging through 10)
 - o Jumps of 10

South African Curriculum Documents

The Foundation Phase CAPS document highlights the use of number lines for performing calculations as techniques (methods or strategies) and for mental strategies.

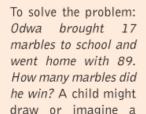
Empty number lines for calculating are used as examples in this extract from page 82 of the Foundation Phase Numeracy Handbook.

The Assessment Standard that deals with the *ordering and comparing* of numbers in each Grade, support the development of **Level 3** number sense through encouraging the development of the mental number line described in Unit 3. The way in which children will break down and build up numbers is guided by a number of things which include the actual numbers in the problem that they are solving and the operation that they are performing. A mental (or written) number line is crucial in this regard.

+60

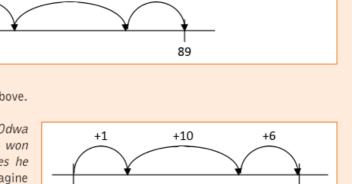
+3

17



number line and use it as illustrated above.

By contrast in solving the problem: Odwa brought 89 marbles to school and won another 17. How many marbles does he have now? A child might draw or imagine a number line and use it as illustrated alongside.



+9

 \rightarrow 63 + 7 \rightarrow 70 + 2 \rightarrow 72

106

In order to make a plan to solve these two problems the child would, in the first example, need to have a sense of the relative positions on the number line of the start and end points and then be able to count on in efficient ways, whereas in the second problem the child needs to have a sense of the starting value and must be able to break down the other number in response to the starting number in order to achieve the answer. Both solutions rely on the child's ability to order and compare numbers.

Figure 3: Extract from Foundation Phase Numeracy Handbook page 82

Strategies to use on the empty number line

Although there are many calculation strategies, in this session we discuss three strategies that are commonly used for mental calculations of addition and subtraction:

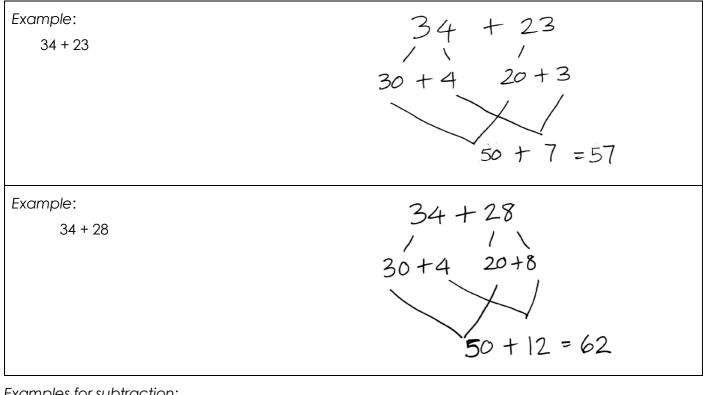
- 1. Splitting
- 2. Jumps of 10
- 3. Next friendly number (also known as bridging through 10)

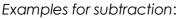
1. SPLITTING (DECOMPOSING)

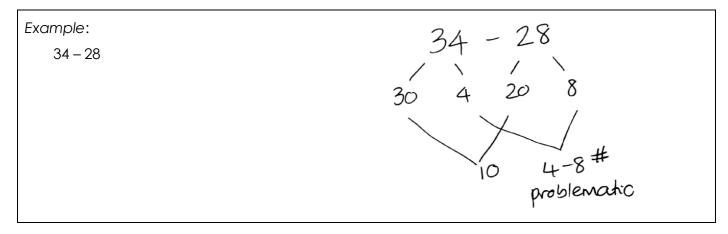
This is a strategy which learners develop once they start to learn about place value. The numbers are split into friendlier pieces such as hundreds, tens and units.

NOTE: Although this strategy is much more efficient than counting in ones or using tally marks, it is not so easy to work with as a mental strategy. It can also be problematic for subtraction (as seen in the example below).

Examples for addition:







2a. JUMPS OF TEN – ADDITION (without crossing a ten)

With this strategy, one number is kept whole and jumps of 10 and then units are added to it. It makes sense for the whole number to be the larger number.

Example:

23 + 34

Which is the larger number?	34 The sum can be re-written as 34 + 23
Count on in jumps of 10 and jumps of 1 (for the units)	+10 +10 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1
Help children to use fewer jumps by:	
Adding 3 units in one jump (using the known fact 4 + 3 = 7)	+10 $+10$ $+334 44 54 57$
Adding the 2 tens in one jump	+20 +3 34 54 57 34 54 57

NOTE:

11

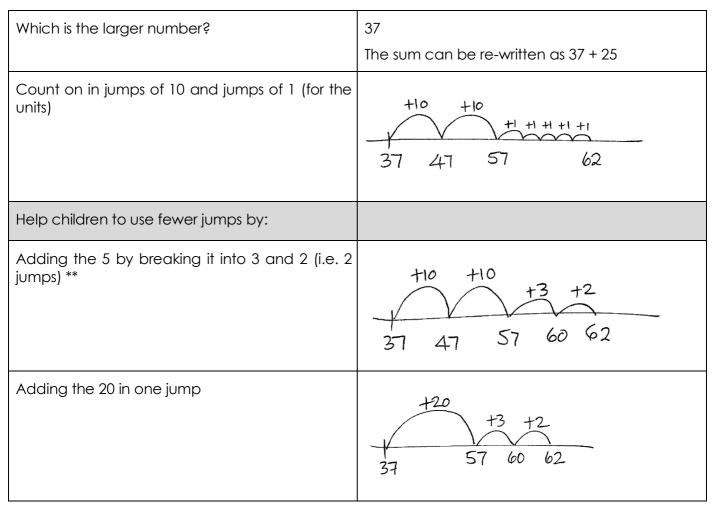
** This does not work for subtraction

2b. JUMPS OF TEN – ADDITION (crossing a ten)

With this strategy, one number is kept whole and jumps of 10 and then units are added to it. Again, it makes sense for the whole number to be the larger number.

Example:

25 + 37



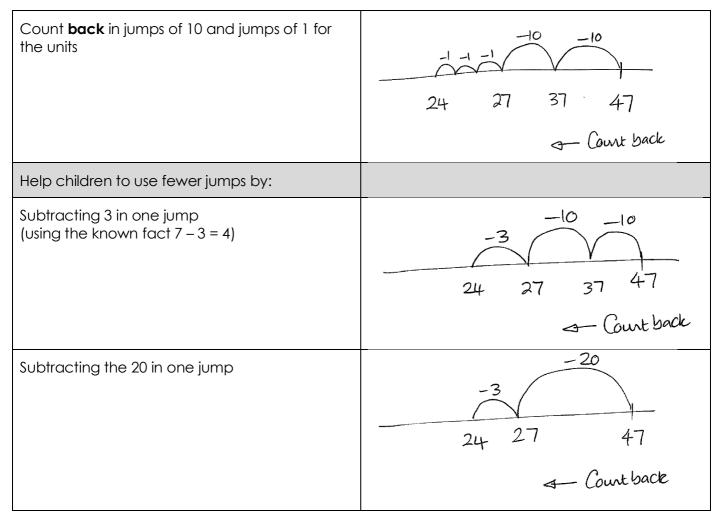
** This also uses another strategy called **Next Friendly Number**, which is discussed below.

2c. JUMPS OF TEN – SUBTRACTION (without crossing a ten)

With this strategy, one number is kept whole and jumps of 10 and then units are added to it.

Example:

47 - 23

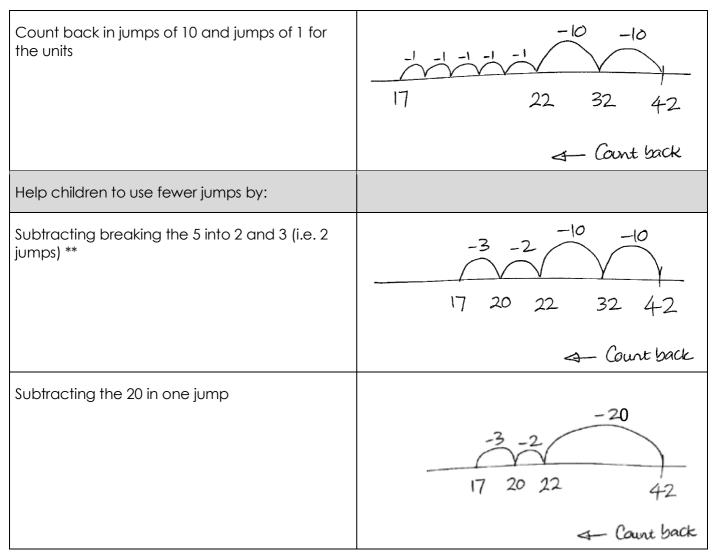


2d. JUMPS OF TEN – SUBTRACTION (crossing a ten)

With this strategy, one number is kept whole and jumps of 10 and then units are added to it.

Example:

42 - 25



NOTES:

- ** This also uses another strategy called **Next Friendly Number**, which is discussed below.
- # Researchers recommend jumping as it works equally well for both addition and subtraction. Splitting (decomposing) can create problems for learners when it comes to subtraction as we saw above.

3a. NEXT FRIENDLY NUMBER (BRIDGING THROUGH 10) – ADDITION

This strategy is the opposite of Jumps of 10 in that the units are dealt with first and then the 10s. The idea of using the units first is to jump forwards to the next friendly number.

Example:

38 + 26

Count forward to next friendly number which is 40 with a jump of 2.	38 + 26
The 6 is broken into 2 and 4. A jump of 4 is next to 44 Then 2 jumps of 10 to 64	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
Help children to use fewer jumps by:	
Adding the 20 in one jump	38 + 26 /\ 2 4
	+20 +20

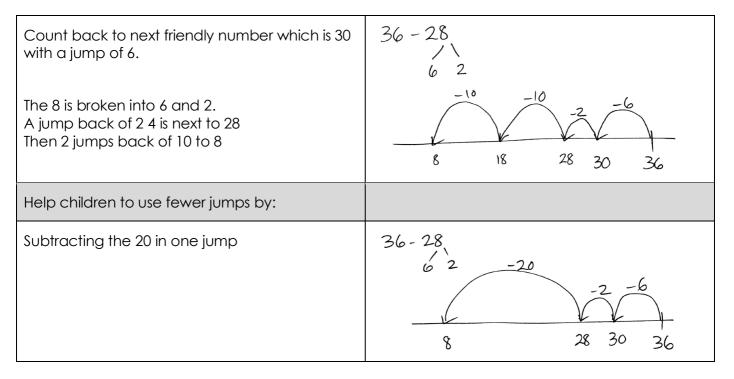
3b. NEXT FRIENDLY NUMBER (BRIDGING THROUGH 10) - SUBTRACTION

This strategy is the opposite of Jumps of 10 in that the units are dealt with first and then the 10s. The idea of using the units first is to jump backwards to the next friendly number.

Example:

15

36 - 28



Introducing the empty number line

Step One: Practice moving using the fewest number of jumps

When first using the empty number line, explore and practice moving to given numbers in the fewest number of jumps.

A possible question might be: How can we go from 0 to 39 in the fewest number of jumps of tens and ones?

Learners should be encouraged to share different strategies and discuss which strategy uses the fewest number of jumps.

For example, when jumping from 0 to 39 one learner could make 3 jumps of ten and nine jumps of one, while another may make 4 jumps of ten to 60 and then jump back one to 39.

Step Two: Use the empty number line to calculate (solve addition and subtraction problems)

The next step is to use the number line to model how learners solve addition and subtraction problems. Learners use the number line to explain their thinking process. Again, it is important for learners to share different strategies to encourage them to see how to use the empty number line more efficiently.

Possible questions include:

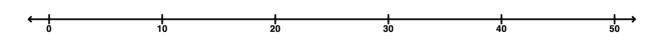
- How can we go from 27 to 53 in a small number of jumps? Who has another way?
- How can we go from 62 to 45 in a small number of jumps? Who has another way?

Teaching notes

One of the interesting things about using the empty number line is that it shows that we don't all think the same way. The empty number line allows learners to see that there are many ways to solve the same problem.

You model learners' thinking using the number line

You can start doing these activities using a semi-structured number line if you wish, as we have shown in the examples below. The number line provided in session 8 is useful for doing this with the whole class.



Then move to **hand-drawn lines** (without any numbers or partitions) when you think your learners are ready. You can use these to model learner's methods for the rest of the class. The idea is that the number line helps to translate the learners' words into something that other learners can understand and engage with. It is not intended that the number line is an accurate representation of what is in the learner's head.

This method is more **flexible**, as you can use it to solve any problem. You use an empty number line to capture learners' thinking

Sample teaching sequences are shown on pages 27 to 24.

Learners start to use empty number lines themselves to solve problems.

You need to give learners plenty of opportunities to explain their ideas while you show their thinking on the empty number line. Over time, learners will start to use these themselves to solve problems.

Lines do not need to be drawn with a ruler – they are meant to help learners in their thinking. Encourage learners to show their working on a number line. Emphasise that numbers do not have to be exactly positioned.

Note: Try to avoid writing the problem vertically as learners will tend to use the vertical procedure to solve the problem.

Reflection Activity



Get into groups of 3-5 or work with the teachers at your table. Reflect on your use of the following activities from the last session.

Thinking about Number Talks and Semi-Structured Number Lines

- Did you do any Number Talks with your class this month? If so, what did you do? How did you find the experience? Did you learn anything about how your learners think?
- 2. Reflect on your experiences of using semi-structured number lines.
- 3. Did you make any adaptations to the activities? If so, show / explain to the members of your group.
- 4. What were the learner experiences of the activities?
- 5. Where in your school did you draw your number line?
- 6. Did you find anyone to sponsor a permanent number line in your school?

NOTES:

This section provides details of the activities that are presented in this workshop. Every workshop will have a similar section so you know where to look in the handbook.

Number talk prompts

Page: 20

Empty Number Line Activities

Page: 27

Resources

Number Talks: Which one doesn't belong?



Why use "which one doesn't belong?"

They are useful for encouraging creative and critical thinking in learners. The prompts that we have provided also work with other representations (e.g. dot patterns, 10 frames, dice, cards etc) that we have used in this eNICLE programme.

The activities do not have a single correct answer. They are designed to be interpreted in different ways in order to encourage deep thinking and classroom discussion (in small groups, whole class, or both). Encourage learners to think in different ways and to participate in discussion.

GUIDELINES	 Do not suggest answers All learners should participate Allow many solution strategies 	
OBJECT OF LEARNING	Learners explain their thinking: HOW they SEE it and WHY it makes SENSE	Learners begin to: • Reason abstractly • Speak mathematically

Working with the Number Talk Prompts

The prompts below are for Grade 1 and 2 learners.

PREPARATION

Select a number talk prompt from the options on pages 22 to 24 that has a familiar representation for your learners and is relevant to where your learners are in their school year.

- Work through the prompt yourself (or with a group of colleagues/family members) so that you know how you can solve it. The prompts have been set up so that there are always at least two different ways to look at it. Think about colour, shape, size, patterns and so on.
- Decide how to present the prompt to the learners in your classroom. Will you:
 - Draw it on the board?
 - Project it?
 - Photocopy pages for pairs of learners to work with?

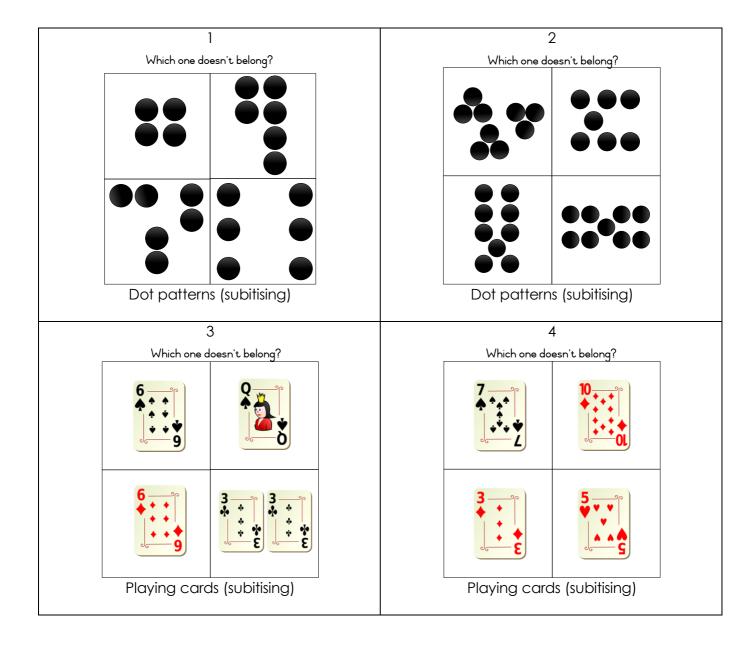


DOING THE NUMBER TALK WITH LEARNERS

- 10 minutes should be enough time for the learners to work with each prompt.
- Show learners four images.
- You ask: "Which one doesn't belong? Think about why you would say that."
- Learners use their mathematical and reasoning skills to decide which of the four items does not belong and to justify their choice.
- Use the Think-Pair-Share technique:
 - Learners have a few minutes to think on their own
 - Then they work with a partner to share their ideas for 1 minute
 - Then gather contributions from the class as a whole and discuss the different ways learners have suggested.
- Remember to encourage the learners to give reasons using mathematical language.
- Here are some example follow-up questions:
 - Explain why you think that x doesn't belong?
 - Are there other ways Learner A could have used to decide that x does not belong?
 - How can Learners A, B, and C all valid answers even though they each chose different options?

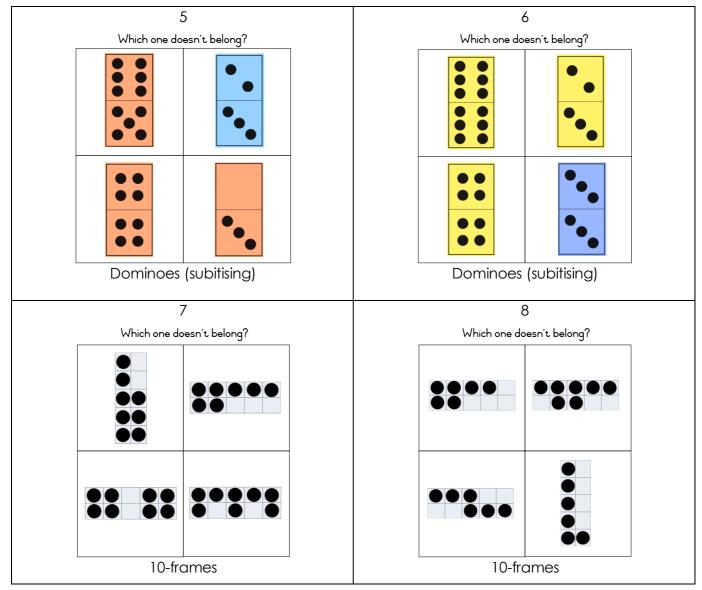


Which one doesn't belong? Prompts



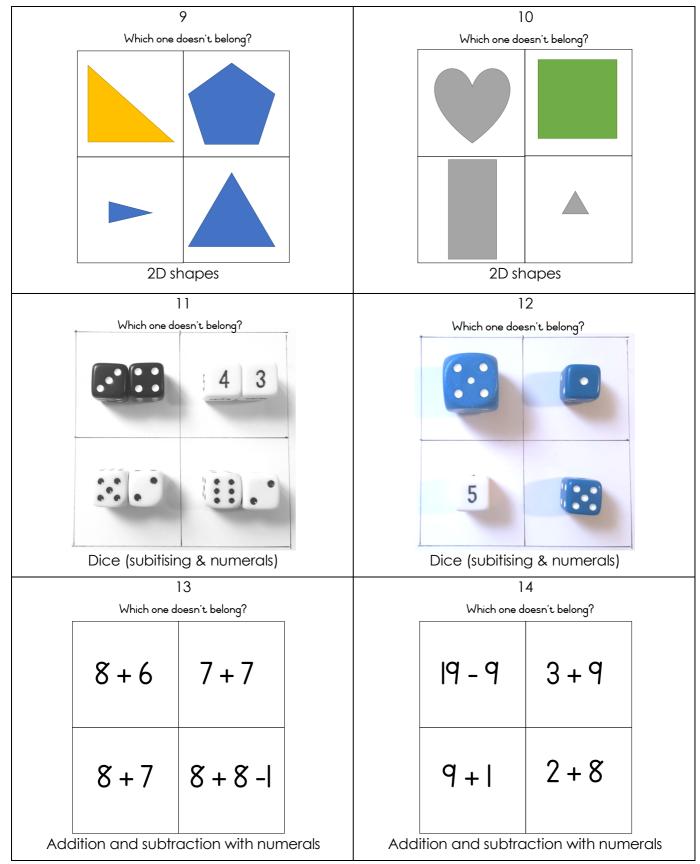


Prompts continued





Prompts continued





Solutions

Think about the ways each is different and list the differences. Think about colour, shape, size, patterns and so on. Try to find at least two for each prompt.

The first two have been done for you.

 Top left doesn't belong because all others are 6-dot patterns Bottom right doesn't belong because they are far apart. All other dots are arranged in pairs of dots, close together. 	 2 Top left doesn't belong because all others have the standard five-dot shape in the pattern Top right doesn't belong because all the other dot patterns have 9 dots
3	4
5	6
7 7	8



Solutions continued



Empty Number Lines: A sample teaching sequence for addition

Next friendly number / bridging through 10

This teaching sequence works with the next friendly number (bridging through 10) strategy.

Mathematical object of learning:	You need:	Work with:
 Using empty number lines to solve addition problems using the next friendly number (bridging through ten) strategy 	 Chalk Classroom blackboard/whiteboard OR Flip chart paper 	The whole class if you have space, or smaller groups

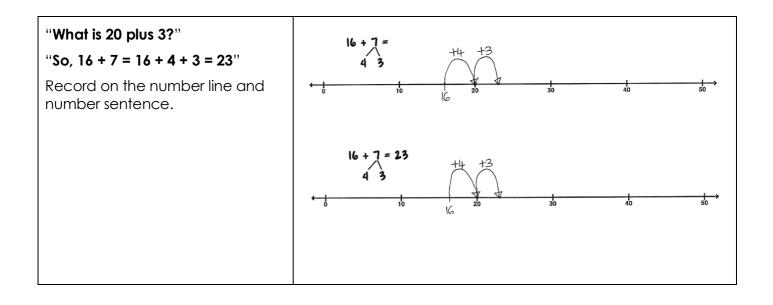
Teaching sequence

This example uses a **semi-structured number line** to solve this addition problem:

16 + 7 =

Record 16 + 7 = on the board	16 + 7 =				
"Where is 16 on this number line?"	← <u> </u> 0 10			40	→ 50
Ask a learner to come and mark the line.					
"We have to jump 7 forwards.	16 + 7 =	+4			
Let's make one jump to the next ten rather than jumping in 1s.	↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓	16 20	4 30	40	 50
What is the next ten after 16?"					
Learners to answer.					
"16 plus what gives 20?"					
Learners to answer.					
Record on the number line.					
"We have added 4. We need to add 7.	16 + 7 =	#			
7 splits into 4 and what?"	← ¦	16 20		4 0	
Learners to answer.		Ŷ			
Split the 7 in 16 + 7 into 4 and 3					
"How many more do we need to add?"					
Learners to answer.					
Record on the number line.					

Empty Number Lines: A sample teaching sequence for addition continued



Empty Number Lines: A sample teaching sequence for addition continued

Next friendly number / bridging through 10

This teaching sequence works with the next friendly number (bridging through 10) strategy.

1	Mathematical object of learning:	You need:	Work with:
•	Using empty number lines to solve addition problems using the next friendly number (bridging through ten) strategy	Learners need: • Scrap paper or till roll paper • Pencils	The whole class if you have space, or smaller groups

Follow on

Remind learners how they solved 16

16 + 7?

Let learners say the method and how it works or show on the board.

Use the number line to model at the board using bridging through 10 to solve:

17 + 8 =

INDIVIDUAL TASKS

Ask learners to solve this sum by bridging through ten.

27 + 8 =

- Encourage learners to show their working on a number line emphasise that numbers do not have to be exactly positioned.
- Encourage mental working method of jumping NOT counting in 1s.
- Encourage learners to explain their work/thinking to a partner.

More follow on

Use the number line to model at the board using bridging through 10 to solve:

15 + 8 =

INDIVIDUAL TASKS

Ask learners to solve these sums by bridging through ten.

25 + 8 =

16 + 9 =

If any learners finish these two sums quickly, give them more to practice.

Empty Number Lines: A sample teaching sequence for addition continued

More follow on

INDIVIDUAL TASKS

Ask learners to solve these sums by bridging through ten.

16 + 8 =

25 + 8 =

17 + 6 =

If any learners finish these three sums quickly, give them more to practice.

Empty Number Lines: A sample teaching sequence for subtraction

Next friendly number / bridging through 10

This teaching sequence works with the next friendly number (bridging through 10) strategy.

Mathematical object of learning:	You need:	Work with:
 Using empty number lines to solve subtraction problems using the next friendly number (bridging through ten) strategy 	 Chalk Classroom blackboard/whiteboard OR Flip chart paper 	The whole class if you have space, or smaller groups

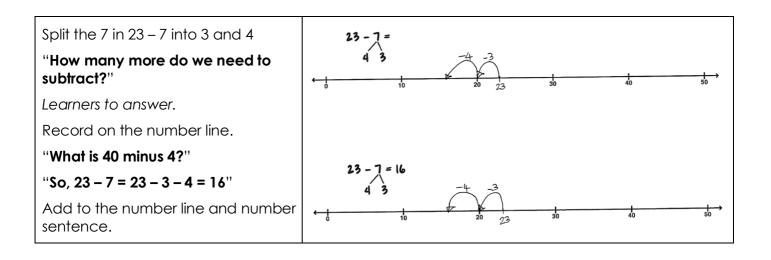
Teaching sequence

This example uses a **semi-structured number line** to solve this subtraction problem:

23 – 7 =

Record 23 – 7 = on the board " Where is 23 on this number line? " A learner to come and mark the line.	23 - 7 =		30 40	
"We have to jump 7 backwards. Let's make one jump to the ten before 23 rather than jumping in 1s. What is the ten before 23?" Learners to answer "23 minus what gives 20?" Learners to answer Record on the number line.	23 - 7 = ← 1 0 10	20 23	-	
 "We have subtracted 3. We need to subtract 7. 7 splits into 3 and what?" Learners to answer 	23 - 7 = 43	20 23	-	50 →

Empty Number Lines: A sample teaching sequence for subtraction continued



Empty Number Lines: A sample teaching sequence for subtraction continued

Next friendly number / bridging through 10

This teaching sequence works with the next friendly number (bridging through 10) strategy.

	Mathematical object of learning:	You need:	Work with:
•	 Using empty number lines to solve subtraction problems using the next friendly number (bridging through ten) strategy 	Learners need: • Scrap paper or till roll paper • Pencils	The whole class if you have space, or smaller groups

Follow on

Remind learners how they solved 23 – 7?

Let learners say the method and how it works or show on the board.

Use the number line to model at the board using bridging through 10 to solve:

17 – 8 =

INDIVIDUAL TASKS

Ask learners to solve this sum by bridging through ten.

27 – 8 =

- Encourage learners to show their working on a number line emphasise that numbers do not have to be exactly positioned.
- Encourage mental working method of jumping NOT counting in 1s.
- Encourage learners to explain their work/thinking to a partner.

More follow on

Use the number line to model at the board using bridging through 10 to solve:

25 – 8 =

INDIVIDUAL TASKS

Ask learners to solve these sums by bridging through ten.

35 – 8 =

26 – 9 =

If any learners finish these two sums quickly, give them more to practice.

More follow on

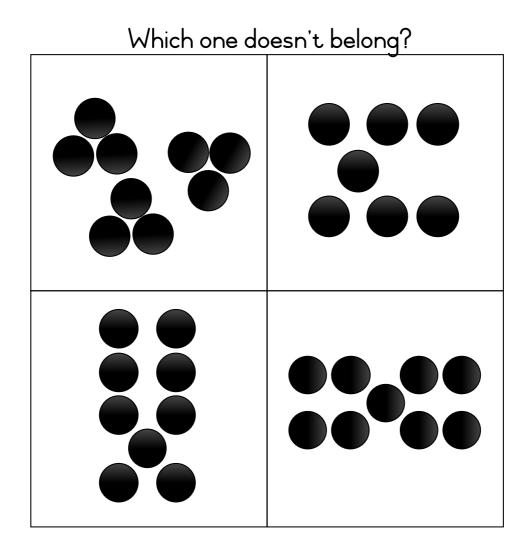
INDIVIDUAL TASKS

Ask learners to solve these sums by bridging through ten.

26 – 9 =

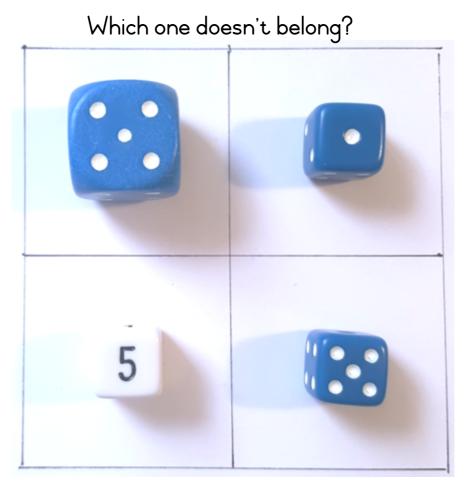
35 – 8 =

22 – 6 =



Reflection questions:

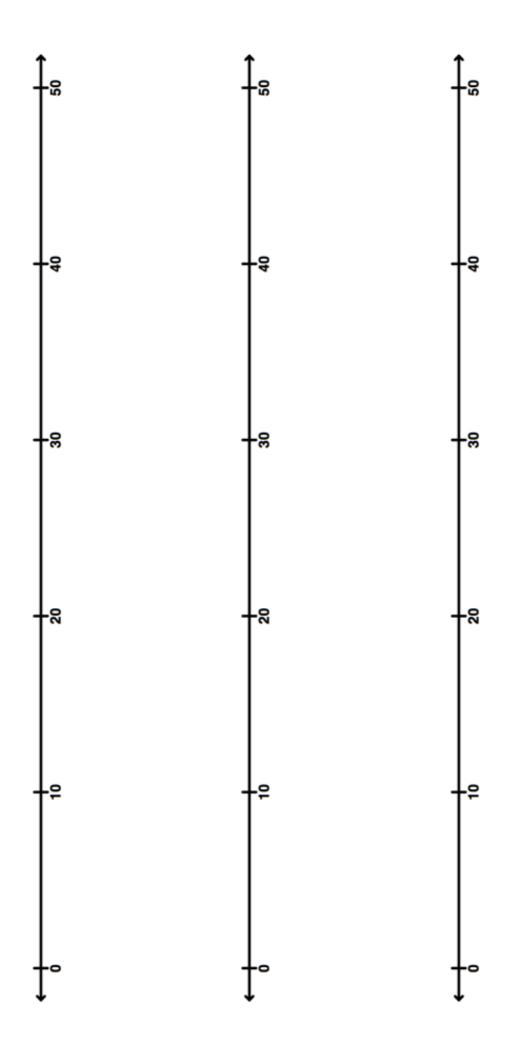
• How many ways did your table come up with to find the one that doesn't belong?



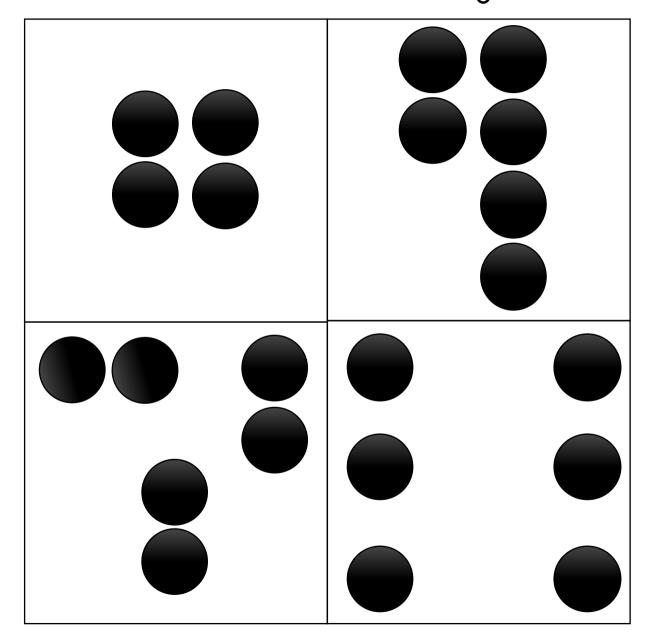
Reflection questions:

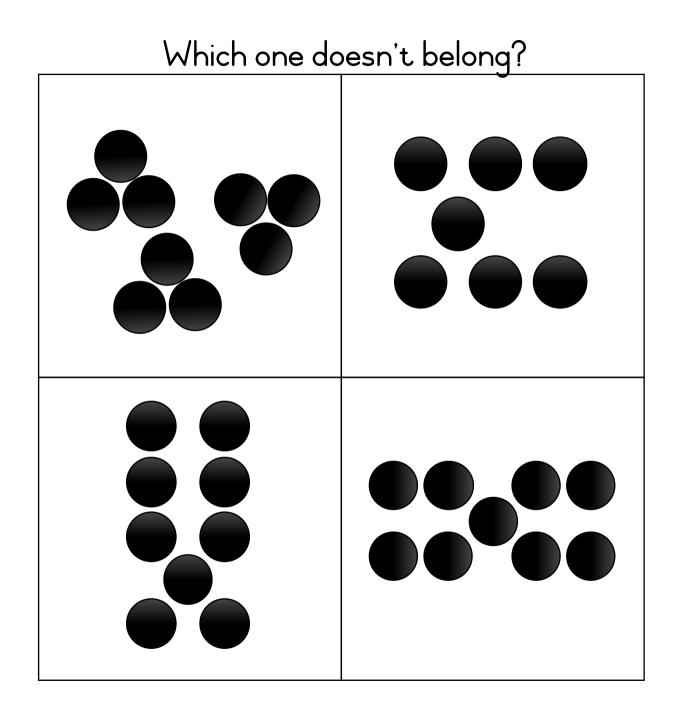
• How many ways did your table come up with to find the one that doesn't belong?

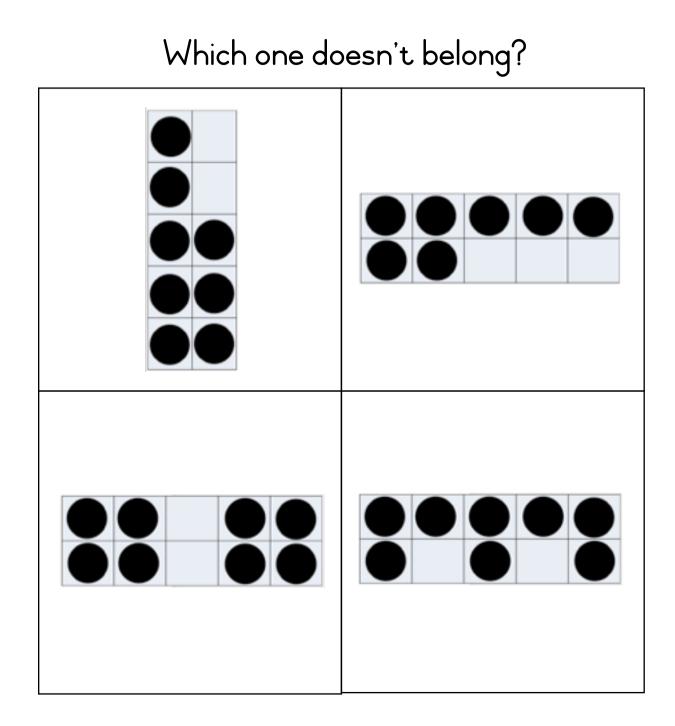
- Semi-structured number lines 1 to 50
- "Which one doesn't belong?" number talk prompts



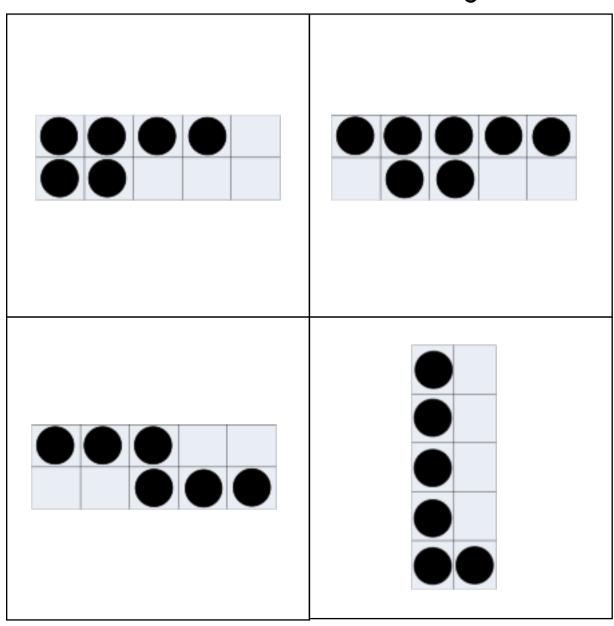
Which one doesn't belong?

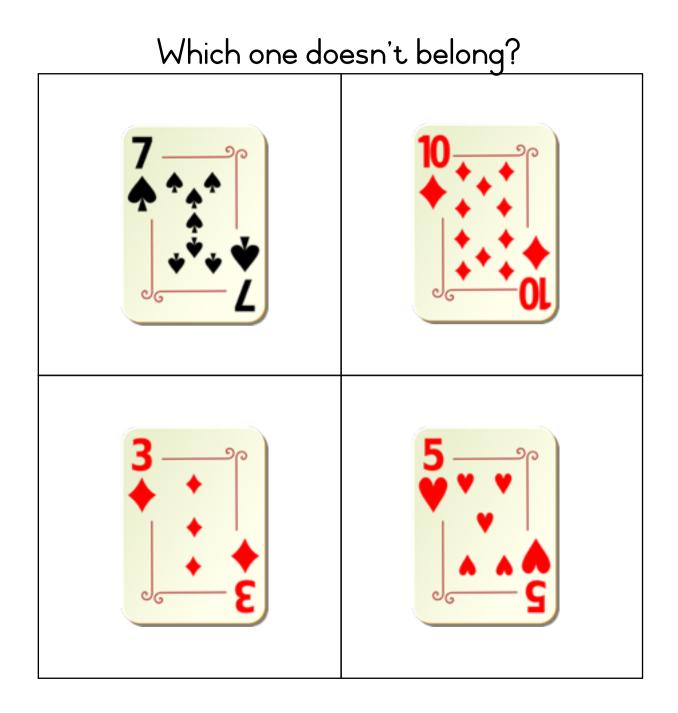


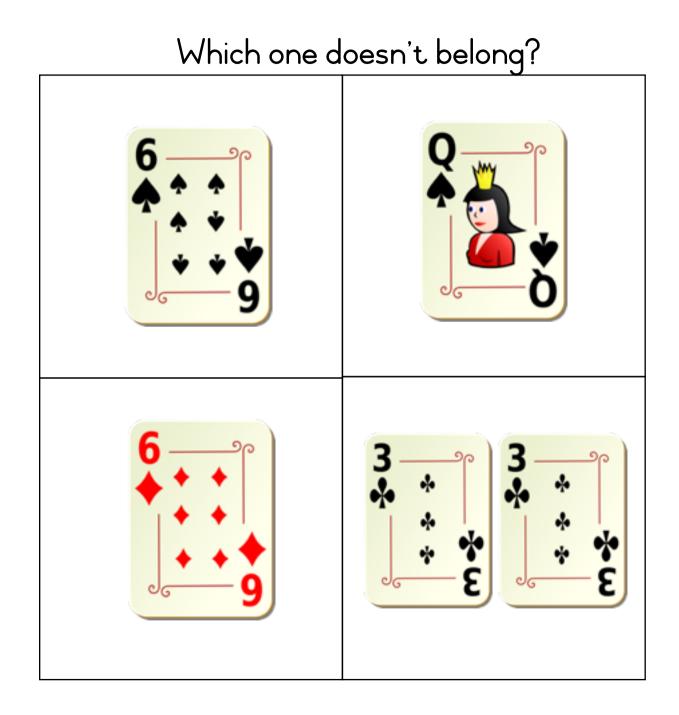


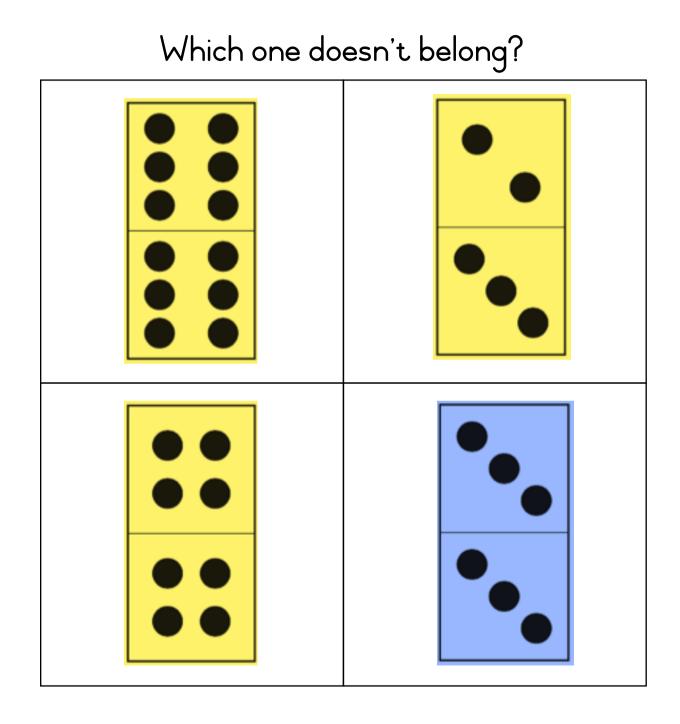


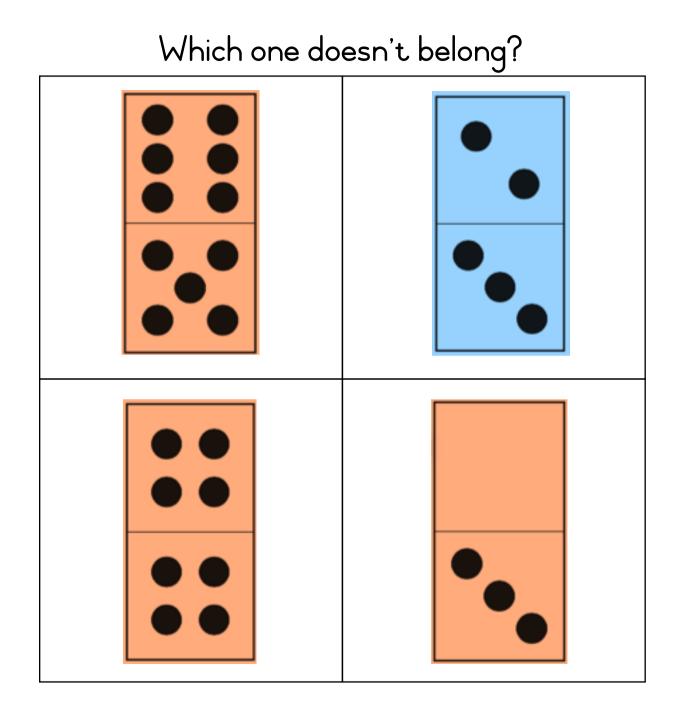


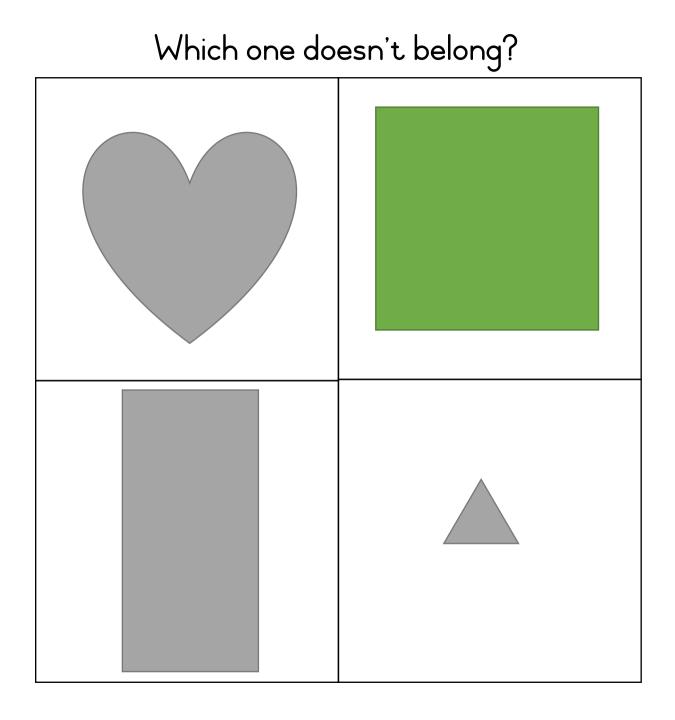


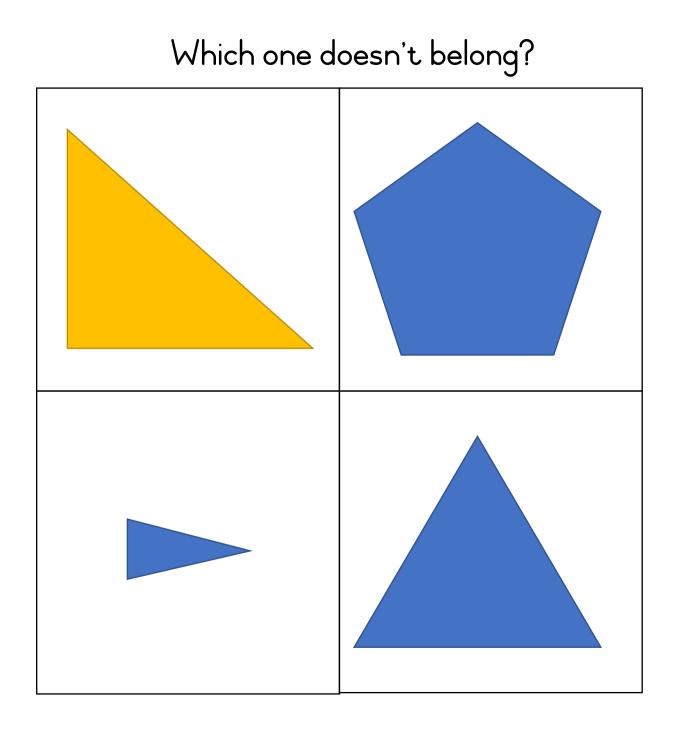


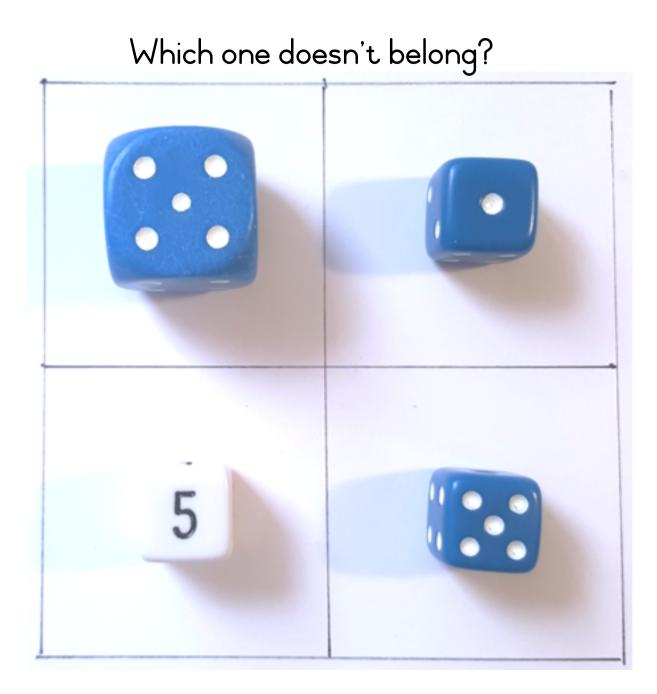














Which one doesn't belong?

